Averages of rates of return and β's

- Preliminary result before discussion of Lund (2002).
- How find an average rate of return or average β?
- Suppose a cash flow next year is sum of n elements.
- Have shown: Can find n values separately, then sum.
- But how does this relate to rates of return for each element?
- For simplicity consider only $n = 2$, but easy to generalize.
- Will show: Total rate of return is value-weighted average.
- Then also: Total β is value weighted average of β's.
- One plus rate of return is:

$$\frac{a\tilde{P}_1 + b\tilde{P}_2}{V(a\tilde{P}_1 + b\tilde{P}_2)} = \frac{a\tilde{P}_1}{aV(\tilde{P}_1) + bV(\tilde{P}_2)} + \frac{b\tilde{P}_2}{aV(\tilde{P}_1) + bV(\tilde{P}_2)}$$

$$= \frac{aV(\tilde{P}_1)}{aV(\tilde{P}_1) + bV(\tilde{P}_2)} \cdot \frac{\tilde{P}_1}{V(\tilde{P}_1)} + \frac{bV(\tilde{P}_2)}{aV(\tilde{P}_1) + bV(\tilde{P}_2)} \cdot \frac{\tilde{P}_2}{V(\tilde{P}_2)}.$$

Let $w_1 \equiv \frac{aV(\tilde{P}_1)}{aV(\tilde{P}_1)+bV(\tilde{P}_2)}$, the value weight of element 1, and w_2 similarly. These are non-stochastic and factor out of the covariance:

$$\text{cov}\left(w_1\tilde{r}_1 + w_2\tilde{r}_2, \tilde{r}_M\right) = w_1 \text{cov}\left(\tilde{r}_1, \tilde{r}_M\right) + w_2 \text{cov}\left(\tilde{r}_2, \tilde{r}_M\right).$$

Taxation, uncertainty, and the cost of equity

Motivation

- How are investment decisions affected by taxes on corporations?
  - Effect subject to much political debate.
  - Required pre-tax rate of return determines which investments are made.
  - Often assumed: High taxes imply high required pre-tax rate of return. Distortive effect of taxes.
  - Will modify this view. Depends on details of tax system.
- How are required expected after-tax returns affected by taxes?
  - "Capital market determines required after-tax return."
  - Under full certainty, this is simply the interest rate, r.
  - Typical question: How much must the pre-tax rate of return exceed r in order to yield r after tax?
  - But under uncertainty, this is misleading.
  - The requirement from the market is not r, but the CAPM equation. Requirement as function of β.
  - Lund (2002): Taxes affect riskiness of after-tax return.
  - Taxes affect required expected after-tax rate of return.
  - This is not the important variable for distortive effects, but important to get correct investment criterion in firms.
  - Firms should not use same required expected after-tax return under different tax systems.

Corporations financed by equity and loans

- Third question: How are required expected rates of return affected by borrowing done by the corporation?
  - Interplay of borrowing and taxes is important.
  - Also, result on borrowing alone provides intuition.
  - Borrowing: Consider situations with no risk of bankruptcy.
  - The lender will only require the risk free interest rate.

Overview of cases

- Borrowing, uncertainty, no tax.
- Tax, no borrowing, full certainty.
- Both taxes and borrowing, uncertainty, more general tax system, with special cases:
  - Typical corporate income tax.
  - Tax on gross income without any deductions.
  - Cash flow tax (with immediate deductions).
  - Modified cash flow tax: Postpone deduction with interest.

Define measure of distortion in pre-tax rate of return

- Model of project $(-I, \tilde{P}Q)$ with zero net value after tax.
- Dividing line between what is profitable and what is not.
- Investor is interested in return after taxes, after borrowing.
- CAPM gives values, as seen from time 0, of time 1 cash flows.
- Thus CAPM determines required expected after-tax returns.
- In each case we then calculate required expected pre-tax return.
- Shows how taxes (+ borrowing) affect real investment decisions.
- Under taxation, in order to give the required after-tax expected return, the pre-tax expected rate of return, $E(\tilde{P})Q/I - 1$, typically must exceed what it would need to be without taxation.
- Could measure distortion as required $E(\tilde{P})Q/I$, but this depends also on β.
- Instead, observe that $E(\tilde{P})Q/I = [E(\tilde{P})/V(\tilde{P})][V(\tilde{P})Q/I]$.
- Define $\gamma \equiv V(\tilde{P})Q/I$ as the required pre-tax extra value factor needed to achieve zero after-tax value in each case.
- (Equation (5) in Lund (2002); φ means the same as V.)
- This γ measures the distortion from a no-tax situation.
- In a no-tax situation, the requirement is $V(\tilde{P})Q = I$, so $\gamma = 1$.

Effect of borrowing on required rates of return

- Consider first a loan in a situation without taxes.
- For borrower, loan has cash flow B now, $-B(1+r_f)$ at time 1, written as $(B, -B(1+r_f))$.
- The loan has a net value of zero, since the obligation to pay $-B(1+r_f)$ has a present value of $-B$.
- If a project $(-I, \tilde{P}Q)$ is marginal without the loan, then the cash flow to equity, $(-I+B, \tilde{P}Q - B(1+r_f))$, is marginal with the loan.
- The loan in itself creates no distortion in investment, $\gamma = 1$.
- But the beta of the cash flow is affected. Let η be the relative equity financing of I, so that $B = I(1-\eta)$, $\eta = (I-B)/I$.
- Then the CAPM β of equity is proportional to

$$\text{cov}\left(\frac{\tilde{P}Q - B(1+r_f)}{I-B} - 1, \tilde{r}_M\right) = \text{cov}\left(\frac{\tilde{P}Q}{I-B}, \tilde{r}_M\right) = \frac{1}{\eta}\text{cov}\left(\frac{\tilde{P}Q}{I}, \tilde{r}_M\right)$$

- The last of these covariances would matter when $B = 0$.
- β of equity is inversely proportional to the equity share η.
- Borrowing makes equity more risky (both systematic and total risk).

Taxes under full certainty, no borrowing

- Assume investment I yields profit PQ next period.
- Assume a fraction t is taxed away, so that $PQ(1-t)$ is left.
- One plus rate of return after this gross tax is $PQ(1-t)/I$.
- Assume market requires after-tax rate of return of r.
- Implies $PQ(1-t)/I = 1+r$; solve for $PQ/I = (1+r)/(1-t)$.
- This is required rate of return before taxes, plus one.
- E.g., if $r = 0.05$ and $t = 0.3$, then $(1+r)/(1-t) = 1.5$.
- Distortion from tax system: Compare with no-tax situation.
- Find $\gamma \equiv V(\tilde{P})Q/I = [PQ/(1+r)]/I = 1/(1-t)$.

Introduce deduction at time 1 for investment cost

- Cash flow at time 1 is $PQ - t(PQ - cI)$ with c constant.
- Depreciation allowance, but reduced to one period: $c = 1$.
- (More realistic: Deduction of I stretched over many periods.)
- One plus rate of return after this tax is $(PQ(1-t) + tcI)/I$.
- Requirement after tax is $(PQ(1-t) + tcI)/I = 1 + r$; solve for $PQ/I = (1 + r - tc)/(1-t)$.
- This is required rate of return before taxes, plus one.
- E.g., if $r = 0.05$, $c = 1$, $t = 0.3$, then $(1+r-ct)/(1-t) \approx 1.07$.
- Again, compare with no-tax situation.
- $\gamma = [PQ/(1+r)]/I = (1 + r - tc)/(1-t)(1+r)$.

Effects of taxes on γ

- On previous page, first model is special case of last, letting $c = 0$.
- This means that increasing c from 0 to 1 implies reduced γ.
- If $r = 0.05$ and $t = 0.3$, then γ goes from 1.43 to 1.02.
- May get rid of distortion, solve for $\gamma = 1$, find $c = 1 + r$.
- Maintain present value of deduction; let it accumulate interest.
- Will soon show: Also $\gamma = 1$ if instead deduct I at time 0.

More general model: Uncertainty, tax, borrowing

- Cash flow to equity at time 0 is $-I + B + taI$.
- Invest I, borrow B, get immediate deduction of aI in tax base.
- This also reduces the financing need to $(1 - ta)I$, so now $B = (1 - \eta)(1 - ta)I$.
- Cash flow at time 1 is $\tilde{P}Q - t(\tilde{P}Q - cI - Br_f) - B(1 + r_f)$.
- Interest on loan is Br_f, deductible in tax base.
- Case considered is like sect. 2.1 in Lund (2002), but simpler:
  - Assume here interest on loan is always deductible ($g = 1$).
  - Assume here no impact of personal tax on CAPM ($\theta = 1$).
- $\tilde{P}$ is uncertain, but assume tax base is positive for sure.
- Tax base < 0 is possible in Lund (2002) sect. 3, not this course.

Different corporate tax systems; parameters a and c

- Corporations in most OECD countries pay corp. income tax.
- Rules differ between countries, but typically:
  - Net financial income is in tax base, so $r_f B$ is deductible.
  - The nominal investment cost is deductible, but timing varies.
  - No compensation for inflation, no interest accumulation.
- "Accelerated depreciation"; $a > 0$; deduct part of I in year 0.
- Not all countries have this, often $a = 0$.
- Remaining part, $1-a$, often deductible over a number of years.
- According to "depreciation schedule" to reflect loss of value.
- Can be linear loss, depreciation $\frac{1}{n}, \frac{1}{n}, \frac{1}{n}, \ldots$ for n years.
- Or exponentially decreasing value, $\delta(1-\delta)^t$ (with $0 < \delta < 1$).
- In two-period model, represent depreciation as $c = 1 - a$.
- Three other systems have theoretical and/or practical interest:
  - Gross income taxation, $a = c = 0$ (see top of p. 6 above).
  - Cash flow taxation, $a = 1, c = 0$, requires refund (negative tax in year 0) or other tax base in which I can be deducted.
  - Interest compensation, $a = 0, c = 1 + r_f$ (see top of p. 7).

Distortion (γ) in the more general model

- Value at time zero of claim to cash flow at time one is

$$V\left(\tilde{P}Q - t(\tilde{P}Q - cI - Br_f) - B(1+r_f)\right)$$
$$= V(\tilde{P})Q(1-t) + \frac{tcI - (1 + r_f(1-t))B}{1+r_f}.$$

- For the marginal project, this is equal to financing need after borrowing and taxes, $\eta I(1-ta)$.
- For the marginal project, $\gamma = V(\tilde{P})Q/I$, implying

$$\eta I(1-ta) = \gamma I(1-t) + \frac{tcI - (1+r_f(1-t))B}{1+r_f}.$$

- Substitute in $B = (1-\eta)(1-ta)I$ and solve for

$$\gamma = \frac{1}{1-t}\left\{(1-ta)\left[\eta + \frac{1+r_f(1-t)}{1+r_f}(1-\eta)\right] - \frac{tc}{1+r_f}\right\}.$$

- Interpretation: When $\eta = 1$ this reduces to

$$\gamma = \frac{1}{1-t}\left\{(1-ta) - \frac{tc}{1+r_f}\right\} = \frac{1}{1-t}\left\{1 - t\left(a + \frac{c}{1+r_f}\right)\right\}.$$

– Distortion ($\gamma \neq 1$) if present value of (a, c) is not equal to one.

- If $\eta < 1$, $[\eta + \frac{1+r_f(1-t)}{1+r_f}(1-\eta)]$ reflects subsidy to borrowing.

CAPM β in the more general model

- The β of equity (i.e., the β of shares) is proportional to

$$\text{cov}\left(\frac{\tilde{P}Q(1-t)+t(cI+Br_f)-B(1+r_f)}{I(1-ta)(1-\eta)}-1,\tilde{r}_M\right).$$

- Want to sort out the impact of borrowing and taxation.
- Expression before comma is one plus the rate of return to equity.
- Only $\tilde{P}$ is uncertain in this rate of return.
- Use previous result: Value-weighted average of covariances.
- Here, two covariances, $\text{cov}\left(\frac{\tilde{P}}{V(\tilde{P})},\tilde{r}_M\right)$ and zero.
- The covariance on top of this page can be rewritten as:

$$\frac{V(\tilde{P}Q(1-t))}{V(\tilde{P}Q(1-t))+V(t(cI+Br_f)-B(1+r_f))}\text{cov}\left(\frac{\tilde{P}}{V(\tilde{P})},\tilde{r}_M\right)$$

Plug in from previous page:

$$=\frac{\gamma I(1-t)}{\eta I(1-ta)}\text{cov}\left(\frac{\tilde{P}}{V(\tilde{P})},\tilde{r}_M\right)=\frac{\gamma(1-t)}{\eta(1-ta)}\text{cov}\left(\frac{\tilde{P}}{V(\tilde{P})},\tilde{r}_M\right),$$

which implies equation (12) in Lund (2002), the β of equity is

$$\beta_X=\frac{\gamma(1-t)}{\eta(1-ta)}\beta_P.$$

Interpretation of result on β and taxes

$$\beta_X = \frac{\gamma(1-t)}{\eta(1-ta)}\beta_P.$$

- If $t = 0$, then also $\gamma = 0$, and $\beta_X = \beta_P/\eta$, see p. 5.
- A corporation that borrows, gets a higher β of equity.
- Consider now cases with $t > 0$, but $\eta = 1$, so that $\gamma(1-t) = 1 - t(a + c/(1+r_f))$, and $\beta_X = \frac{1-t(a+c/(1+r_f))}{1-ta}\beta_P$.
- Then a higher c (deduction in year 1) will reduce β_X.
- Given $cI > 0$, a higher t will reduce β_X since

$$\frac{\partial}{\partial t}\left(\frac{1-t(a+c/(1+r_f))}{1-ta}\right) = -\frac{c/(1+r_f)}{(1-ta)^2} < 0.$$

- But what with a cash flow tax or deduction with interest?
- (Maintain assumption $\eta = 1$ for simplicity.)
- A cash flow tax ($a = 1, c = 0$) has $\gamma = 1$ and $\beta_X = \beta_P$.
- A tax with interest accumulation ($a = 0, c = 1 + r_f$) has $\gamma = 1$ and $\beta_X = (1-t)\beta_P$.
- Deduction of $(1 + r_f)I$ in year 1 has same value as I in year 0.
- But postponement with interest is like loan to government.
- Has opposite effect of borrowing by corporation: Reduced β.

Some implicit assumptions, shortcomings

- Have assumed that debt is always repaid.
- Must rely on firm always having sufficient income.
- Implies lower bound on outcomes for $\tilde{P}$.
- Similar assumption: Tax base always positive.
- When considering changes in tax rates:
- Have assumed that r_f and the probability distribution of $(\tilde{P}, \tilde{r}_M)$ is unchanged.
- Not a full general equilibrium model. Partial equilibrium.
- Pre-tax β_P was assumed exogenous, independent of tax.
- Relevant if consider tax in small country or sector.
- Capital market equilibrium determined in larger economy.
- Have neglected inflation (or assumed inflation risk free).
- Difference between lecture and Lund (2002): θ in CAPM.
- Also: Assumed here interest expenditures always deductible.
- Parameter θ is relevant if alternative rates of return are taxed.
- Relevant e.g. due to personal taxation in larger economy.
- But no effect on conclusions mentioned here.
- Main effect in model: If $\theta > 1 - t$, it pays to borrow.
- Could save taxes by reducing η towards zero.

Conclusion

- Standard practice, with required expected rate of return to equity independent of taxes, is strongly misleading
- Except:
  - OK if firm operates under only one tax system: All betas are then tax-distorted in the same way
  - If market works according to theory, observation of shares in the firm will give correct equity beta
- Crucial part of paper: Characterizing after-tax marginal project
- Valuation parameters will then depend on tax rates
- Why are the results important?
  - Firms may make wrong decisions if they apply same required expected rate of return under different tax systems, which many firms do.
  - If authorities consider tax reform, the (theoretical) effects of a reform can only be identified if a consistent theory of firms' behavior is applied.
  - Although much of the necessary theory existed, many firms used old-fashioned rules of thumb.
- Shortcoming of analysis: Partial equilibrium
- What if tax system is applied to all firms in equity market?