Administrative

- Please check course web site often (messages, exercises, etc.):
- http://www.uio.no/studier/emner/sv/oekonomi/ECON4515/v08/
- 13 lectures of 90 minutes, once weekly, Mondays 10:15–12
- 10(?) seminars of 90 minutes, starting in the week beginning with 28 January (week 5)
- Seminars in one group or in two parallel groups, one on Tuesdays 16:15–18
- Grade based only on final exam; three hours, closed book
- In week 13, 24–30 March, there will be no lecture or seminar, instead exercise ("term paper") to hand in, will be returned and discussed in a lecture on 5 May
- Apart from this compulsory term paper, all lectures, seminars and exercises are optional
- But: Essential to work with seminar exercises to learn material and prepare for exam
- Lecture notes (like these) are distributed on web site before each lecture, before 10:00 hrs. on Fridays
- Many diagrams missing in notes, to be drawn during lectures
- Lectures in English, but Norwegian translation when asked for
- You may ask questions in Norwegian during lectures, will be translated, then answered

Finance Theory: Overview

- Main topic: What are the values of various assets?
- Both financial and real assets: Securities (shares of stock, bonds, options, etc.), investment projects, property
- Central feature of theories: Uncertainty about future income streams connected to the assets, or their values in the future
- Equilibrium models: Supply and demand determine values
- Applications in firms and business:
  - Determine values for trading assets
  - Decision tool for investment projects
  - Answer questions like: Should firms diversify?
- Applications in government:
  - Privatization
  - Decision tool for investment projects
  - Regulation of markets
  - Taxation of firms

Overview, contd.

- You will *not* learn how to make money in the markets
- In fact, you will learn why that is very difficult
- You will learn basic theory about what determines (and what does *not* influence) security equilibrium prices
- You will also learn about the role of these markets in the economy
  - Desynchronize (separate consumption from income) in time
  - Desynchronize between outcomes (states of nature)
  - Welfare consequences

Topics to be found in other courses:

- This course does not cover control of firms, or conflicts due to asymmetries of information between management, shareholders, and lenders. Those topics: ECON4245 Economics of the Firm
- This course does not discuss in any detail the pricing of options and similar securities. That is done in ECON4520 Finance Theory 2

Required background and overlap

- This course builds on mathematics at the level of ECON3120/4120; those who do not have it, should take that course in parallel
- This course builds on undergraduate statistics at the level of ECON2130
- More math, such as ECON4140/4145, and statistics, such as ECON4130 or ECON4135, is an advantage
- This course overlaps with ECON2130, ECON3210/4210 and ECON1810 on the topic of decisions under uncertainty, "expected utility", but full credit is given anyhow

Consequences of overlap, and of parallel math course

- Will start at fairly basic level mathematically
- Will increase math level when, e.g., integration and linear algebra are taught in ECON3120/4120
- Perhaps somewhat confusing: Will soon jump to p. 93 in Danthine & Donaldson
- Lectures will be self-contained, and will cover pp. 41–92 later
- Start with theory of choice under uncertainty, in particular with choice based only upon mean (expectation) and variance

Choice under uncertainty

- In order to construct theoretical model of asset markets: Need theory of people's behavior in these markets
- "Choice under uncertainty" since choice between uncertain (risky) alternatives
- Example:
  - May buy government bonds and earn interest at a known rate
  - May alternatively buy shares in the stock market with risky returns
  - E.g., invest everything in one company, such as Norsk Hydro
  - One certain, one uncertain alternative
- In reality many uncertain alternatives: Shares in different companies
- May also diversify: Invest some money in one company, some in another
- May also invest outside of asset markets, "real investment" projects
- Outcome one year into the future of each choice is uncertain
- Assume the outcome in each alternative can be described by a probability distribution
- Exist also theories of choice under "total uncertainty" without probabilities, but much more difficult

Choice under uncertainty, contd.

- Choice between probability distributions of consumption in future periods
- Simplification in finance: Only one good, money (but theory in chapters 1 and 8 D&D can deal with vectors of different goods)
- To begin with: Uncertainty in *one period* only
- Choices are made now (often called period zero), with uncertainty about what will happen next (period one)
- Only one future period: Consumption = wealth in that period
- Each choice alternative gives one probability distribution of outcomes in period one
- *All consequences* and the total situation of the decision maker should be taken into consideration when choices are described; for instance:
  - Choose between (a) keeping \$10 and (b) spending it on a lottery ticket with 1 per cent probability of winning \$1000 and 99 per cent of loss
  - This is *different* from the problem of choosing between \$10010 on one hand and on the other a 1 per cent probability of \$11000 and 99 per cent probability of \$10000

Expected utility versus choice based on mean and variance

- Some of you know a theory of choice under uncertainty proposed by John von Neumann and Oscar Morgenstern (1947)
- Known under name "expected utility"
- Will return to this later in the course
- Start instead with simplifying assumption:
- Individuals choose between different probability distributions based on two characteristics of these, the mean (also known as the expected value) and the variance
- These are important characteristics of a probability distribution, but those two numbers alone do not fully characterize the distribution
- The assumption is thus only a useful simplification, and has its limitations
- A person may very well strictly prefer one distribution to another, even though both have the same mean and the same variance
- But in the beginning of this course, we shall neglect that possibility
- Assume also all individuals prefer higher mean ("non-satiation") and lower variance ("risk aversion")

About individuals' mean-variance preferences

- Is it better to consume 1000 with certainty than an uncertain consumption with mean 1200 and variance 10 000?
- Perhaps mean 1400 and variance 30 000 is better than both of these?
- Assume each individual is able to choose between such alternatives
- For each individual: Indifference curves in mean-variance diagram
- By convention draw diagram with mean (of consumption) on vertical axis
- Instead of variance, use standard deviation $= \sqrt{\text{variance}}$
- Reason for using standard deviation: Will simplify some later discussions
- Indifference curves are increasing curves in diagram
- Assume they are convex (more on this later in course)

Mean-variance portfolio choice

- One individual, mean-var preferences
- Has a given wealth W_0 to invest at $t = 0$
- Regards probability distribution of future ($t = 1$) values of securities as exogenous; values at $t = 1$ include payouts like dividends, interest
- Today also: Regards security prices at $t = 0$ as exogenous
- Later: Include this individual in equilibrium model of competitive security market at $t = 0$

Notation: Investment of W_0 in n securities:

$$W_0 = \sum_{j=1}^{n} p_{j0} X_j = \sum_{j=1}^{n} W_{j0}$$

Value of this one period later:

$$\tilde{W} = \sum_{j=1}^{n} \tilde{p}_{j1} X_j = \sum_{j=1}^{n} \tilde{W}_j = \sum_{j=1}^{n} p_{j0} \frac{\tilde{p}_{j1}}{p_{j0}} X_j$$

$$= \sum_{j=1}^{n} p_{j0}(1 + \tilde{r}_j) X_j = \sum_{j=1}^{n} W_{j0}(1 + \tilde{r}_j)$$

$$= W_0 \sum_{j=1}^{n} \frac{W_{j0}}{W_0}(1 + \tilde{r}_j) = W_0 \sum_{j=1}^{n} w_j (1 + \tilde{r}_j) = W_0(1 + \tilde{r}_p),$$

where the w_j's, known as portfolio weights, add up to unity

Mean-var preferences for rates of return

$$\tilde{W} = W_0 \sum_{j=1}^{n} w_j(1+\tilde{r}_j) = W_0 \left(1 + \sum_{j=1}^{n} w_j \tilde{r}_j\right) = W_0(1+\tilde{r}_p)$$

- $\tilde{r}_p$ is rate of return for investor's portfolio
- If each investor's W_0 fixed, then preferences well defined over $\tilde{r}_p$, may forget about W_0 for now
- Let $\mu_p \equiv E(\tilde{r}_p)$ and $\sigma_p^2 \equiv \text{var}(\tilde{r}_p)$; then

$$E(\tilde{W}) = W_0(1+E(\tilde{r}_p)) = W_0(1+\mu_p),$$

$$\text{var}\,\tilde{W} = W_0^2\,\text{var}(\tilde{r}_p),$$

$$\sqrt{\text{var}(\tilde{W})} = W_0\sqrt{\text{var}(\tilde{r}_p)} = W_0\sigma_p$$

Increasing, convex indifference curves in $(\sqrt{\text{var}(\tilde{W})}, E(\tilde{W}))$ diagram imply increasing, convex indifference curves in (σ_p, μ_p) diagram

But: A change in W_0 will in general change the shape of the latter kind of curves ("wealth effect")

Equilibrium models vs. arbitrage pricing (D&D ch. 2)

Theoretically two very different approaches to asset pricing

Equilibrium model:

- Determine prices by supply and demand
- The equilibrium prices depend on everything in the model, such as the preferences of the agents, their endowments, perhaps some exogenous variables (typically: the risk free interest rate)
- However, will be able to give some fairly simple results

Arbitrage pricing:

- Determines prices by showing the correspondence with other existing assets
- Argument: Since this asset gives the same future cash flow as some other (set of) asset(s), it must have the same value today
- If not, there would be opportunities of arbitrage, making money by buying and selling at observed market prices
- Conceptual problem: If we find how a price must relate to some other price(s), what if these change? (Equilibrium?)

In this course: Will define arbitrage more carefully, and give some theory (D&D ch. 10), but concentrate on equilibrium models

Surprisingly, the practical difference between the two types of models does not need to be big

Arbitrage pricing particularly useful for options and similar securities, whose prices obviously depend on prices of other securities (typically stocks)

Preview of practical results (D&D sect. 2.2)

- Practical focus: Formulae $V(\tilde{X})$ giving asset value
- Background: Why not just take present value of $E(\tilde{X})$?
- One particular principle will be important: Value additivity
- $V(\tilde{X}_1 + \tilde{X}_2) = V(\tilde{X}_1) + V(\tilde{X}_2)$
- With this in mind, what does $V()$ function look like?
  - Alt. 1: Risk-adjusted discount rate,
    $$\frac{E(\tilde{X})}{1 + r_f + \pi},$$
    π is risk premium added to risk-free interest rate r_f
  - Alt. 2: Present value (PV) of risk-adjusted expectation,
    $$\frac{E(\tilde{X}) - \Pi}{1 + r_f}$$
    where r_f is used to find PV, but a deduction Π is made in $E(\tilde{X})$
  - Alt. 3: Expected present value based on adjustment in probability distribution,
    $$\frac{\hat{E}\tilde{X}}{1 + r_f}$$
    where $\hat{E}$ represents those adjusted probabilities
  - Alt. 4: Pricing based on state-contingent outcomes, $X(\theta)$,
    $$\Sigma_\theta \; q(\theta)X(\theta)$$
    where $q(\theta)$ is value of claim to one krone in state θ