ECON 4515 Finance theory 1 Diderik Lund, Fall 2004

Exercises for seminars 1 – 2 November

Instructions:

(Repeated from previous exercises, except the *emphasized part.*)

For all students: Please try to solve these exercises before the seminar. It is generally a better idea to work a little with all questions than to work a lot with only a few of them.

For those who have volunteered to present a written suggested solution: You are asked please to make copies of your solution for all participants in your seminar group. There are currently about 15 students in each group. *Make 20 copies.* You can borrow a copy card for this purpose in the department reception office, room 1241 ES. The intention is that all students as well as the lecturer should have the opportunity to read your answers before the seminars. For the Monday group this means that the copies should be available before noon on the preceding Friday. For the Tuesday group it means that the copies should be available before noon on the preceding Monday. There are boxes, one for each seminar group, in the shelves in the students' area on the 12th floor. You are free to come and ask me (Diderik Lund) about the problems before you produce your solution.

An answer can be hand written or computer typed. The problem with typing is that many of you are not familiar with producing formulae and diagrams on a computer. This is the main reason why we do not encourage electronic submissions, although you will have to learn some of this for writing a master's thesis.

Questions/problems

Consider the function $U(C) \equiv ae^{bC} + d$, where a, b, and d are constants, and $e \approx 2.78$ is the well-known constant. C is consumption.

(a)

What condition(s) must be satisfied by a, b, and d in order for $E[U(C)]$ to properly represent the preferences of a risk averse person who maximizes von Neumann-Morgenstern expected utility? What are the coefficients of absolute and relative risk aversion, $R_A(C)$ and $R_R(C)$, for this U function?

Consider an individual planning for a future period, with the given U function with that/those conditions you stated in part (a). The individual has wealth Y_0 to be divided between two financial investments, Y_f (risk free) and Y_r (risky). The future consumption is equal to the sum of the future values of these investments. Y_f will increase by the factor $R_f \equiv 1 + r_f$, where r_f is the risk free interest rate. Y_r will increase similarly by the factor $\tilde{R} \equiv 1 + \tilde{r}$, where $\tilde{r}$ is a risky rate of return (— this may in fact be a decrease for low outcomes of $\tilde{r}$). The individual regards Y_0, R_f, and the probability distribution of $\tilde{R}$ as exogenously given.

In what follows, you should discuss both the case in which short selling is allowed and the case in which it is not.

(b)

Describe the individual's maximization problem and its solution.

(c)

Discuss the statement "Optimal Y_r does not depend on the size of Y_0."

Assume in the following that $\tilde{R}$ has only two possible outcomes, R_1 and R_2, and that $R_1 > R_2$. The probability of R_1 is p.

(d)

Describe the solution to the maximization problem in this case. Show that under some conditions the optimal Y_r can be written as

$$Y_r^* = \frac{\ln \frac{p(R_1 - R_f)}{(1-p)(R_f - R_2)}}{-b(R_1 - R_2)}.$$

(e)

Find out whether Y_r^* as given by the formula above is increased or decreased by changes in p, R_f, and b. Try to give intuitive explanations for these effects.

(f)

Could Y_r^* from the formula exceed Y_0? What would the individual do then if borrowing is not allowed?

(g)

Could Y_r^* from the formula be negative? What would the individual do then if short selling is not allowed?

(h)

What will happen if $R_1 < R_f$? What will happen if $R_2 > R_f$? Can these situations occur?

(i)

What would be the solution for parts (a) – (d) if the individual is instead attracted to risk?