Problem set 1: Risky investments and savings behavior

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Exercise 1: Risk aversion

(a) Consider the function $U(C) = ae^{bC} + d$, where a, b, and d are constants, and $e \approx 2.718$ is the well-known constant. C is consumption. What condition(s) must be satisfied by a, b, and d in order for $E[U(C)]$ to properly represent the preferences of a risk averse person who maximizes von Neumann-Morgenstern expected utility?

What are the coefficients of absolute and relative risk aversion, $R_A(C)$ and $R_R(C)$, for this utility function?

Hint: There are conditions on the signs of U' and U''.

(b) Find the coefficients of absolute and relative risk aversion for the shifted power utility function $U(C) = \frac{\rho}{1-\rho}\left(\frac{C-\xi}{\rho}\right)^{1-\rho}$

Exercise 2: Savings behavior under risk

Consider an individual who has some wealth today that she plans to invest for consumption tomorrow. The individual has wealth Y_0 to be divided between two financial investments, Y_f (risk free) and Y_r (risky). The future consumption is equal to the sum of the future values of these investments. Y_f will increase by the factor $R_f \equiv 1 + r_f$, where r_f is the risk free interest rate. Y_r will increase similarly by the factor $\tilde{R} \equiv 1+\tilde{r}$, where $\tilde{r}$ is a risky rate of return (— this may in fact be a decrease for low outcomes of $\tilde{r}$). The individual regards Y_0, R_f, and the probability distribution of $\tilde{R}$ as exogenously given.

In what follows, you should discuss both the case in which short selling is allowed and the case in which it is not.

(a) Write down the agent's budget constraint in period 1. Her income will be the return on the investments made in period 0. Then write down the expected utility maximization problem as a function of Y_r instead of $\tilde{C}$, and state the first order condition.

(b) Draw four graphs in the $E(U(\tilde{C})), Y_r)$ diagram that describes possible solutions of the optimization problem above. You want to distinguish between the cases where there is a global maximum and where there is not.

Hint: This is essentially a graph of the utility function as a function of Y_r. What is the sign of U' when $Y_0 = 0$?

Let the agent have a negative exponential utility function with $\alpha, \beta > 0$

$$U(C) = -\alpha e^{-\beta C} + d$$

Also, assume in the following that $\tilde{r}$ has only two possible outcomes, r_1 and r_2, and that $r_1 > r_2$. The probability of r_1 is p and the probability of r_2 is $1 - p$. You can now get rid of the expectations operator in the first order condition.

(c) Show that the optimal Y_r now can be written as

$$Y_r^* = \frac{\ln \frac{p(r_1 - r_f)}{(1-p)(r_f - r_2)}}{\beta(r_1 - r_2)}$$

What assumptions do we have to make on r_1, r_2 and r_f for this to be possible?

Hint: It's in the math

(d) Does the optimal Y_r depend on Y_0? Which of the four cases laid out in part (b) are we in now?

(e) Is the optimal Y_r increasing or decreasing in p, r_f and β? Assume for this question only that the agent invests a positive amount in the risky asset. (Why assume this?) Give an intuitive explanation of the effects.

(f) Could Y_r^* from the formula exceed Y_0? What would the individual do then if borrowing is not allowed?

Hint: Corner solution

(g) Could Y_r^* from the formula be negative? What would the individual do then if short selling is not allowed?

Hint: Corner solution

(g) Consider the cases where $r_2 < r_1 < r_f$ and where $r_f < r_2 < r_1$. Can these situations occur?

Hint: Think about whether these situations could be financial market equilibria.