Exercises for seminars 13 – 14 September

Instructions:

For all students: Please try to solve these exercises before the seminar. It is generally a better idea to work a little with all questions than to work a lot with only a few of them.

For those who have volunteered to present a written suggested solution: You are asked please to make copies of your solution for all participants in your seminar group. There are currently about 12 students in each group. Make 15 copies. You can borrow a copy card for this purpose in the department reception office, room 1241 ES. The intention is that all students as well as the lecturer should have the opportunity to read your answers before the seminars. For the Monday group this means that the copies should be available before noon on the preceding Friday. For the Tuesday group it means that the copies should be available before noon on the preceding Monday. There are boxes, one for each seminar group, in the shelves in the students' area on the 12th floor. You are free to come and ask me (Diderik Lund) about the problems before you produce your solution. But please, try for yourself first!

An answer can be hand written or computer typed. The problem with typing is that many of you are not familiar with producing formulae and diagrams on a computer. This is the main reason why we do not encourage electronic submissions, although you will have to learn some of this for writing a master's thesis.

In the texts which follow, the notation is sometimes inconsistent. For instance, the risk free interest rate can be r_f in problem (1), but R_f in problem (2). The happens because the problems are collected from various sources, and you just have to get used to this. But within each problem, there should be consistency.

(1a)

Consider the standard version of the CAPM, illustrated in a (σ, μ) diagram. Assume that the "risk free asset" is really represented by a bank, which offers deposits and loans at the same interest rate r_f. Illustrate in the diagram what kind of preferences will induce a person to borrow from the bank, and what kind of preferences will induce deposits.

(1b)

Assume instead that the bank offers risk free deposits with an interest rate of r_d and risk free loans with an interest rate of r_ℓ, which exceeds r_d. Show that in this situation the investors will divide themselves into three different groups, depositors, borrowers, and some who prefer not to use the bank.

(2)

Consider an economy in which the Capital Asset Pricing Model holds. The economy has only two types of shares with risky future values. Share i has a rate of return of $\tilde{R}_i$, for $i = 1, 2$. In addition the agents may save or borrow at a risk free interest rate R_f. It is not necessary to derive the model results.

(2a) Find the market portfolio's expected rate of return, and the variance of this rate of return, when the following information is given:

$$E(\tilde{R}_1) = 0.1, \quad E(\tilde{R}_2) = 0.2, \quad \mathrm{var}(\tilde{R}_1) = 0.06^2 = 0.0036,$$

$$\mathrm{var}(\tilde{R}_2) = 0.18^2 = 0.0324, \quad \mathrm{cov}(\tilde{R}_1, \tilde{R}_2) = 0.002,$$

the total market value of all shares of type 1 is equal to the total market value of all shares of type 2.

(2b) Show that the given information is sufficient to determine the magnitude of R_f, and try to find this magnitude.

(3)

In the lecture notes for 1 September, p. 8, it was shown that adding noise to a risky rate of return does not affect its β, when noise is interpreted as a random variable with no correlation with the market portfolio. The underlying assumption is that the asset in question is so small in relation to the market portfolio that there is no noticeable effect on the rate of return on the market portfolio.

You are now asked to show that the same is true for multiplicative noise. This can, e.g., be interpreted as technical uncertainty in produced quantity, so that a firm's future value is the product of an output price (correlated with the market portfolio) and a quantity (uncorrelated with the market portfolio).

Remember that the covariance can be defined through

$$\mathrm{cov}(\tilde{X}, \tilde{Y}) = E(\tilde{X} \cdot \tilde{Y}) - E(\tilde{X}) \cdot E(\tilde{Y}).$$

Use this definition to show that if $\tilde{\varepsilon}$ is stochastically independent of $(\tilde{r}_j, \tilde{r}_M)$, and $E(\tilde{\varepsilon}) = 1$ and $\mathrm{var}(\tilde{\varepsilon}) > 0$, then the β of $\tilde{\varepsilon} \cdot \tilde{r}_j$ is the same as the β of $\tilde{r}_j$. Show also that the variance of $\tilde{\varepsilon} \cdot \tilde{r}_j$ exceeds the variance of $\tilde{r}_j$.