ECON4510 Finance Theory
Lecture 8

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Arbitrage Pricing Theory & Risk/Return Multifactor Models

- Why do we need multi-factor models?
- How are the multi-factor models grounded in the CAPM/APT?
- What is the APT?
- How does the APT differ from the CAPM?
- How can we understand the Fama-French factor models?

Why multi-factor models?

- As a conceptual model, the CAPM is very useful. However, in practice, with the data and methods that are available to us to measure market beta, it is not sufficiently useful to compute required rates of return and expected returns or to discover mispriced securities.
- Multifactor models are useful in this context.
- These models introduce uncertainty stemming from multiple sources, whereas the CAPM, in principle, limits risk to one source – covariance with the market portfolio.

Using the CAPM as a basis for factor models

- In the CAPM, the only relevant factor is the market model. However, the market factor itself could be composed of different macroeconomic factors, e.g. business cycle uncertainty , interest rates, inflation etc.
- Suppose the market is composed of two factors F_1 and F_2, i.e. $R_m = sF_1 + (1-s)F_2$, where even though we can't observe R_m, we *can* observe F_1 and F_2.
- Then we can rewrite the CAPM as:
  $E(R_i) = R_f + \beta[R_m-R_f] = (1-\beta)R_f + \beta E(R_m) = (1-\beta)R_f + \beta[sE(F_1) + (1-s)E(F_2)]$; rewriting, $E(R_i) = R_f + \beta s\,[E(F_1) - R_f\,]+ \beta(1-s)[E(F_2) - R_f]$.
- This suggests a regression of R_i on $(F_1 - R_f)$ and $(F_2 - R_f)$:
  $E(R_i) = R_f + \gamma_{i1}[E(F_1)-R_f] + \gamma_{i2}[E(F_2)-R_f]$.
- The two sources of risk here are exposure to the two factors; comparing the two equations, we also see that $\gamma_{i1}+\gamma_{i2}=\beta$.
- In this formulation, we don't need to observe the market portfolio any more. (Although we would be able, in principle, to estimate s, and hence the market portfolio, as well.)
- However <u>all</u> factors must be observable, and our market portfolio decomposition must be assumed to be correct.

The APT approach: Exposition

- The following description assumes two factors, but there could be many factors in principle.
- Assume that returns can be generated as in a quasi-market model: $R_i = a_i + b_{i1}F_1 + b_{i2}F_2$ + error. Note that we are no longer assuming any connection between these factors and the market portfolio.
- If there are enough assets that are sufficiently different from each other, it should be possible to create three well-diversified portfolios that had no uncertainty other than their exposure to F_1 and F_2 (i.e. the error term would be zero).
- Then, it will be true that the expected return on these three portfolios can be described by a linear combination of their b's:
  $E(R) = c_0 + c_1\, b_{i1} + c_2\, b_{i2}$.
- We can show this by means of an example.

The APT approach: Exposition

- Suppose these three portfolios – A, B and C – were described by the following parameters:

Portfolio	Expected Return	b_{i1}	b_{i2}
A	15	1.0	0.6
B	14	0.5	1.0
C	10	0.3	0.2

- Then, the APT says: $E(R) = c_0 + c_1 b_{i1} + c_2 b_{i2}$.
- That is, $15 = c_0 + 1c_1 + 0.6c_2$
  $14 = c_0 + 0.5c_1 + 1.0c_2$ and
  $10 = c_0 + 0.3c_1 + 0.2c_2$.
- Solving this system of three equations for c_0, c_1 and c_2, we find $c_0 = 7.75$, $c_1 = 5$ and $c_2 = 3.75$.
- The claim is that this pricing equation ($E(R) = 7.75 + 5\, b_{i1} + 3.75\, b_{i2}$) can be used to price any security in the economy.

The APT approach: Proof

- Suppose a security, D, exists with $b_{i1} = 0.6$ and $b_{i2} = 0.6$. Since we have enough securities, we can create a diversified portfolio with almost no idiosyncratic error that would also have the same values for b_{i1} and b_{i2}. So let's assume that our security has no idiosyncratic error.
- Then, according to our equation, its expected return should be 7.75 + 5 (0.6) + 3.75 (0.6) = 13%. Suppose, however, that the expected return for D is 15%.
- Then we can create a combination of securities A, B and C such that this new portfolio, E, has the same b-values as D.
- This can be done by solving the system of equations: $x_1(1.0) + x_2(0.5) + x_3(0.3) = 0.6$; $x_1(0.6) + x_2(1.0) + x_3(0.2) = 0.6$; $x_1+x_2+x_3=1$, where the x_is are the portfolio weights for portfolio E. In this case, the solution is $x_1=x_2=x_3=1/3$. This portfolio would have the same b-values as D, but would have an expected return of (15+14+10)/3 = 13%.
- Hence an arbitrage opportunity exists.
- In general, the exercise of such arbitrage opportunities will force all security prices to be such that expected returns are described by our single pricing equation, $E(R) = c_0 + c_1 b_{i1} + c_2 b_{i2}$.

The APT approach: Identification

- We now simplify this pricing equation by identifying c_0, c_1 and c_2.
- Note that a portfolio with no b-risk will have to earn c_0; hence c_0 = the risk-free rate.
- A portfolio with b_1 risk equal to 1 and no b_2 risk will earn $c_0 + c_1$. Similarly, a portfolio with $b_2=1$ and $b_1=0$ will earn c_0+c_2.
- Hence our pricing equation becomes:
  $E(R) = R_f + [E(R_{F1})-R_f]b_{i1} + [E(R_{F2})-R_f]b_{i2}$,
  where $E(R_{F1})$ is the expected return on a security or portfolio that has $b_1=1$, $b_2=0$, and $E(R_{F2})$ is the expected return on a security/portfolio that has $b_1=0$, $b_2=1$.

The APT

- We can't guarantee that at all times, all assets will satisfy this equation, since it's not an equilibrium condition anymore, but rather a no-arbitrage condition.
- On the other hand, the advantage of the APT is that we don't need to worry any more about Roll's critique – that the market portfolio is not observable.
- The APT is a purely empirical model – as such, it can be used for a subset of assets in the economy.
- So, if we can assert that returns on our subset of assets can be described as $R_i = a_i + g_{i1}F_1 + g_{i2}F_2 + \text{error}$ for some F_1 and F_2, then the APT holds, as long as there are sufficiently many assets in this microcosm.
- The problem is that the underlying model could change at any time; we have no guidance as to what generates the underlying model!

CAPM versus APT

APT

- If the APT holds, then there will be no arbitrage opportunities.
- APT pricing relationship is quickly restored even if only a few investors recognize an arbitrage opportunity.
- The expected return–beta relationship can be derived without using the true market portfolio.

CAPM

- Model is based on an inherently unobservable "market" portfolio.
- Rests on mean-variance efficiency. The actions of many small investors restore CAPM equilibrium.
- CAPM describes equilibrium for all assets.

Factor Models

- Chen, Roll, and Ross used industrial production, expected inflation, unanticipated inflation, excess return on corporate bonds, and excess return on government bonds.
- The Fama-French-Carhart (FFC) model uses firm characteristics – it specifies four different factor portfolios.
  - The market portfolio
  - A self-financing portfolio consisting of long positions in small stocks financed by short positions in large stocks – the SMB (small-minus-big) portfolio.
  - A self-financing portfolio consisting of long positions in stocks with high book-to-market ratios financed by short positions in stocks with low book-to-market ratios – the HML (high-minus-low) portfolio.
  - A self-financing portfolio consisting of long positions in the top 30% of stocks that did well the previous year financed by short positions the bottom 30% stocks – the PR1YR (prior 1-yr momentum) portfolio.
- The resulting factor-pricing equation is:

$$E[R_s] = r_f + \beta_s^{Mkt}(E[R_{Mkt}] - r_f) + \beta_s^{SMB}E[R_{SMB}] + \beta_s^{HML}E[R_{HML}] + \beta_s^{PR1YR}E[R_{PR1YR}]$$

- Since the last three portfolios are self-financing, there is no investment and the risk-free return does not figure in the formula.

Using the FFC Model

Factor Portfolio	Average Monthly Return (%)	95% Confidence Band (%)
$\text{Mkt} - r_f$	0.64	± 0.35
SMB	0.17	± 0.21
HML	0.53	± 0.23
PR1YR	0.76	± 0.30

Data Source: Kenneth French.

- We see above estimates of expected risk premiums for the four FFC factors.
- Let us now consider how to use the FFC model in practice. Suppose you find yourself in the situation described below:

You are considering making an investment in a project in the food and beverages industry. You determine that the project has the same level of non-diversifiable risk as investing in Coca-Cola stock. Determine the cost of capital by using the FFC factor specification.

Using the FFC Model

Solution

You decide to use data over the past five years to estimate the factor betas of Coca-Cola stock (ticker: KO). You therefore regress the monthly excess return (the realized return in each month minus the risk-free rate) of your company's stock on the return of each portfolio. The coefficient estimates are the factor betas. Here are the estimates of the four factor betas based on data from years 2000 through 2004:

$$\beta_{KO}^{Mkt} = 0.158$$
$$\beta_{KO}^{SMB} = 0.302$$
$$\beta_{KO}^{HML} = 0.497$$
$$\beta_{KO}^{PR1YR} = -0.276$$

Using these estimates and the current risk-free monthly rate of 5%/12 = 0.42%, you calculate the monthly expected return of investing in Coca-Cola stock:

$$\begin{aligned} E[R_{KO}] &= r_f + \beta_{KO}^{Mkt}(E[R_{Mkt}] - r_f) + \beta_{KO}^{SMB}E[R_{SMB}] + \beta_{KO}^{HML}E[R_{HML}] + \beta_{KO}^{PR1YR}E[R_{KO}] \\ &= 0.42\% + 0.158 \times 0.64\% + 0.302 \times 0.17\% + 0.497 \times 0.53\% - 0.276 \times 0.76\% \\ &= 0.626\% \end{aligned}$$

The annual expected return is 0.626% × 12 = 7.512%. The annual cost of capital of the investment opportunity is about 7.5%.

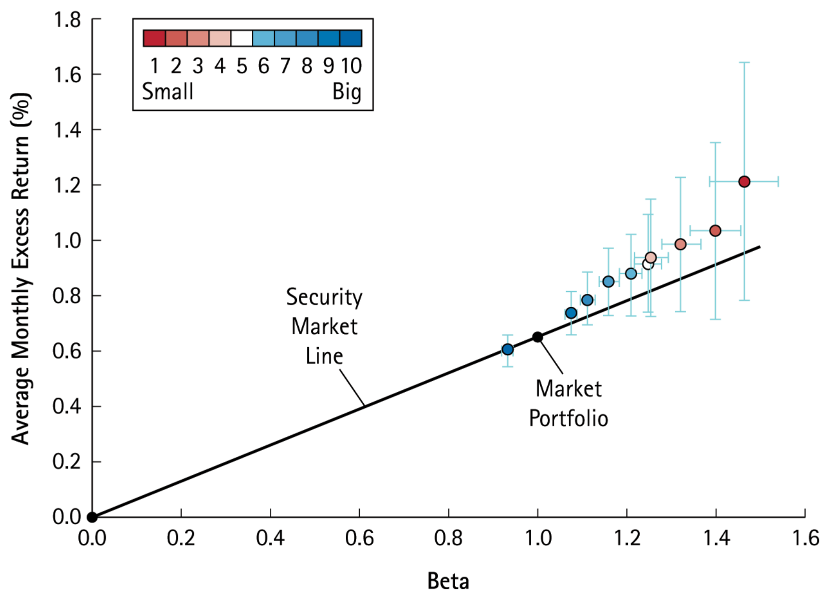
The Fama-French model

- The three factor Fama-French model includes only the market, the SMB and the HML factors.
- As points of reference, the historical average from July 1926 to July 2002 of the annual SMB factor has been approximately 3.3%1; and in a lecture in 2003, Ken French stated that he believes the annual SMB premium to be in the range of 1.5-2.0%.
- Over the time period from 1926 to 2002, the premium for value stocks (HML factor) has averaged approximately 5.1% annually, and was cited by Ken French in 2003 as having a then current value of approximately 3.5-4.0%.

CAPM vs other models

- We have seen that the CAPM could be true at the same time as multi-factor models.
- The CAPM is not testable, in principle, because it requires the observability of the market portfolio.
- Hence tests of the CAPM could "fail." On the other hand, since the factor pricing models, such as the Fama-French model or the Chen-Roll-Ross models are empirical, they could "succeed."
- But, as we saw, they could simply be multi-factor versions of an underlying CAPM.
- Furthermore, some of the successes of alternative asset-pricing models could be spurious.
- For example, the Fama-French model says that size is a proxy for risk. Some researchers (e.g. Banz) constructed portfolios of stocks and ordered them by the size of the stocks they contained and checked to see if all such portfolios lay on the Security Market Line.
- They found that they did not – in fact, portfolios of small stocks tended, on average, to earn higher returns than portfolios of larger stocks.

Excess Return of Size Portfolios, 1926–2005

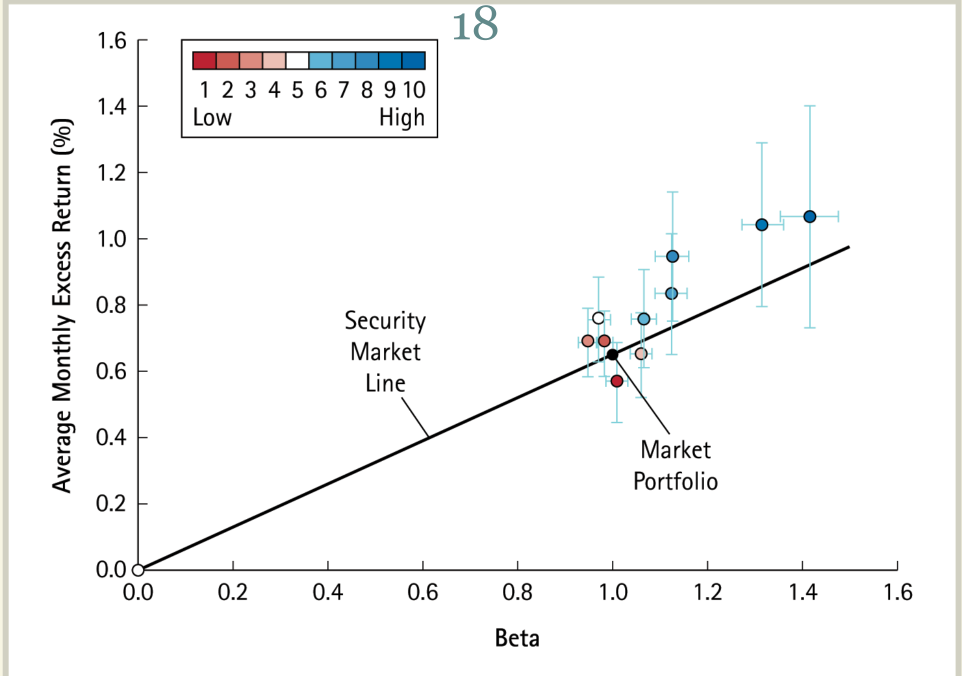


The plot shows the average excess return (the return minus the three-month risk-free rate) for ten portfolios formed in each month over 80 years using the firms' market capitalizations. The average excess return of each portfolio is plotted as a function of the portfolio's beta (estimated over the same time period). The black line is the security market line. If the market portfolio is efficient and there is no measurement error, all portfolios would plot along this line. The error bars mark the 95% confidence bands of the beta and expected excess return estimates.

Size Anomalies

- Why should there be such a pattern?
- One answer is that it's due to data-snooping – that is, given enough characteristics, it will always be possible ex-post to find some characteristic that by pure chance happens to be correlated with the estimation error of average returns.
- Another answer is that if the market portfolio is inefficient, then some assets would be overpriced and some assets would be underpriced. The overpriced assets would tend to be larger since their market values are larger than what they should be according to the CAPM. Similarly, underpriced assets would tend to be smaller. Since underpriced (overpriced) assets would tend over time to realize higher (lower) returns, we would expect to see patterns like those of Banz.
- In fact, it turned out that portfolios consisting of stocks that had high book-to-market ratios (perhaps a proxy for underpriced stocks) had higher average returns than portfolios consisting of stocks with low book-to-market ratios.

Excess Return of Book-to-Market Portfolios, 1926–2005



The plot shows the average excess return (the return minus the three-month risk-free rate) for ten portfolios formed in each month over 80 years using the stocks' book-to-market ratios. The average excess return of each portfolio is plotted as a function of the portfolio's beta (estimated over the same time period). The black line is the security market line. If the market portfolio is efficient and there is no measurement error, all portfolios would plot along this line. The error bars mark the 95% confidence bands of the beta and expected excess return estimates.

Empirical Tests of the CAPM

- The second possibility is that the conventional tests of the CAPM fail because our proxy for the market portfolio leaves out important sources of risk.
- When Jagannathan and Wang (1996) included proxies for business cycle risk and labor income risk (non-traded human capital), they found that both sources of risk were priced.
- Our standard measures of beta assume constancy over the business cycle and our market portfolio leaves out human capital. Jagannathan and Wang's model might be compensating for these risk sources.

Does the Fama-French model make sense?

- One problem with the Fama-French model is that if we don't know why assets with high scores on their factors are rewarded, we won't know when the model is no longer empirically valid (because we use historical data to test models); furthermore, we won't be able to use the model to computer required rates on return on new assets or non-traded assets.
- Liew and Vassalou (2000) show that returns on style portfolios (based on size or book-to-market factors) are positively related to future macroeconomic growth and hence may proxy business risk.
- Petkova and Zhang (2005) also relate the higher average return on high book-to-market (value) portfolios to risk premiums.
- Value firms have, on average, high amounts of tangible capital. Since such investment cannot be easily withdrawn, these firms are committed and hence at greater risk if there is an economic downturn; on the other hand, growth firms can defer investment more easily. As a result, it makes sense that such firms should earn higher average returns, as the Fama-French model says.
- In fact, they find that the beta of the HML portfolio is negative in good economies, but positive in recessions. This means that the CAPM beta of value portfolios would be less than that of growth portfolios in good times, but higher in bad times. Thus, the HML beta is a kind of adjustment to the CAPM model that allows betas to vary across the business cycle, as in Jagannathan and Wang.

Liquidity, Momentum and Asset Pricing

- We have seen that sensitivity to momentum factors is also positively correlated with higher average returns; i.e. stocks that move in sync with market-wide momentum have higher average returns. Could this be related to liquidity?
- One indicator of illiquidity is price reversals. Asset prices will dip when there are large sell orders, then recover.
- Pastor and Stambaugh (2003) find that firms that have done well when market liquidity is high (high liquidity betas) have higher average returns. This is consistent with the notion that investors need more wealth when liquidity is lowest (e.g. to meet margin calls or to have funds for consumption without having to sell too many assets).
- Although they weren't able to link firm liquidity with market returns (because estimating individual stock liquidity requires much more data), the two notions are consistent with each other.
- They also find that premiums to a momentum factor can mostly be accounted for by such liquidity premiums.

The moral of the story

- Unless we have an empirically successful asset pricing model that is also theoretically meaningful, we have to be careful in using it.
- To the extent that the risk premiums implied by the empirical model make sense, we can rely on it more.
- The CAPM model, once its implementation is extended to take into account liquidity risks and non-traded human capital seems to be conceptually able to account for a lot of the empirical results.
  - Note that, in theory, the CAPM includes human capital; however, in practice, the measure of the market portfolio that is usually used excludes non-traded assets, such as human capital.
- Above all, be cautious when using asset pricing models. Take into account risk due to model errors!