Note on optimal income tax rate in the top income bracket.

Suppose that the marginal tax rate in the top income bracket beyond the income level $\overline{Z}$ is set equal to τ . Let the average income in the bracket be *Z* and let *N* be the number of taxpayers in this interval. Define

$$e = \frac{dZ}{d(1-\tau)}\frac{1-\tau}{Z} = -\frac{dZ}{d\tau}\frac{1-\tau}{Z}$$

$$\frac{dZ}{d\tau} = -\frac{Z}{1-\tau}e$$

Let *g* be the average welfare weight assigned to the taxpayers in the top bracket, where *g* is measured in terms of government revenue.

Increase τ by $d\tau$. The increase in the taxpayments in the top bracket neglecting behavioural changes is then $\left(Z-\overline{Z}\right)d\tau N$. The welfare effect of transferring this tax payment from the taxpayers to the government is then $\left(1-g\right)\left(Z-\overline{Z}\right)d\tau N$ .The behavioural effect on tax revenue is

$$\frac{dZ}{d\tau}N\tau d\tau = -\frac{Z}{1-\tau}eN\tau d\tau = -\left(Z-\overline{Z}\right)\frac{\tau}{1-\tau}eN\frac{Z}{Z-\overline{Z}}d\tau = -\left(Z-\overline{Z}\right)\frac{\tau}{1-\tau}eaNd\tau$$

where by definition $a = \dfrac{Z}{Z-\overline{Z}}$.

The overall welfare effect is then

$$\left(1-g\right)\left(Z-\overline{Z}\right)d\tau N - \left(Z-\overline{Z}\right)\frac{\tau}{1-\tau}eaNd\tau$$

$$= \left(Z-\overline{Z}\right)d\tau N\left[1-g-ea\frac{\tau}{1-\tau}\right]$$

The first order condition for the optimal value of τ is then

$\left(Z-\overline{Z}\right)d\tau N\left[1-g-ea\dfrac{\tau}{1-\tau}\right] = 0$, that is,

$$1-g-ea\frac{\tau}{1-\tau} = 0$$

Then $\tau = \dfrac{1-g}{1-g+ae}$