## Problem set 5:

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## Exercise 1: Firm valuation and capital structure in the CAPM

A limited-liability company produces a good that will be sold one period of time from now (in period 1). The firm has chosen its optimal amount of investment, $I^{*}$, and will with certainty produce an amount $Q=F(I)$ to be sold at a price $\tilde{P}$. The price is uncertain at the time of the investment.

The firm decides to sell shares after the investment is undertaken. That is, a share is a claim to the future cash flow $\tilde{P} Q$.
(a) If priced according to the CAPM, the return on a share in this company $(k)$ is given by

$$
1+\tilde{r}_{k}=\frac{\tilde{P}}{V(\tilde{P})}
$$

Explain why.
(b) Show that the CAPM- $\beta$ of this firm can be written as

$$
\beta_{k}=\frac{\left(1+r_{f}\right)}{\sigma_{M}^{2} \frac{E(\tilde{P})}{\operatorname{cov}\left(\tilde{P}, \tilde{r}_{m}\right)}-E\left(\tilde{r}_{m}\right)+r_{f}}
$$

Give an interpretation of this expression. Why doesn't the $\beta$ depend on the initial investment?

Assume that a fraction $1-a$ of the investment was financed by debt that with full certainty will be paid back with interest rate in period 1.
(c) What is the Modigliani-Miller irrelevance result?
(d) The investment decision is made in period 0 . How does the
decision to partly loan-finance the project affect the investment decision? (Remember that the firm knows its optimal $I^{*}$ ).
(e) How does the loan-financing affect the CAPM- $\beta$ of the project?

Go back to the case with full equity-financing, and assume that the company considers to merge with another company $(j)$. This second company will produce a product in quantity $Y$ that will be sold next period at an uncertain price $P^{\prime}$. The necessary investments are already made.
(f) What is the CAPM- $\beta$ of the merged company? Give an interpretation of the relationship between the $\beta$ of the merged company and the $\beta$ of the two companies by themselves.

## Exercise 2: Using the CAPM

(a) You observe the following situation:

| Security | Beta | Expected return |
| :--- | :---: | :---: |
| Renewable Energy Corp | 1.3 | 0.23 |
| Statoil | 0.6 | 0.13 |

Assume these securities are correctly priced. Based on the CAPM, what is the expected return on the market? What is the risk-free rate?
(b) Assume the annual risk-free rate is $r_{f}=1 \%$ and that the annual mean and standard deviation of the return on the Oslo Børs OBX index are $6 \%$ and $10 \%$, respectively. Using stock prices over the last three years you find that the return on Telenor stocks have had an annualized covariance with the OBXindex of $\operatorname{cov}\left(\tilde{R}_{1}, \tilde{R}_{M}\right)=0.006$. A share in Entra is currently priced at 100 NOK per share.

Calculate an estimate of the market $\beta$ of Entra.
(c) Use the information from (b) for this exercise.

Suppose the CAPM is true. What is the expected price of an Entra stock one year from now?

## Exercise 3: Arbitrage pricing theory

(a) You observe the following three portfolios and their exposure to the factors $f_{1}$ and $f_{2}$.

| Portfolio | Expected return | $f_{i 1}$ | $f_{i 2}$ |
| :--- | :---: | :---: | :---: |
| 1 | 7.3 | 0.2 | 2.5 |
| 2 | 6.2 | 0.8 | 0 |
| 3 | 7.3 | 1.0 | 1.0 |

If the APT holds and the factors above are sufficient to describe the pricing of assets in the economy, the following equation can be used to price any security,

$$
E\left(R_{i}\right)=c_{0}+c_{1} f_{i 1}+c_{2} f_{i 2}
$$

Find the parameters of this equation.
(b) Assume there is a security S with expected return $9.2 \%, f_{S 1}=$ 0.9 and $f_{S 2}=0.5$. Is there an arbitrage opportunity?

