Problem Set 7

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Exercise 1: The binomial share and option pricing model

Assume that the binomial share and option pricing model is valid: Consider a share which for sure does not pay any dividend in the period we focus on. All agents know that if the share price at time t is S, then the share price at $t+1$ will be uS or dS, where $u > d$.

There are no restriction on short-sales and no taxes or transaction costs. All investors believe in the same values of u and d. The realization of next-period share price is history-independent.

(a) Draw the tree that shows the possible values of the share in time t, $t+1$ and $t+2$. Calculate the values at each node if $S = 10.00$, $u = 1.2$ and $d = 1.0$.

Assume there is also a European call option with two periods left to maturity. The exercise price of the option is $K = 13.19$. i.e, the owner of the option can choose to buy the share for 13.19 in $t+2$. The values of u and d are the same as for the share. The one period interest rate factor is $e^r = 1.1$.

(b) Draw the corresponding tree for the option. Calculate the value of the option at each node in $t+2$.

(c) To price the option in time t and $t+1$ you only need an assumption about absence of arbitrage opportunities. For each node: Create a portfolio that holds B units of the risk-free asset and Δ units of the share. Which Δ and B is required for the portfolio to have the same value as the option at both possible future nodes? Now use the absence of arbitrage assumption, the price of this portfolio today has to be the same as the price of the option.

(d) Why is the price of the option increasing with time?

(e) Assume that we instead of the theoretical price calculated above observe a price of 0.50 for the option. Show that this creates a risk-free arbitrage opportunity, and show how to take advantage of this during the time to maturity.

Exercise 2: Arbitrage pricing theory

Note: this is exercise 3 from problem set 6.

(a) You observe the following three portfolios and their exposure to the factors f_1 and f_2.

Portfolio	Expected return	f_{i1}	f_{i2}
1	7.3	0.2	2.5
2	6.2	0.8	0
3	7.3	1.0	1.0

If the APT holds and the factors above are sufficient to describe the pricing of assets in the economy, the following equation can be used to price any security,

$$E(R_i) = c_0 + c_1 f_{i1} + c_2 f_{i2}$$

Find the parameters of this equation.

(b) Assume there is a security S with expected return 9.2%, $f_{S1} = 0.9$ and $f_{S2} = 0.5$. Is there an arbitrage opportunity?