

## Seminar 6 in Public Economics 4620 – 6<sup>th</sup> May

**Problem 1** Define the following concepts:

- a) Lump sum tax
- b) Poll tax
- c) Marginal tax
- d) Progressive income tax (Show that the tax is progressive if the marginal tax exceeds the average tax.)
- e) First best allocation
- f) Second best allocation
- g) Tax distortion

**Problem 2** Consider an economy with  $n$  identical individuals. Each has a utility function

$$u^h(x^h, l^h, G)$$

where  $x^h$  is consumption of private goods,  $l^h$  is the consumption of leisure ( $H - l^h = L^h$  is the amount of labour household  $h$  supplies) and  $G$  are the units of public goods it consumes (same for all households). Suppose there is a linear technology in the economy and the price of leisure (wage) is numeraire. The overall resource constraint in the economy is

$$p \sum_h x^h + p_G G = \sum_h L^h$$

Since all households are identical we can write

$$pHx + p_G G = HL$$

( $x$  is each household's consumption of goods and  $L$  its supply of labour)

Suppose the public good can be financed by a combination of a lump sum tax  $T$  on each household, and a per unit tax  $t$  on the consumption good.

1. Write down the budget constraint for the household (assume labour income is the only income)
2. Find the first order conditions for the optimal choice of labour and consumption for the household.
3. The government maximizes the sum of individual welfare,  $Hu$  (identical households). Suppose first that the public good can be financed by a lump sum tax ( $T$ ). Show that optimal supply of public goods implies that the sum of the marginal substitution rate between the public good and some private good (leisure for example) should be

equal to the marginal transformation rate between these goods ( $p_G$  if we compare with leisure). This is the Samuelson rule for optimal public good provision.

4. How is this condition changed when the public good is financed through a distortionary tax on consumption.

**Problem 3** Consider an economy where the government sets a flat tax at rate  $\tau$  on earnings to raise revenue. Individual  $i$  earns gross income  $y_i = y_i^0(1 - \tau)^\varepsilon$ , where  $y_i^0$  independent of taxation and is called potential income.  $\varepsilon$  is a positive parameter equal for all individuals in the economy.

1. Show that  $\varepsilon$  is the elasticity of income with respect to the net-of-tax rate  $1 - \tau$
2. Is this the compensated or uncompensated elasticity?
3. Show that the tax rate maximizing total tax revenue is equal to  $\tau^* = \frac{1}{1+\varepsilon}$  and explain why it decreases in  $\varepsilon$ .