## Exam 4620 spring 2022

1. Short true, false or uncertain statements - explain briefly your answer. (total $30 \%$, same weight on each)
(a) Standard economic models imply that capital taxes fully fall on labor. Wrong. The extent to which capital taxes fall on labor depends on the elasticity of capital supply, the elasticity of labor supply, and the elasticity of substitution between capital and labor.
(b) It is optimal for the government to impose high commodity taxes on price inelastic goods.
True if the goal is to minimize the efficiency loss associated with commodity taxation. But distributional concerns may induce the government to set lower taxes on inelastic goods
(c) A government that maximizes a utilitarian social welfare function would impose a tax schedule such that every person obtains the same level of consumption.
Only true if individual income is fixed. If income is chosen by individuals that trade off leisure against consumption a income tax will create efficiency losses that must be traded off against equal consumption
(d) Suppose individuals (indexed $i$ ) have private information about their risk $0<\gamma_{i}<1$ of loosing their income. This risk is exogenously given (no moral hazard). In this case it is always better to have public mandatory social insurance than private voluntary insurance. This is true if there are no administrative costs associated with social insurance. With administrative costs mandatory insurance may lead "over insurance". The loss of insuring individuals who have a willingness to pay that is lower than the costs must be compared with under insurance due to adverse selection in a private market. The private solution may then be better
(e) The government should always carry out projects where the aggregate willingness to pay is larger than the costs of the project.
Not true. Rich individuals have a low marginal value of money and are willing to pay a lot for a project A that benefits them but that imposes a cost on poor individuals who are - because they have a high marginal value of money - are not willing to pay a lot to not have A. A government may want to weigh down the rich individuals willingness to pay.
2. Income taxation ( $50 \%$, weights indicated after each question) Consider a static (one period) society in which a person $i$ who earns income $z_{i}=w_{i} l_{i}$ pays an income tax $T\left(z_{i}\right)$, where $w_{i}$ is the wage rate and $l_{i}$ is the hours a person works. The wage $w_{i}$ is given, equal to the innate ability of a person, and $l_{i}$ is chosen by $i$. People are born with different abilities
(wages). Suppose all individuals have the same utility function $u\left(c_{i}, z_{i}\right)$ with $c_{i}=z_{i}-T\left(z_{i}\right)$. The utility function is increasing and concave in $c$ and decreasing and concave in $z$.
(a) Draw indifference curves for a person $H$ with a high wage $\left(w_{H}\right)$ and a person $L$ with low wage $\left(w_{L}<w_{H}\right)$ in a diagram with $c$ on the vertical axis and $z$ on the horizontal axis. Explain why for any point in this diagram that $H$ has flatter indifference curves than $L$. (10pts) This is the single crossing condition. Consider a point $c^{0}, z^{0}$ in this plane and ask how much $c$ an individual would require to stay at the same utility level if she had to increase income with one unit. A $H$-individual would require a lower increase in $c$ than a $L$-individual since $H$ has to increase labour supply with $\frac{1}{w_{H}}$ unit while a $L$-individual would have to increase labour with $\frac{1}{w_{L}}>\frac{1}{w_{H}}$.
(b) Draw a progressive income tax schedule $T(z)$ in this diagram. (5 pts) With a progressive tax scheme the amount of taxes payed increases (weakly) with the pre-tax income, that is $\frac{T(z)}{z}$ increases in $z$. Figure
(c) The government wants to redistribute income from $H$ to $L$. Does it matter whether the government can observe $w_{i}$ or only can observe $z_{i}$ ? Explain your answer. ( 15 pts )
Matters a lot. If the government observes exogenously given $w$ they can redistribute without thinking about incentive effects of the tax. If only the endogenously chosen $z$ can be observed there is efficiency losses associated with taxation.
(d) Suppose the tax schedule $T(z)$ is piecewise linear. It has three income brackets $j=(1,2,3)$. For income from 0 to $z_{1}>0$ the tax rate $\tau_{1}=0$. For income between $z_{1}$ and $z_{2}>z_{1}$ the tax rate is $\tau_{2}>0$ and for the top bracket $z>z_{2}$ the tax rate is $\tau_{3}>\tau_{2}$. Draw this tax schedule in a diagram with $c$ on the vertical axis and $z$ on the horizontal axis. (5 pts)
Figure
(e) Suppose the government wants to increase the top bracket from $\tau_{3}$ to $\tau_{3}+\triangle$. Illustrate this reform in a diagram and explain how this will change the social welfare in the economy. (hint use the perturbation argument and show that there are effects on government revenue and on the income and utility of income earners and on social welfare) (45 pts)
Reform: Introduce a small increase in $\tau_{3}$ to $\tau_{3}+\triangle$ for income above $z_{2}$, suppose government use revenue to redistribute lump sum. The effect of $\triangle$ can be decomposed into two effects. First there is an effect on government budget (on the tax income generated), second there is an effect on social welfare, coming from the fact that those at the top have to pay more taxes.

The effect on government budgets can be decomposed into a mechanical effect (a higher tax will increase tax income if behavior is not changed) and a behavioral effect coming from the fact that the top earners will change their pretax income $z$ is a function of $(1-\tau)$. The effect on government budgets. An increase in the tax rate at the top The effect on government revenue of small $\triangle>0$ reform above $z_{2}$ can be decomposed into two effects

$$
d R=d M+d B
$$

i. Mechanical increase in tax revenue (effect if individuals do not change behavior)

$$
d M=\triangle\left(z^{m}-z_{2}\right) N
$$

where $z^{m}$ is the average income of those with income above $z_{2}$. And $N$ is the number of individuals in this bracket.
ii. Behavioral response comes from the fact that those who face the increase in taxes from $\tau_{3}$ to $\tau_{3}+\triangle$ will adjust their income ( $\varepsilon$ is the compensated and uncompensated elasticity - no income effect)

$$
d B=\tau d z^{m} N=-\tau \frac{d z^{m}}{d(1-\tau)} d \tau N=-\varepsilon z^{m} \frac{\tau}{1-\tau} d \tau N
$$

Welfare effect, two terms
1 Reduce utility of those who have to pay more taxes
Those who pay more tax have chosen their labour supply optimally $\Longrightarrow$ there is no first order utility loss of changing their labour supply a little bit
$\Longrightarrow$ the top earners loose utility only through the direct effect (they have to pay more taxes)
$\Longrightarrow$ The monetary value of their utility loss is equal to $d M$ by the envelope theorem
2 How this matters for welfare depend on the relative welfare weight these individuals are givenThe government values this loss at $\bar{g}$

$$
d W=-\bar{g} d M=-\bar{g}\left(d \tau\left(z^{m}-\bar{z}\right) N\right)
$$

(f) Assume alternatively that the government makes a small increase in $\tau_{2}$, would that effect the behavior of those who prior to the reform earn income above $z_{2}$ ? ( 20 pts )
They do not experience a change in the marginal tax, so no price effect. But they will experience a loss in income. A loss in income will - if leisure is a normal good - induce them to increase their earnings (increase labor supply)
3. Distributional effects of wealth taxation. (20\%) The graph below shows the distribution of the annual wealth tax as share of gross income for four different rankings: yearly individual income, yearly household income, household income over a longer time period (8-19 years), and household income over the lifetime.
(a) Household income is obtained by the use of an equivalence scale. Explain what an equivalence scale is.
Equivalence scale is used to compare the economic wellbeing in households of different size when there are economies of scale in the household: heating, one TV, etc. The students have be introduced to one possible representation of a scale with the following form: $S=\left(n_{a}+\theta n_{c}\right)^{\alpha}$, where $n_{a}$ is the number of adults, $n_{c}$ the number of children and $\theta$ is the weight attached to children. $\alpha \in[0,1]$ represents the assumption of economies of scale in the household. If we do not differentiate between adults and children and use $\alpha=0.5$, we get that the economic wellbeing for a single-adult household with income NOK 100,000 is assumed to be equivalent to household income of NOK 200,000 for a 4-person household $(200000 / \sqrt{4}=100000)$. This formula is referred to in Herault and Creedy (2012), on the reading list.
(b) The graph shows the distribution of the same measure of the tax burden, annual wealth tax over gross income, for four different income concepts. The tax burden for decile 1 according to the "annual individual income"concept (annual income ranking) is relatively high. Explain why the tax burden is higher when the ranking is based on "annual individual income"than for the other income concepts.
The is graph has been presented and discussed during one of the lectures. The key to a good answer here is to demonstrate that one has understood that rankings differ with respect to the different income concepts, i.e., that persons/households found in decile 1 vary. So why is the tax burden so high in decile 1 for annual individual income? An obvious reason is that one finds a person who has relatively high wealth but also a "bad year" in terms of income. For example a business-owner, with high wealth and a corresponding wealth tax burden, may have low annual income due to losses that particular year. For household income, the person can be moved out of decile 1 , because the income of the other household members determine the ranking. Then the wealth tax share in decile 1 goes down. For income over a longer period or lifetime income, the person with a "bad year" (who pays the wealth tax) may well be higher ranked (and found in other deciles), as his/her permanent income likely is higher. Another example of a person who could be found in decile 1, according to annual income, is a person with relatively low pension income. During the lifetime, the person has accumulated wealth to the extent that he/she pays wealth tax, but yearly (pension) income could be low. He/she shares wealth and income with a person with higher income (for example, a pensioner with a higher pension), and could therefore move out of decile 1 when income is
measured at the household level. Also, the household may have relatively high income over a longer time period, which means that they are not found in decile 1 for the other income concepts. A third group in decile 1 (annual invidual income) could be a relatively young person with low income and high wealth. Then wealth (most likely) has been inherited and income over time is higher.


Fig. 2. Annual wealth tax as a share of income, for rankings (in deciles) by different income concepts

