# Exercises for seminars 11 – 12 October

## Instructions:

(Repeated from previous exercises, except the *emphasized part.*)

For all students: Please try to solve these exercises before the seminar. It is generally a better idea to work a little with all questions than to work a lot with only a few of them.

For those who have volunteered to present a written suggested solution: You are asked please to make copies of your solution for all participants in your seminar group. There are currently about 15 students in each group. Make 18 copies. You can borrow a copy card for this purpose in the department reception office, room 1241 ES. The intention is that all students as well as the lecturer should have the opportunity to read your answers before the seminars. For the Monday group this means that the copies should be available before noon on the preceding Friday. For the Tuesday group it means that the copies should be available before noon on the preceding Monday. There are boxes, one for each seminar group, in the shelves in the students' area on the 12th floor. You are free to come and ask me (Diderik Lund) about the problems before you produce your solution. *Observe that I am away from my office on Thursday and Friday, 7–8 October.*

An answer can be hand written or computer typed. The problem with typing is that many of you are not familiar with producing formulae and diagrams on a computer. This is the main reason why we do not encourage electronic submissions, although you will have to learn some of this for writing a master's thesis.

## (1)

Consider the following two alternative stochastic processes, suggested as descriptions of a stock price.

$$P_t = P_{t-1} \cdot u_t \quad \text{with } u_t \text{ i.i.d.} \tag{1}$$

$$P_t = P_0 \cdot e^{a \cdot t} \cdot v_t \quad \text{with } v_t \text{ i.i.d.} \tag{2}$$

You can assume that $a$ is a positive constant, and that $E_{t-1}(v_t) = 1$. You may also, as a simplification, assume that the stock has a beta value of zero.

Discuss whether the two processes have the same expected time path (for some values of $a$ and $E_0(u_t)$). Discuss for each process whether it could describe stock prices in an efficient stock market. If not, how could you device a trading rule to make profits based on observing outcomes? (Moderately difficult: Suggest a rule, and explain in words why it will work. More difficult: Suggest a rule, and prove with calculations that the expected logarithm of the return will represent an excess return in relation to the risk.)

## (2)

In the lecture of 29 September, Shiller's test of market efficiency was presented as follows: If a stock price is an unbiased, optimal forecast of the expected present value of dividends, then it can be written as

$$P_t = \sum_{\tau=1}^{\infty} \frac{E_t(D_{t+\tau})}{(1+r_a)^{\tau}}, \tag{3}$$

under the assumption that the risk adjusted discount rate $r_a$ is constant. The realized present values of dividends can be written as expected values plus an error term $U_t$ with expectation equal to zero,

$$\sum_{\tau=1}^{\infty} \frac{D_{t+\tau}}{(1+r_a)^\tau} = \sum_{\tau=1}^{\infty} \frac{E_t(D_{t+\tau})}{(1+r_a)^\tau} + U_t.$$

Shiller observes that the optimal forecast should be uncorrelated with the error term. Thus he arrives at the hypothesis

$$\text{var}\left[\sum_{\tau=1}^{\infty} \frac{D_{t+\tau}}{(1+r_a)^\tau}\right] = \text{var}\left[P_t\right] + \text{var}\left[U_t\right] > \text{var}\left[P_t\right],$$

which is rejected by the data.

**(a)** Suppose instead that the market makes some unsystematic mistakes, so that

$$P_t = \sum_{\tau=1}^{\infty} \frac{E_t(D_{t+\tau})}{(1+r_a)^\tau} + V_t,$$

where $V_t$ are i.i.d., with expectation zero. Discuss whether this could explain the variance result which Shiller finds. Discuss whether this could be used to device a trading rule in order to make profit based on observations of the outcomes.

## (3)

Introduce a growth rate, $g$, in equation (3) above, and show that it can be reformulated into the formula on top of page 74 in Malkiel's article. (Depending on the details, you may get a factor $(1+g)$ too much.) If the required expected rate of return did *not* change as suggested by Malkiel, how much would $g$ have to change instead in order to justify a 33 percent drop in share prices?

## (4)

Suppose that you live in an economy with efficient capital markets. At $t = 0$ you have one piece of information which is not known to the market: One of your fellow students, who is very clever and (for some reason) got a master's in engineering, not in economics, started right after university to work for a company which is traded in the stock market. A year later it is announced that the company has developed a new technology which will save costs significantly. Another year later it is announced that the company has sold licenses for this new technology to all its competitors. Discuss at what points in time it is a good idea for you to buy and/or sell shares in the company.