Problem set 2:

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Exercise 1: Safe real investment opportunity

Consider an individual who maximizes expected utility and is risk averse. The individual has a given wealth W to invest in order to provide for consumption next period. The wealth is divided between two investment opportunities. One is a financial asset with uncertain rate of return, $\tilde{r}$. Define $\tilde{R} = 1 + \tilde{r}$. The other opportunity is a real investment project. If $I > 0$ is invested now, the project will produce $f(I) > 0$ next period (with an output price of unity), with no uncertainty. Assume that f is increasing and concave for all $I \geq 0$.

(a) Write down the agent's budget constraint in period 1.

(b) Write down the agent's optimization problem and derive the first order condition. Show that it can be written on the form

$$f'(I)E[U'(\tilde{C})] = E[U'(\tilde{C}) \cdot \tilde{R}]$$

and equivalently

$$f'(I) = E(\tilde{R}) + \frac{\text{cov}[U'(\tilde{C}), \tilde{R}]}{E[U'(\tilde{C})]} \tag{1}$$

(c) Interpret equation (1) above. When is the covariance negative?

Assume now that $\tilde{R}$ can take only two values, R_1 and R_2, with $R_1 > R_2 > 0$, and $\Pr(\tilde{R} = R_1) = p \in (0, 1)$.

(d) Show what the simplified first-order condition looks like. Show that it can only be satisfied if $R_2 < f'(I) < R_1$.

(d) Can there be cases in which it is impossible to choose I to satisfy the first order condition?

Hint: what happens if short-selling is prohibited? Draw marginal return of investment I in a $(f'(I), I)$ diagram.

Exercise 2

(a) Consider the function $U(C) = aC^b$, with a and b being constant, real numbers. The function is only defined for $C \geq 0$. What conditions must be satisfied for $E[U(C)]$ to represent the utility of a von Neumann-Morgenstern type person who is risk averse? What are the measures of absolute and relative risk aversion for this utility function?

Assume that the agent with the utility function mentioned here has some wealth W. This wealth needs to be invested to provide for consumption in the future (in $t = 1$). There is no additional income. A bank offers risk free borrowing and saving at a given interest rate r_f. There is also a risky asset with a rate of return $\tilde{r}$

(b) Write down the agent's budget constraint for future consumption when an amount v is invested in the risky asset and $W - v$ is invested in the risk-free asset. Formulate the agent's optimization problem and derive the first order condition.

(c) Assume that $\tilde{r}$ has only two possible outcomes, r_1 and r_2, with numbers chosen such that $r_1 > r_2$, and with $\Pr(\tilde{r} = r_1) = p$. Show that under some assumptions, the optimal amount to invest in the risky asset is

$$v^* = \frac{W(1 + r_f)(X - 1)}{r_1 - r_f + X(r_f - r_2)}$$

where X is given by

$$X = \left[\frac{p(r_1 - r_f)}{(1-p)(r_f - r_2)} \right]^{\frac{1}{1-b}}$$

Why do you need the assumption $r_2 < r_f < r_1$?

(d) Based on the assumptions in (c), show that if $E(\tilde{r}) = r_f$, then $v^* = 0$. Assuming that this is the case, what would be the effect on v^* of an increase in p? Give an economic interpretation.

Hint: there is only one way that the left hand side of the expression for v^* can be equal to zero.

(e) Now assume that $v^* > 0$. What is the effect on v^* of an increase in the individual's risk aversion? Give an economic interpretation.