

Problem set 5:

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Exercise 1: Project valuation using the capital asset pricing model

(a) Use the CAPM equation

$$\mathbb{E}(\tilde{r}_j) = r_f + \frac{\sigma_{Mj}}{\sigma_M^2} (\mathbb{E}(\tilde{r}_M) - r_f)$$

together with the definition of stochastic return

$$1 + \tilde{r}_j = \frac{\tilde{p}_{j1}}{p_{j0}}$$

to derive the valuation equation

$$p_{j0} = \frac{1}{1 + r_f} (\mathbb{E}(\tilde{p}_{j1}) - \lambda \text{cov}(\tilde{p}_{j1}, \tilde{r}_m)) \equiv V(\tilde{p}_{j1})$$

where $\lambda \equiv \frac{E(\tilde{r}_M) - r_f}{\sigma_M^2}$, \tilde{p}_{j1} is the price (*the value*) of the asset tomorrow and p_{j0} is the price today. Remember to rewrite σ_{Mj} in the CAPM equation in terms of the stochastic price tomorrow.

Interpret the equation.

There is a limited-liability firm with shares traded in the stock market. There are no taxes and the company does not need to borrow. The company now considers to undertake its first real investment project.

Today, the company needs to decide the output quantity, $Q \geq 0$, which will require a quantity $C(Q)$ of an input factor. $C(Q)$ is increasing and convex, and you can assume that $C(0) = 0$. There are no other costs. Both quantities Q and $C(Q)$ will be known with full certainty as soon as Q is chosen, and there is no way to change

these later.

The output will be ready for sale one period later, at a price \tilde{P} that is uncertain as seen from period zero.

- (b) Write down the total cash flow of the firm *in terms of period 1-value* when the firm pays a given price W_0 for each unit of the input factor in period 0. The cash flow should look like a regular profit function with uncertainty, although the interpretation is somewhat different.
- (c) Write down the market value of the cash flow using the capital asset pricing model (remember, an asset is a claim to some cash flow). Show that the market value is additively separable and that the market value of a non-stochastic variable is equal to the present value of that quantity.
- (d) The firm maximizes the market value of the real investment project. Write down the maximization problem and derive a condition that pins down the optimal choice of Q . Assume there exists an inner solution.
- (e) How does the optimal choice of Q depend on
 - (i) the risk aversion of the owners of the firm
 - (ii) the riskiness of the output price
- (f) Now assume that the firm has to pay for the input factor in period 1 and that the price per unit \tilde{W} is uncertain at period zero. Redo the firm's optimization problem in this case and find the new optimality condition (assuming there is an interior solution).
- (g) How does the choice of Q now depend on the riskiness of the output price?

Exercise 2: Firm valuation and capital structure in the

CAPM

A limited-liability company produces a good that will be sold one period of time from now (in period 1). The firm has chosen its optimal amount of investment, I^* , and will with certainty produce an amount $Q = F(I)$ to be sold at a price \tilde{P} . The price is uncertain at the time of the investment.

The firm decides to sell shares after the investment is undertaken. That is, a share is a claim to the future cash flow $\tilde{P}Q$.

- (a) If priced according to the CAPM, the return on a share in this company (k) is given by

$$1 + \tilde{r}_k = \frac{\tilde{P}}{V(\tilde{P})}$$

Explain why.

- (b) Show that the CAPM- β of this firm can be written as

$$\beta_k = \frac{(1 + r_f)}{\sigma_M^2 \frac{E(\tilde{P})}{\text{cov}(\tilde{P}, \tilde{r}_m)} - E(\tilde{r}_m) + r_f}$$

Give an interpretation of this expression. Why doesn't the β depend on the initial investment?

Assume that a fraction $1 - a$ of the investment was financed by debt that with full certainty will be paid back with interest rate in period 1.

- (c) What is the Modigliani-Miller irrelevance result?
- (d) The investment decision is made in period 0. How does the decision to partly loan-finance the project affect the investment decision? (Remember that the firm knows its optimal I^*).
- (e) How does the loan-financing affect the CAPM- β of the project?

Go back to the case with full equity-financing, and assume that the company considers to merge with another company (j). This second company will produce a product in quantity Y that will be sold next period at an uncertain price P' . The necessary investments are already made.

- (f) What is the CAPM- β of the merged company? Give an interpretation of the relationship between the β of the merged company and the β of the two companies by themselves.