Corrected version of p. 10 of lecture notes of 5 May 2008

The suggested answer to part 4(b) was misleading at one point: The interpretation of the effect of p_1 on optimal c_{i1}. The question was:

4(b)

Assume both individuals have a utility function of the form $U_i(c_{is}) \equiv -e^{-\alpha_i c_{is}}$, where α_i is an individual-specific, positive constant. Show that under the assumption that there is an interior solution, the optimal c_{i1} can be written as

$$c_{i1} = \frac{\hat{Y}_i - \frac{p_2}{\alpha_i}\ln\left(\frac{\pi_2 p_1}{\pi_1 p_2}\right)}{p_1 + p_2}$$

Answer

(There are no changes in the derivation of optimal consumption:)

We have $U_i'(c_{is}) \equiv \alpha_i e^{-\alpha_i c_{is}}$. Plug this into the f.o.c.:

$$\frac{\pi_1 \alpha_i e^{-\alpha_i c_{i1}}}{\pi_2 \alpha_i e^{-\alpha_i c_{i2}}} = \frac{p_1}{p_2}$$

Rewrite several times in order to arrive at the required expression:

$$e^{\alpha_i(c_{i2}-c_{i1})} = \frac{p_1 \pi_2}{p_2 \pi_1},$$

$$\ln\left(\frac{p_1\pi_2}{p_2\pi_1}\right) = \alpha_i(c_{i2} - c_{i1}) = \frac{\alpha_i}{p_2}(\hat{Y}_i - (p_1 + p_2)c_{i1})$$

This can easily be transformed into the required expression.

(Here comes a better interpretation of the effect of p_1 on optimal c_{i1}:)

We need to use a standard result from consumer theory, that demand is homogeneous of degree zero in prices. This is true in models where there is no given monetary budget. The whole budget consists of the revenue from selling endowments at prices which are exogenous to each agent. If all prices are doubled (or more generally, changed by the same factor), this will not affect demand.

In our model the budget constraint of the consumer is

$$\hat{Y}_i \equiv p_1 Y_{i1} + p_2 Y_{i2} = p_1 c_{i1} + p_2 c_{i2}.$$

This is equivalent to

$$\frac{p_1}{p_2}Y_{i1} + Y_{i2} = \frac{p_1}{p_2}c_{i1} + c_{i2},$$

which shows that relative prices are all that matters to the consumer. The $\hat{Y}_i$ is *not* exogenous in the model, but depends on the prices, and is defined as $p_1 Y_{i1} + p_2 Y_{i2}$. In this case there is only one relative price, $\frac{p_1}{p_2}$. In the answer to 4(b), it is helpful to rewrite optimal consumption as

$$c_{i1} = \frac{\frac{\hat{Y}_i}{p_2} - \frac{1}{\alpha_i}\ln\left(\frac{\pi_2 p_1}{\pi_1 p_2}\right)}{\frac{p_1}{p_2} + 1}.$$

At this point it is useful to get rid of the variable $\hat{Y}_i$, and to define the relative price $p = p_1/p_2$. This gives,

$$c_{i1} = \frac{pY_{i1} + Y_{i2} - \frac{1}{\alpha_i}\left(\ln\left(\frac{\pi_2}{\pi_1}\right) + \ln(p)\right)}{p+1},$$

Here we have written the optimal c_{i1} in terms of the relative price and the exogenous parameters of the model, and we can ask what happens if the relative price changes while the exogenous parameters are kept constant.

A partial derivative shows the effect.

$$\frac{\partial c_{i1}}{\partial p} = \frac{1}{(p+1)^2}\left[Y_{i1} - Y_{i2} - \frac{1}{\alpha_i}\left(\frac{p+1}{p} - \ln(p)\right) + \frac{1}{\alpha_i}\ln\left(\frac{\pi_2}{\pi_1}\right)\right].$$

The difference $Y_{i1} - Y_{i2}$ represents the income effect, as it is the effect which comes from the first term in the numerator in the expression for c_{i1}, the budget $(pY_{i1} + Y_{i2})$. The income effect could be positive or negative, depending on the sign of $Y_{i1} - Y_{i2}$. If you will have a high endowment in state 1, a higher relative price for state 1 implies that the income effect leads you to consume more in state 1. The next terms, which involve α_i and the probabilities, represent the substitution effect. One would think that the substitution effect was clearly negative, i.e., a higher p should lead to lower c_{i1} if we disregard the income effect. But the logarithm of p may be large, and the logarithm of the ratio of probabilities may take any positive or negative value. So if p or π_2/π_1 is large enough, and the risk aversion parameter α_i is not too large, we may get a positive overall effect: A higher p_1/p_2 gives a higher c_{i1}, somewhat paradoxically.