ECON4515 Finance theory 1 Diderik Lund, 13 May 2008

Extra problem set for those who handed in a compulsory term paper which was not accepted

To be handed in to the Department's reception office before 11:00 hrs. on Thursday, 15 May. Beware that the office closes at 11:00 hrs., so you need to be there a few minutes before.

The problem set consists of three pages with parts (1), (2a), (2b), (2c), (3a), (3b), (3c), (3d), and (3e).

You may answer in English or Norwegian. If any part is hand-written or hand-drawn: Please do so legibly!

Introduction

Consider a model with only two points in time, time zero and time one. Consider two assets which both give uncertain rates of return between time zero and time one, $\tilde{r}_1$ and $\tilde{r}_2$, respectively. The expected values of these two variables are $\mu_1 = 0.15$ and $\mu_2 = 0.10$, respectively. The variances of the two variables are $\sigma_1^2 = 0.08^2 = 0.0064$ and $\sigma_2^2 = 0.06^2 = 0.0036$, respectively. The two variables have a covariance of $\sigma_{12} = 0.0040$.

In some questions below you are asked to draw diagrams. You may save work by not calculating too many points exactly. It is sufficient that the curves have roughly the right curvatures and slopes, and that their intersections and/or tangency points are located correctly in relation to other such points.

(1)

What is the minimum variance which can be achieved by forming a portfolio using only these two assets? Show how you arrive at your answer. Sketch, in a suitable diagram, the opportunity set of all portfolios which can be created from the two assets. (In order to save work: You do not need to identify more than three points exactly.)

(2)

Consider two investors who have preferences over rates of return based on expected value (μ) and variance (σ^2). More specifically, they both rank such rates of return according to utility functions given as

$$U_i(\mu, \sigma^2) = \mu - K_i\sigma^2,$$

where K_i is a constant in each function. The investors are indexed by $i, i = 1, 2$, and have different values of the constant, $K_1 = 12.5$ and $K_2 = 25$.

(2a)

If the investors are not allowed to form portfolios, but only to choose between investing in asset 1 or asset 2 (described in the introduction), which asset would be preferred by investor 1? Which would be preferred by investor 2? Illustrate the different choices with indifference curves in the same kind of diagram that you have sketched in part (1).

(2b)

If the investors are allowed to form portfolios of the two assets, what would be the optimal portfolio for investor 1? What would be optimal for investor 2? Sketch these optimality conditions in the same kind of diagram.

(2c)

What is the economic meaning of the constant K_i? What would happen to the optimal portfolio (the choice made by investor i) if $K_i \to \infty$? What would happen to the optimal portfolio if $K_i \to 0$?

(3)

Consider now the possibility that the two assets, mentioned above, exist in an economy where the zero-beta CAPM (see Roll (1977)) is valid. There may be many other assets in the economy. Below there will be five different, alternative specifications of the relationship between the two assets and concepts which are defined in that model. You are not asked to find out whether these specifications could be true simultaneously, but to consider each of them separately. Some of the specifications are not consistent with the model. You are asked to identify those which are not consistent with the model, and to explain why not in each of those cases. (In the other cases, no explanation is asked for.)

Observe that the word "efficient" is used a bit differently by Roll than in the rest of the course. In this problem, as in the rest of the course, we only use "efficient" about portfolios which maximize expected return for some given value of the variance of return. We use "frontier portfolio set" about the set of portfolios which minimize variance of return for each given expected return.

Warning: There may be information given which is irrelevant for answering the question.

(3a)

One of the assets is located on the efficient set, while the other is not. Consistent with the zero-beta CAPM? If not, why not?

(3b)

Both assets are located on the efficient set. One of them is the zero-beta portfolio in relation to the other. Consistent with the zero-beta CAPM? If not, why not?

(3c)

None of the assets are located on the frontier portfolio set. The covariances with the rate of return on the market portfolio are $\text{cov}(\tilde{r}_1, \tilde{r}_m) = 0.001$ and $\text{cov}(\tilde{r}_2, \tilde{r}_m) = 0.002$, respectively. Consistent with the zero-beta CAPM? If not, why not?

(3d)

None of the assets are located on the frontier portfolio set. The covariances with the rate of return on the market portfolio are $\text{cov}(\tilde{r}_1, \tilde{r}_m) = 0.001$ and $\text{cov}(\tilde{r}_2, \tilde{r}_m) = 0$, respectively. Consistent with the zero-beta CAPM? If not, why not?

(3e)

Both of the assets are located on the frontier portfolio set. The covariances with the rate of return on the market portfolio are $\text{cov}(\tilde{r}_1, \tilde{r}_m) = 0.001$ and $\text{cov}(\tilde{r}_2, \tilde{r}_m) = 0$, respectively. Consistent with the zero-beta CAPM? If not, why not?