Solution to selected seminar questions

Seminar 1, question 7 First, the problem was a bit sloppy formulated. I apologize for that. In (a) I wanted you to set up the problem to find the compensated demand for leisure (labour). In (d) I wanted you to use a Taylor approximation to find an expression of the relative excess burden.

Solution An individual derives utility by consuming commodities (c) and leisure (l); $u(c,l)$. The wage rate is w, the price of goods is 1, time endowment for leisure and work (L) is T and y is non-work income. The primal problem is to maximize $u(c,l)$ subject to $c = w(T-l)+y$. The dual problem is to minimize $y = c + wl - wT$ subject to $u(c,l) = U$. Substituting the optimal choice of leisure (labour) into the minimand gives the expenditure function $E(w,U)$.

(a) The compensated demand for leisure is denoted $l^c(w,U)$ and we find it by taking the derivative of the expenditure function with respect to w (we then get $l^c - T = -L^c$)

(b) The government imposes a tax on (the fixed) wage. Denote the initial wage w^0, the after tax wage is $(1-t)w^0 = w^1$. Suppose the consumer obtains utility U^1 after the tax is imposed. The equivalent variation is given by $EV = E(w^1,U^1) - E(w^0,U^1)$. Since the derivative of the expenditure function is equal to $-L^c(w,U^1)$ we can write $EV = -\int_{w^1}^{w^0} L^c(w,U^1)dw$.

(c) Draw

(d) Using a first order Taylor approximation to express the compensated labour supply we obtain

$$L^c(w,U^1) \approx L^c(w^1,U^1) + \frac{L^c(w^1,U^1)}{dw}\Big|_{w=w^1}(w-w^1)$$

Using this in the expression of the equivalent variation and solving the integral

$$EV \approx L^c(w^1,U^1)(w^0-w^1) + \frac{L^c(w^1,U^1)}{dw}\Big|_{w=w^1}(w-w^1)\ \frac{(w^0-w^1)^2}{2}$$

using the fact that $w^0 - w^1 = tw^0$

$$EV \approx L^c(w^1,U^1)tw^0 + \frac{L^c(w^1,U^1)}{dw}\Big|_{w=w^1}(w-w^1)\ \frac{(tw^0)^2}{2}$$

We know that the excess burden is equal to the EV minus the tax income given the compensated labour supply.

$$EB \approx L^c(w^1,U^1)tw^0 + \frac{L^c(w^1,U^1)}{dw}\Big|_{w=w^1}(w-w^1)\ \frac{(tw^0)^2}{2} - L^c(w^1,U^1)tw^0$$

The first and the last term cancel (explain why). If we express EB as a percentage of tax revenue we get

$$\frac{EB}{L^c(w^1,U^1)tw^0} \approx \frac{\frac{L^c(w^1,U^1)}{dw}|_{w=w^1}(w-w^1)\ \frac{\left(tw^0\right)^2}{2}}{L^c(w^1,U^1)tw^0}$$

Rearranging and using the formula for the compensated elasticity we obtain

$$\frac{EB}{L^c(w^1,U^1)tw^0} \approx \frac{1}{2}\varepsilon_L^c \frac{t}{1-t}$$

Hence of the compensated elasticity is 0.64 and the tax rate is 0.4 the excess burden is 21% of the revenue collected.

Seminar 2, question 2 Let us first rewrite the formula for optimal taxation with a representative household. The first order condition for optimal taxes can be written as (recall that $q_i - p_i = q_i - 1 = \tau_i$)

$$-\sum_j \tau_j \frac{\partial h_j}{\partial q_i} = \frac{\theta}{\lambda} x_i \ \forall i \tag{1}$$

The left hand side in (1) is equal to the increase in excess burden of taxation of introducing a tax on good i (it is equal to $\frac{dEB}{d\tau_i}$ when there is no initial tax on good i). The marginal increase in revenues by introducing a tax on good i is given by $\frac{dR}{d\tau_i} = x_i + \sum_j \tau_j \frac{\partial h_j}{\partial q_i} \implies x_i = \frac{dR}{d\tau_i} - \frac{dEB}{d\tau_i}$, we can therefore write the right hand side of equation (1) as $\frac{\theta}{\lambda}\left(\frac{dR}{d\tau_i} - \frac{dEB}{d\tau_i}\right)$ this then means that a first order condition for the optimal tax structure imply that the excess burden per NOK in revenue is equal across all taxed goods

$$\frac{\frac{dEB}{d\tau_i}}{\frac{dR}{d\tau_i}} = \frac{\lambda}{\lambda + \theta}.$$

(this is quite intuitive: if the excess burden of collecting one NOK in revenue was lower for i than for j one should increase the tax rate on i and lower it on j, it is equivalent to optimality in consumer theory where the marginal utility of per NOK spent on a good should be equal across all goods ($\frac{\frac{\partial U}{\partial x_i}}{p_i} = \alpha \ \forall i$)

Heterogeneous househholds This rule is gone be modified when distributional concerns are introduced, that is, when households differ and a change in their consumption is assigned different social value. When the government evaluates a tax policy according to the welfare functions $W\left(V^1(\mathbf{q}), V^2(\mathbf{q})...V^H(\mathbf{q})\right)$ we should expect that the optimal tax structure will be modified by the fact that different households have different marginal utility of money ($\alpha^j \neq \alpha^k$) and different welfare weights ($\frac{\partial W}{\partial V^j} \neq \frac{\partial W}{\partial V^k}$) .

If a good i is consumed disproportionally much by individuals who have a high marginal utility of money (because they are poor) we expect the tax rate to be adjusted downwards compared to the optimal policy when only efficiency matters. This is exactly what will happen.

With heterogenous households the first order condition for optimal tax structure is given by

$$\sum_h \frac{\partial W}{\partial V^h}\frac{\partial V^h}{\partial q_i} + \lambda\left[\sum_h x_i^h + \sum_j \tau_j \sum_h \frac{\partial x_j}{\partial q_i}\right] = 0 \tag{2}$$

Using the envelope result $\frac{\partial V^h}{\partial q_i} = -\alpha^h x_i^h$, the Slutsky equation (decomposing the price effect on demand into a substitution and income effect (z is income) and letting $\sum_h \frac{\partial h_j^h}{\partial q_i} = H_{ji}$ and $X_i = \sum_h x_i^h$ we can write (2) as

$$-\sum_j \tau_j H_{ji} + X_i\left[\lambda - \left(\frac{\sum_h x_i^h\left(\frac{\partial W}{\partial V^h}\alpha^h + \lambda\sum_j \tau_j \frac{\partial x_j^h}{\partial z^h}\right)}{X_i}\right)\right] = 0 \tag{3}$$

The term $\frac{\sum_h x_i^h\left(\frac{\partial W}{\partial V^h}\alpha^h + \lambda\sum_j \tau_j \frac{\partial x_j^h}{\partial z^h}\right)}{X_i} = \beta_i$ is the social marginal welfare of income associated with good i. It is the social value of a marginal increase in income for household h :$\beta^h = \left(\frac{\partial W}{\partial V^h}\alpha^h + \lambda\sum_j \tau_j \frac{\partial x_j^h}{\partial z^h}\right)$ times the households share of the consumption of this good $\frac{x_i^h}{X_i}$. We can now write (3) as

$$-\sum_j \tau_j H_{ji} = \frac{\lambda - \beta_i}{\lambda} X_i \tag{4}$$

Comparing (4) with (1) we can see that the right hand side is no longer independent of i: It depends on the social marginal welfare of income associated with good i: β_i. Hence if we rewrite this equation as the ratio between the marginal excess burden of an increase in tax i and the marginal revenue of an increase in tax i we get

$$\frac{\frac{dEB}{d\tau_i}}{\frac{dR}{d\tau_i}} = \frac{\lambda - \beta_i}{\beta_i}$$

Which means it is no longer optimal to set the marginal excess burden equal for all sources of revenue; with heterogenous households we will adjust taxes according to which households that consume the good; goods with a high correlation between $\frac{x_i^h}{X_i}$ and β^h will have a high β_i and a lower tax is optimal.

Try to find an expression for the modified Ramsey rule (taxes expressed as a function own and cross compensated price elasticities) in a case with three goods (consult Boadway for a more complete exposition). Hint: Use the symmetry of the Slutsky matrixes and the fact that compensated demand is homogenous of degree 0)