Exercises for seminars 30 − 31 August

(1)

(Part of exam question for ECON460, fall 2002, slightly rewritten.)

In this exercise you are asked to sketch two different opportunity sets in the same diagram, and then to consider what can be said about what choices an investor would make. Observe that in this text, as in many others you will come across, there is no indication of which variables are stochastic, i.e., the risky rates of return are written as r_j, not $\tilde{r}_j$.

Consider a pair of risky securities, 1 and 2, with rates of return $r_j, j = 1, 2$, with the properties $\mu_1 = E(r_1) = 0.16$, $\sigma_1^2 = \text{var}(r_1) = 0.2^2 = 0.04$, $\mu_2 = E(r_2) = 0.08$, and $\sigma_2^2 = \text{var}(r_2) = 0.3^2 = 0.09$.

Two additional pieces of information are needed in order to define the opportunity set of portfolios from the two securities. Short selling may or may not be allowed (see below). With regard to the covariance, you are asked to consider two cases:

  Case (i):   The covariance between the rates of return is $\sigma_{12} = \text{cov}(r_1, r_2) = 0.045$.

  Case (ii):   The covariance between the rates of return is $\sigma_{12} = \text{cov}(r_1, r_2) = 0.015$.

1(a)

For case (i): Determine the composition of that portfolio of the two securities which has the minimum variance, and then show that this variance is $\sigma_{mv}^2 = 0.63/16 \approx 0.198^2$. Do likewise for case (ii), with the resulting $\sigma_{mv}^2 = 0.54/16 \approx 0.184^2$.

1(b)

Sketch the two hyperbolae in the same (σ, μ) diagram. Remember that each hyperbola is symmetric around a horizontal line at the μ level which gives the minimum variance (and minimum standard deviation).

1(c)

What are the two opportunity sets (for cases (i) and (ii), resp.) if short selling is allowed? If you only know that an investor is risk averse with mean-variance preferences, can you determine whether the investor would prefer to have the opportunity set given by case (i) or that given by case (ii)?

1(d)

What are the two opportunity sets if short selling is *not* allowed? If you only know that an investor is risk averse with mean-variance preferences, can you determine whether the investor would prefer to have the opportunity set given by case (i) or that given by case (ii)?

2

The lecture notes for 25 August describe short selling of a risky security on p. 6. Discuss whether this description is relevant (in practice, not only in theory) for two other forms of investment: (i) A risk free security. (ii) A risky real investment project.

3

The lecture notes for 25 August consider situations with one or more risky assets, but there is never more than one risk free asset. Explain why a situation with many different risk free assets is not an interesting situation to consider.

4

In the lecture notes for 25 August it was implicitly assumed that the risky assets have different expected rates of return. What can you say about the opportunity set in a situation with two risky assets with the same expected rate of return?