Note: Symbols and equation numbering are the same as in Grossman, chapter 7 in Handbook, with the exception that I replace G_t for ϕ_t (since $G_t \equiv \partial h_t / H_t$ the use of ϕ_t is superfluous).

Health capital (H) at time t the result of
history (H_{t-1})
depreciation $(\delta_t H_{t-1})$ – age, accident, disease ...
investments in health (I_{t-1}), exercise, medical care ...
Note: H the stock of health at the beginning of the period.

$$\begin{aligned} H_t &= H_{t-1} - \delta_{t-1} H_{t-1} + I_{t-1} \\ &\Updownarrow \\ I_{t-1} &= \underbrace{H_t - H_{t-1}}_{\textit{net} \text{ investments}} + \underbrace{\delta_{t-1} H_{t-1}}_{\text{depreciation}} \end{aligned}$$

As with investments in general: There is a cost today (later we denote this π_{t-1}), and a later gain (that lasts for some time), in terms of an increase in $H_t, H_{t+1}, H_{t+2}, \ldots$ and associated benefits.

Benefits from good health
Let Ω be the total number of days (full time), TL be number of sick days, h the number of healthy days.
The higher health (H):
the more of healthy days, $h_t = G_t H_t$, where $G_t > 0$,
and the fewer number of sick days $TL = \Omega - h_{t.}$
In the model, all the effects of H goes through h.

1. Direct utility:
$$U(h_t, Z_t),$$
where $\partial U / \partial h_t = uh_t \cdot \frac{\partial h_t}{\partial H_t} = uh_t \cdot G_t$
2. More time for work (and leisure). (Non-sick leisure time is exogenous, so if $H \nearrow$, then more time for work at home or in the marketplace $\longrightarrow$ higher consumption.)

Household production functions
(3) $I = i(M_t, TH_t; E_t)$ – investments in health
(4) $Z = z(X_t, T_t; E_t)$ – other consumption goods
with inputs: market goods (M_t, X_t) and own time (TH_t, T_t).
You may think of M_t as medical care.

Resource constraints:

The time constraint

$$\underset{\text{full time}}{\Omega} = TW_t + \underbrace{TH_t + T_t}_{\text{time spent on household production}} + \underset{\text{time sick}}{TL_t}$$

How does $H \nearrow$ improve production/earnings opportunities? Only by reducing TL_t.
Using $h_t = \Omega - TL_t$, the time constraint can be written as

$$TW_t = h_t - TH_t - T_t$$

The income constraint

$$\Sigma_t \left(P_t M_t + Q_t X_t\right) / (1+r)^t = \Sigma \left(W_t TW_t\right) / \left(1+r\right)^t + A_0$$

Combining the time and the income constraints gives

$$\Sigma_t \left(P_t M_t + Q_t X_t\right) / (1+r)^t = \Sigma \left(W_t \left(h_t - TH_t - T_t\right)\right) / \left(1+r\right)^t + A_0 \quad \text{(5')}$$

Some **(restrictive?) assumptions**

No uncertainty
Perfect knowledge
Constant preferences
No joint production. For example, $\partial I / \partial X = \partial Z / \partial M = 0$. The actions that affect health, do not matter for other consumption goods.
Can freely choose TH, TW, M and X given $(5')$.

The individual as a **rational** producer of own health chooses to:
Maximize (1') given (5') and using equations (2-4).

$$\begin{aligned} L(...) \quad \equiv \quad & \Sigma_{t=0}^{n} u\left(h_t, Z_t\right) + \\ & \lambda \Sigma_{t=0}^{n} \frac{1}{\left(1+r\right)^t} \left[W_t \left(h_t - TH_t - T_t\right) - \left(P_t M_t + Q_t X_t\right)\right] \end{aligned} \quad (*)$$

Discounts future income streams - a krone today $(t-1)$ is more valuable than a krone later $(t, t+1, t+2, ...)$ because of $r > 0$.

Maximization – of interest to us: the determinants of I: M and TH (cf. equation 3).

The first order condition for gross health investment in period $t-1$, equations (9) and (10) follow from two first order conditions, one for M and one for TH.

Differentiate L wrt I, thinking of M as a function of I.

Differentiate L wrt I, thinking of TH as a function of I.

When using M_t as the choice variable:

$$\frac{1}{(1+r)^{t-1}} P_{t-1} \frac{\partial M_{t-1}}{\partial I_{t-1}} = \frac{W_t G_t}{(1+r)^{t-1}} + ... + \frac{Uh_t}{\lambda} G_t + (1-\delta_t) \ldots (1-\delta_{n-1}) \frac{Uh_n}{\lambda} G_n$$

Define $\pi_{t-1} = P_{t-1} \frac{\partial M_{t-1}}{\partial I_{t-1}}$. In equilibrium:

$$\pi_{t-1} = P_{t-1} \frac{\partial M_{t-1}}{\partial I_{t-1}} = W_{t-1} \frac{\partial TH_{t-1}}{\partial I_{t-1}}. \tag{10}$$

The derivation of the user cost of health

Then it is convenient to look at the optimal health stock in period t (H_t) rather than the gross investments.

The relation between H_t and $I_1, ..., I_n$:

$$\begin{aligned} I_{t-1} &= H_t - (1-\delta_{t-1}) H_{t-1}, \\ I_t &= H_{t+1} - (1-\delta_t) H_t \end{aligned}$$

while H_t does not enter in the expression for I_{t-j},for $j \neq 0, 1$.
That is, $\partial I_{t-1}/\partial H_t = 1$, and $\partial I_t/\partial H_t = -(1-\delta_t)$
(and $\partial I_{t-j}/\partial H_t = 0$ for $j \neq 0, 1$).

$$\begin{aligned} \partial L_t/\partial H_t = \lambda \frac{1}{(1+r)^{t-1}} &\left(-\underbrace{P_{t-1}\frac{\partial M_{t-1}}{\partial I_{t-1}}}_{\pi_{t-1}} \underbrace{\frac{\partial I_{t-1}}{\partial H_t}}_{=1} \right) + \\ &+ uh_t G_t + \lambda \frac{1}{(1+r)^t} \left(G_t W_t - \underbrace{P_t \frac{\partial M_t}{\partial I_t}}_{\pi_t} \underbrace{\frac{\partial I_t}{\partial H_t}}_{=-(1-\delta_t)} \right) = 0 \end{aligned}$$

$$\frac{\lambda}{(1+r)^{t-1}} \pi_{t-1} = uh_t G_t + \frac{\lambda}{(1+r)^t} (G_t W_t + \pi_t (1-\delta_t))$$

multiply by $(1+r)^t/\lambda$ and rearrange right hand side (RHS):

$$\pi_{t-1}(1+r) = G_t W_t + \frac{uh_t}{\lambda} G_t (1+r)^t + \pi_t (1-\delta_t)$$

$$\pi_{t-1}(1+r) - \pi_t(1-\delta_t) = G_t W_t + \frac{uh_t}{\lambda} G_t (1+r)^t$$

Rearrangeing the LHS: $\pi_{t-1}(1+r) - \pi_t(1-\delta_t) = \pi_{t-1}\left(1 + r - \frac{\pi_t}{\pi_{t-1}} + \frac{\pi_t}{\pi_{t-1}}\delta_t\right) =$
$\pi_{t-1}\left(\frac{\pi_{t-1}}{\pi_{t-1}} + r - \frac{\pi_t}{\pi_{t-1}} + \frac{\pi_t}{\pi_{t-1}}\delta_t\right) = \pi_{t-1}\left(r - \left(\frac{\pi_t}{\pi_{t-1}} - \frac{\pi_{t-1}}{\pi_{t-1}}\right) + \frac{\pi_t}{\pi_{t-1}}\delta_t\right) = \pi_{t-1}\left(r - \widetilde{\pi}_{t-1} + \frac{\pi_t}{\pi_{t-1}}\delta_t\right)$,
where $\widetilde{\pi}_{t-1} = \frac{\pi_t - \pi_{t-1}}{\pi_{t-1}}$. Grossman assumes that $\delta\widetilde{\pi}_{t-1}$ is very close to zero, then $\frac{\pi_t}{\pi_{t-1}}\delta_t \approx \delta_t$, and we arrive at equation (11):

$$\pi_{t-1}(r - \widetilde{\pi}_{t-1} + \delta_t) = G_t W_t + \frac{uh_t}{\lambda} G_t (1+r)^t \tag{11}$$

The RHS in (11) is the gain from a marginal increase in health stock in period t, H_t.
The LHS in (11), $\pi_{t-1}(r - \widetilde{\pi}_{t-1} + \delta_t)$, is the cost of a marginal increase in H_t.

For (11) to maximize utility, the optimal gross investments in all periods $t = 0, ..., n-1$, must be positive. (There must be an inner solution to the Lagrange problem (*).) $\Leftrightarrow$ The individual would never want to reduce his health by $\delta_t H_t$ or more.
Even if health capital cannot be sold (there is no second-hand market for health, in contrast to physical capital), one may then think of the LHS in eq 11 as a user cost of capital.

The true cost is not π_{t-1} (the monetary expense in period $t-1$of investing in H_t), but the sum of the following cost components

- foregone income, $\pi_{t-1} \times r$, from not placing an amount π_{t-1} in the capital market,
- and the monetary loss from depreciation $\pi_{t-1} \times \delta$,
- and, possibly, price changes from $t-1$ to t. Price increases (P or W) from $t-1$ to t, that is $\widetilde{\pi}_{t-1} > 0$, reduce the user cost of H_t.

Dividing by π_{t-1} on both sides of eq. 11,

$$r - \widetilde{\pi}_{t-1} + \delta_t = \underbrace{G_t W_t / \pi_{t-1}}_{\gamma_t} + \underbrace{\frac{uh_t}{\lambda} G_t \left(1+r\right)^t / \pi_{t-1}}_{a_t}. \qquad (24)$$

$r - \widetilde{\pi}_{t-1} + \delta_t$ is the user cost per krone spent on H_t, and $\gamma_t + a_t$ the marginal gain per krone. γ_t is the productive value of more healthy days, and a_t the direct utility gain from better health.

Grossman derives two models from equation 24,
in the **investment model,** health only matters for the individual's productive capacity (the number of days she can work), thus $a_t = 0$,
in the **consumption model**, health has only a direct utility effect ($\gamma_t = 0$).

Comparative statistics: We will mainly use graphs, and concentrate on the investment model.