Taxation, uncertainty, and the cost of equity

- Main question: How do taxes affect required expected rates of return?
- Focus here on taxes paid by corporations.
  - Taxes paid by shareholders may also have effects.
  - In Lund (2002) summarized in one parameter, θ.
- Two concepts of required expected rates of return: Before and after taxes.
- Under uncertainty: Both may be affected by taxation.
- Simple example without uncertainty, only before-tax requirement is affected:
  - Assume investment I yields profit PQ next period.
  - Want to find required rate of return, given some interest rate and some tax system.
  - Assume a fraction t is taxed away, so that $PQ(1-t)$ is left.
  - One plus rate of return after this gross income tax is $PQ(1-t)/I$.
  - Assume market requires after-tax rate of return of r, the interest rate.
  - From $PQ(1-t)/I = 1+r$, can solve for $PQ/I = (1+r)/(1-t)$.
  - This is one plus required rate of return before taxes.
  - Describes distortion from tax system on investment decision.

More on taxation and investment decisions

- Example showed how to determine cutoff point, "marginal investment."
  - May think of a corporation with many investment projects, ranging from very profitable (high PQ/I) to less profitable (low PQ/I).
  - Or think of one technology with decreasing returns to scale.
- Later today: Modify tax system: Some deduction for I in tax base.
- Perhaps also: Shareholders' alternative investment is also taxed, thus required rate of return less than interest rate.
- Most important modification: Uncertainty.
- New in Lund (2002): β itself depends on tax system
  - The covariance of the after-tax rate of return with $\tilde{r}_M$ differs from the covariance of the before-tax rate of return with $\tilde{r}_M$.
  - Only exception: Cash flow tax.
- Cannot use the normal way of reasoning: "A given required rate of return after taxes. Find out what the tax system implies for the required rate of return before taxes." $p = c(r)$, p. 483.
- Instead: "The required expected rate of return after taxes is given by the SML. If we take as exogenous the β of the firm's activity in case there were no taxes, find out what the tax system implies for the required expected rates of return before and after taxes."

Model

- Investment in period 0, production in period 1, only
- Consider only marginal investment project (i.e., that project which has exactly zero net value after tax):
  - Sufficient in order to find required expected rate of return under non-increasing returns to scale
  - Necessary in order to use CAPM, an equilibrium model; only marginal projects are located on the Security Market Line
  - Solve for marginal project endogenously for each case (i.e., each tax system, each assumption on tax position)
- After-tax beta found endogenously
- Before-tax beta exogenous

Assumption 1

The firm maximizes its market value according to a tax-adjusted Capital Asset Pricing Model,

$$E(r_i) = r_f\theta + \beta_i[E(r_m) - r_f\theta].$$

$\theta \in (0, 1]$ reflects differential personal taxation

r_i is the rate of return of shares in firm i

r_f is the riskless interest rate

r_m is the rate of return on the market portfolio

$\beta_i \equiv \text{cov}(r_i, r_m)/\text{var}(r_m)$

E is the expectation operator

Inflation, if any, is non-stochastic

- Our analysis concerned with "foreign" taxes (or taxes in small sector of economy); these do not affect parameters of CAPM equation
- Partial equilibrium; valuation determined in home country, affected by home country taxes only through θ
- Why θ?
  - Not necessary, but more realistic, may have $\theta = 1$
  - In Norway, $\theta = 1 - 0.28 = 0.72$
  - Under classical tax system (U.S.), $\theta = 1$ minus corporate tax rate

Consequence of CAPM

A claim to any uncertain cash flow X, to be received in period 1, has a period-0 value of

$$\varphi(X) = \frac{1}{1 + r_f\theta}[E(X) - \lambda_\theta \operatorname{cov}(X, r_m)],$$

where $\lambda_\theta = [E(r_m) - r_f\theta]/\operatorname{var}(r_m)$.

A product price, P, will most likely not have an expected rate of price increase which satisfies the CAPM. Beta of P must be defined in relation to the return $P/\varphi(P)$,

$$\beta_P = \frac{\operatorname{cov}(\frac{P}{\varphi(P)}, r_m)}{\operatorname{var}(r_m)}.$$

The fraction $P/\varphi(P)$ is one plus the rate of return on a claim to receiving one unit of output one period into the future.

Assumption 2

In period 0 the firm invests an amount I in a project. In period 1 the project produces a quantity $Q > 0$ to be sold at an uncertain price P. The joint probability distribution of (P, r_m) is exogenous to the firm, and $\text{cov}(P, r_m) > 0$. There is no production flexibility; Q is fixed after the project has been initiated.

- No explicit costs in period 1, but easy extension of model if no flexibility (input factors contracted at project initiation)
- $\text{cov}(P, r_m) > 0$ simplifies discussion; easily relaxed

If no taxes:

- Market value in period 0 of claim to revenue in period 1 is $Q\varphi(P)$
- Marginal project has $I = Q\varphi(P)$

Definition: The *relative distortion parameter* is defined as that ratio

$$\gamma_i \equiv \frac{Q\varphi(P)}{I}$$

which makes the net market after-(corporate-)tax value of the project equal to zero in case i below.

Case numbers on subsequent transparencies are different from those in Lund (2002).

Cash flow tax

Important theoretical concept. Cash flow tax essentially means:

- Proportional tax on firm's non-financial cash flows
- Government's tax cash flow is just like a shareholder's cash flow.
- Government pays a share of all investment (and other) costs (negative cash flows).
- Government receives same share of all revenues (positive c.f.).
- Tax base consists of only real (non-financial) cash flows.
- Loans, interest payments/receipts, etc., do not affect tax base.

Contrast: Standard taxation of firms (corporate income tax, CIT):

- CIT is also proportional, levied on "taxable profits".
- Investment costs are not deducted in tax base in one year, but according to "depreciation schedule" over several years.
- Net financial income is part of tax base; interest payments are thus deductible.
- If a year's tax base is negative (a "loss"), this is carried forward for deduction next year(s).

Case 1: Equity financing, no tax, or cash flow tax

Easy:

$$\gamma_1 = 1$$

(no distortion), and beta of equity after tax is

$$\beta_{V1} = \beta_P$$

Case 2: Leverage, no taxes

Assumption 3

A fraction $(1-\eta) \in [0,1)$ *of the financing need in period 0 is borrowed. This fraction is independent of the investment decision and of the tax system. The loan* B *is repaid with interest with full certainty in period 1.*

- The equity fraction of the financing is η
- Financing need is equal to I minus immediate tax relief for investment, if any
- Default-free debt standard in much of tax analysis
- Well-known results:
  - Leverage without taxes increases equity beta, which is inversely proportional to equity share of financing
  - Leverage without taxes gives no distortion

  Second of these modified here if $\theta < 1$
- Gives intuition for main result (case 3 below): Compared with a cash flow tax, a standard CIT postpones deductions (as depreciation deductions). These are assumed to be risk free. Compared with a cash flow tax, this is like a risk free loan from the firm to the tax authorities: The firm gives up a deduction now, receives it back later. Thus it acts risk-wise in the opposite direction of borrowing: It reduces the risk of equity.

Case 2, contd.

Cash-flow to equity in period 1 is

$$V_2 = PQ - (1 + r_f)B$$

Market value of this in period 0 is

$$\varphi(V_2) = Q\varphi(P) - \frac{1 + r_f}{1 + r_f\theta}B$$

For marginal project

- this must equal the financing need after borrowing
- by definition $Q\varphi(P) = \gamma_2 I$

so that

$$\eta I = \varphi(V_2) = \gamma_2 I - \frac{1 + r_f}{1 + r_f\theta}(1 - \eta)I,$$

which implies

$$\gamma_2 = \eta + \frac{1 + r_f}{1 + r_f\theta}(1 - \eta)$$

The beta value of equity is a value-weighted average of the beta values of the elements of the cash flow (of which the riskless element has zero beta), in this case

$$\beta_{V2} = \frac{Q\varphi(P)}{\varphi(V_2)}\beta_P = \frac{\gamma_2}{\eta}\beta_P = \frac{\gamma_2}{\eta}\beta_{V1}$$

Case 3: Tax position known with certainty

Assumption 4

A tax at rate t will be paid with certainty in period 1. The tax base is operating revenue less $(gr_f B + cI)$ There is also a tax relief of taI in period 0. g, c, and a are constants in the interval $[0, 1]$.

Special cases:

- Gross income tax (example p. 1): $a = c = 0$
- Accelerated depreciation: $a > 0$, $a + c = 1$
- Standard depreciation: $a = 0$, $c = 1$
- Interest tax deductible: $g = 1$
- Tax on non-financial cash flows: $g = 0$

Case 3, contd.

Cash flow to equity in period 1 is

$$V_3 = PQ(1-t) - (1+r_f)B + r_f Bgt + tcI$$

Market value of this in period 0 is

$$\varphi(V_3) = Q\varphi(P)(1-t) - \frac{1+r_f(1-tg)}{1+r_f\theta}B + \frac{tcI}{1+r_f\theta}$$

For marginal project

- this must equal the financing need after borrowing and taxes, $\eta I(1-ta)$
- by definition $Q\varphi(P) = \gamma_3 I$

so that

$$\eta I(1-ta) = \varphi(V_3)$$

$$= \gamma_3 I(1-t) - \frac{1+r_f(1-tg)}{1+r_f\theta}(1-ta)(1-\eta)I + \frac{tcI}{1+r_f\theta}$$

which implies

$$\gamma_3 = \frac{1}{1-t}\left\{(1-ta)\left[\eta + \frac{1+r_f(1-tg)}{1+r_f\theta}(1-\eta)\right] - \frac{tc}{1+r_f\theta}\right\}$$

and

$$\beta_{V3} = \frac{Q\varphi(P)(1-t)}{\varphi(V_3)}\beta_P = \frac{\gamma_3(1-t)}{\eta(1-ta)}\beta_P$$

Discussion of γ_3 and β_{V3}

Special cases:

- If $g = 0, a = 1, c = 0$, then tax is on non-financial cash flows with immediate loss offset giving no distortion, $\gamma_3 = 1$, for equity financed projects ($\eta = 1$), and $\beta_{V3} = \beta_{V2}$
- Standard corporate income tax without accelerated depreciation, $g = 1, c = 1, a = 0$, gives

$$\gamma_3 = \frac{1}{1-t}\left[\eta + (1-\eta)\frac{1 + r_f(1-t)}{1 + r_f\theta}\right] - \frac{t}{(1-t)(1+r_f\theta)}.$$

which has

$$\frac{\partial \gamma_3}{\partial t} = \frac{\eta r_f \theta}{(1-t)^2(1+r_f\theta)} > 0$$

thus γ_3 increasing in tax rate.

With no accelerated depreciation ($a = 0$), β_{V3} is decreasing in tax rate, since

$$\frac{\partial \beta_{V3}}{\partial t} = \frac{-\beta_{V1}[1 + r_f(1-\eta)]}{\eta(1+r_f\theta)} < 0$$

In this case:

- After-tax beta of equity roughly proportional to $1 - t$
- Implies: Required risk premium in rate of return roughly proportional to $1 - t$
- Risk premium under 78 percent tax: Less than 1/3 of risk premium under 28 percent tax

Conclusion

- Standard practice, with required expected rate of return to equity independent of taxes, is strongly misleading
- Except:
  - OK if firm operates under only one tax system: All betas are then tax-distorted in the same way
  - If market works according to theory, observation of shares in the firm will give correct equity beta
- Crucial part of paper: Characterizing after-tax marginal project
- Shortcoming of analysis: Partial equilibrium
- What if tax system is applied to all firms in equity market?
- Valuation parameters will then depend on tax rates
- Why is this important?
  - Firms may make wrong decisions if they apply same required expected rate of return under different tax systems, which many firms do.
  - If authorities consider tax reform, the (theoretical) effects of a reform can only be identified if a consistent theory of firms' behavior is applied.
  - Although much of the necessary theory existed, many firms used old-fashioned rules of thumb.