UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Term paper in: **ECON4515 –Finance Theory 1: Portfolio choice and equilibrium models**

Published: Friday, March 14, 2008

To be delivered by: **Monday, April 7, 2008 before 2 p.m.**

Place of delivery: Department office, 12th floor

Further instructions:

- This term paper is **compulsory**. Candidates who have passed the compulsory term paper in a previous semester, do not have the right to hand in the term paper again. This is so, even if the candidate did not pass the exam.
- You must use a printed front page, which is to be found at http://www.oekonomi.uio.no/info/EMNER/Forside_obl_eng.doc
- **Note:** The students can feel free to discuss with each other how to solve the problems, but each student is supposed to formulate her/his own answers. Only single-authored papers are accepted, and papers that for all practical purposes are identical will not be approved.
- It is of importance that the term paper is delivered by the deadline (see above). Term papers delivered after the deadline, **will not be corrected**.*)
- All term papers must be delivered to the place given above. You must not deliver your term paper to the course teacher or send it by e-mail. If you want to hand in your term paper **before** the deadline, please contact the department office on 12th floor.
- If the term paper is not accepted, you will be given a new attempt. If you still do not succeed, you will not be permitted to take the exam in this course. You will then be withdrawn from the exam, so that this will not be counted as an attempt.

*) If a student believes that she or he has a good cause not to meet the deadline (e.g. illness) she or he should discuss the matter with the course teacher and seek a formal extension. Normally extension will only be granted when there is a good reason backed by supporting evidence (e.g. medical certificate).

You may answer in English or Norwegian. If any part is hand-written or hand-drawn: Please do so legibly!

Introduction

Consider an investment project which requires an investment, I, in period 0 and results in production in period 1, only. The production value is $\tilde{P}Q$, where $\tilde{P}$ is an uncertain product price, while the produced quantity is $Q = f(I)$. The production function, $f(I)$, is increasing and concave, with $f(0) = 0$. No other cost than I is required for production.

You are asked to analyze the economics of the projects in two different settings, which should be done separately. In parts 2(c), 2(g), and 2(h) you are asked to compare the two settings.

Warning: There may be given some information below which is not necessary in order to answer the questions.

Part 1

Assume that the project, if started, will be organized as a corporation with shares traded in the stock market. The corporation has no other activity. It is not taxed and does not borrow.

You may in this part use results from the standard CAPM. You are not asked to derive the model, not even to list the assumptions behind it.

1(a)

Assume that the project is undertaken for some exogenously given value of I, which is not specified any more closely. Give an expression for the market value of the project, based on its systematic risk. Interpret the expression, and in particular that part which measures the systematic risk. Why is this a relevant risk measure in this setting?

1(b)

Assume instead that I can be chosen by the corporation. How would it make the choice? Illustrate the choice in a diagram showing the graph of $f(I)$. From the given assumptions, can you be sure that the corporation will decide on a strictly positive I?

1(c)

Will the choice of I depend on the systematic risk? Will the choice of I depend on $\sigma_P^2 \equiv \text{var}(\tilde{P})$? For both of these questions, give a verbal interpretation: If yes, how and why? If no, why not? Is there any connection between the two questions?

1(d)

Assume that the production function takes the form $Q = f(I) \equiv b\sqrt{I}$, where $b > 0$ is a constant. Find an explicit solution for I.

1(e)

Assume now that $r_f = 0.04$, $E(\tilde{r}_m) = 0.15$, $\text{var}(\tilde{r}_m) = 0.1^2 = 0.01$, $E(\tilde{P}) = 100$, $\sigma_P^2 = 5^2 = 25$, $\text{cov}(\tilde{P}, \tilde{r}_m) = 2$, and $b = 2$. (Of course, r_f and $\tilde{r}_m$ have their usual meanings in a CAPM setting.) What will be the optimal values of I and Q in this case? What will be the (gross) market value in period 0 of a claim to the production value in period 1? Before the investment is made, what is net market value, i.e., the value of having the opportunity to make this investment?

1(f)

Go back to part 1(a), and assume that I is exogenous, but use the functional form of part 1(d) and the numbers of part 1(e). What will be the market value, net of the investment cost, of the corporation if $I = 2500$? If $I = 10000$?

Part 2

Assume that the project, if started, is undertaken by a risk averse investor who does not have access to the stock market. The investor is, however, allowed to borrow or save at the risk free interest rate, r_f. Assume that the investor cares only about his/her consumption in period 1, $\tilde{C}$. More specifically, the investor cares only about the mean, $m \equiv E(\tilde{C})$, and the standard deviation, $s \equiv \sqrt{\text{var}(\tilde{C})}$. The investor's preferences can be represented by a utility function $U(m, s) \equiv m - as^2$, where a is a positive constant. Observe that you are not asked to discuss whether this utility function is consistent with the theory of expected utility, to be discussed in the lecture on 31 March.

2(a)

What is the formula for an indifference curve for this investor in an (s, m) diagram? Draw such a diagram with indifference curves for two different levels of U, for the same a. If another investor has a similar U function with a higher a, what does that investor's indifference curves look like? Show them as dashed curves in the same diagram. How do you interpret the difference between the two investors?

2(b)

Assume that the investor has a wealth W in period 0 which will be divided between the real investment, I, and savings (possibly negative) at the risk free interest rate. The real investment is the kind of project mentioned in the introduction. Consumption next period is the sum of the production value and the savings with interest. (Observe that we neglect the need to consume in period 0. The wealth W is spent only on the two types of investment.) Write down the maximization problem for this investor for the choice of I, without invoking the particular functional form $f(I) = b\sqrt{I}$. Derive the first-order condition for an interior maximum.

2(c)

Compare the solution for $1/f'(I)$ with the solution you found in part 1(b). How will the choice of I depend on the riskiness of the project? Compare your answer to the answer in part 1(c), and interpret the differences.

2(d)

Assume again (as in parts 1(d)–1(f)) that $Q = f(I) \equiv b\sqrt{I}$. Show that the opportunity set for the investor in the (s, m) diagram has the shape of an inverse parabola, i.e., it is a second-order polynomial in s with a negative coefficient on the quadratic term. Draw the opportunity set in an (s, m) diagram. (Hint: Write expressions for m and s, both as functions of I. Since the expression for s as function of I is quite simple, you can solve for I as a function of s, and plug this into the expression for m, so that you get one equation, m as function of s.)

2(e)

How does the solution for I from part 2(b) depend on the wealth W? Give a verbal interpretation of the result. From the given assumptions, can you be sure that the investor will decide on a strictly positive I?

2(f)

Assume now that $a = 0.0096$, $W = 1000$, use the functional form from 1(d) and 2(d), and use the numbers from 1(e). What is the optimal I? Illustrate the solution in the (s, m) diagram.

2(g)

Make one change from part 2(f): What happens if we let $a \to 0$? Interpret the result, comparing it to the results from parts 1(e) and 2(f). Would any kind of investor choose $I > 10000$? Explain your answer.

2(h)

Make another change from 2(f) (but keep now $a = 0.0096$): What happens if we let $\sigma_P \to 0$? Interpret the result, comparing it to the results from parts 1(e), 2(f), and 2(g). Is there a thought experiment in 1(e) which gives a similar result? Under the conditions in 1(e), could you get an optimal I which exceeds 10000? Explain your answer.