Optimal taxation of consumer goods

(the optimal structure of indirect taxes)

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The structure of optimal (tax) policy

- A two stage problem
  - Households maximize utility given a tax policy; $\mathbf{x}(\boldsymbol{\tau})$ is the optimal behavioral response to policy $\boldsymbol{\tau}$.
  - A benevolent government maximizes household welfare (efficiency and equity concerns) given a revenue requirement R and the behavioral response $\mathbf{x}(\boldsymbol{\tau})$.

- Two approaches
  - **Ramsey**: Lump sum taxes are ruled out by assumption; what is the optimal linear tax-policy - what is the optimal structure of tax rates on different commodities. (indirect taxes: tax individual indirectly by taxing their transactions)
  - **Mirrlees**: Lump sum taxes in principle possible, but constrained by information problems (cannot observe the income potential of individuals). Find optimal non-linear policy given redistribution and efficiency concerns and *given* information constraints

The Ramsey model

the problem

- $N + 1$ goods. Households derive utility from consuming these goods.
- One good (leisure) cannot be taxed: The problem is to find optimal tax rates on the other goods given a revenue requirement R. (if all goods could be taxed, the policy problem is trivial - equal tax-rate on all goods would leave relative prices unchanged and is equivalent to a non-distortionary lump sum tax).
- It is an important problem; 28% of all government revenues comes from indirect taxation (VAT, excise duties and customs duties)

The Ramsey model

the households problem

- Utility: $U = u(x_1, ..x_N, l)$
- Pre-tax prices are fixed (normalized to 1): $q_i = 1 + \tau_i$ and wage is given w and time endowment H
- The budget constraint $\sum_i (q_i x_i) = z + (H - l)w$

$$Max\ L_H = u(x_1, ..x_N, l) + \alpha \left[z + (H - l)w - \sum_i (q_i x_i) \right]$$

  - The first order conditions:

$$u'_{x_i} - \alpha q_i = 0$$

    - insert optimal choices into utility function to obtain the indirect utility: $V(\mathbf{q}, w, z)$.

The Ramsey model

the governments problem

- A benevolent government

$$Max\ L_G = V(\mathbf{q}, w, z) + \lambda \left[\sum_i (\tau_i x_i) - R \right]$$

FOC:

$$\frac{\partial V}{\partial q_i} + \lambda \left(x_i + \sum_j \tau_j \frac{\partial x_j}{\partial q_i} \right) = 0$$

using $\frac{\partial V}{\partial q_i} = -\alpha x_i$

$$(\lambda - \alpha) x_i + \lambda \sum \tau_j \frac{\partial x_j}{\partial q_i} = 0$$

The Ramsey model

the governments problem

- Slutsky $\frac{\partial x_j}{\partial q_i} = \frac{\partial h_j()}{\partial q_i} - x_i \frac{\partial x_j}{\partial z}$ and rearranging, using the fact that the substitution effect is symmetric $\frac{\partial h_j}{\partial q_i} = \frac{\partial h_i}{\partial q_j}$ gives

$$\frac{1}{x_i} \sum_j \tau_j \frac{\partial h_i}{\partial q_j} = \frac{-\left(\lambda - \alpha - \lambda \frac{\partial}{\partial z}\left(\sum_j \tau_j x_j\right)\right)}{\lambda} \quad (1)$$

- N equations, one for each taxed good: rhs is independent of $i \implies$ same for all i.

- $\lambda - \alpha - \lambda \frac{\partial}{\partial z}\left(\sum_j \tau_j x_j\right) = \theta =$ the effect of introducing a 1 NOK lump sum tax on households ¨

- lhs is (approximately) the percentage drop in demand (discouragement) caused by a tax system ($\boldsymbol{\tau}$)

- Ramsey tax formula says that the indexes of discouragements must be equal across goods at the optimum

Interpretation of the Ramsey model

- the optimal tax rule on elasticity form

$$\sum_j \frac{\tau_j}{1+\tau_j}\varepsilon^c_{ij} \quad = \quad -\frac{\theta}{\lambda}$$

- Suppose there are two taxed good, indexed 1 and 2 and leisure is good 0. We get

$$\frac{\tau_1}{1+\tau_1}\varepsilon^c_{11} + \frac{\tau_2}{1+\tau_2}\varepsilon^c_{12} = -\frac{\theta}{\lambda}$$
$$\frac{\tau_1}{1+\tau_1}\varepsilon^c_{21} + \frac{\tau_2}{1+\tau_2}\varepsilon^c_{22} = -\frac{\theta}{\lambda}$$

$\Rightarrow \frac{T_1}{T_2} = \frac{\varepsilon^c_{22}-\varepsilon^c_{12}}{\varepsilon^c_{11}-\varepsilon^c_{21}} = \frac{\varepsilon^c_{22}+\varepsilon^c_{10}+\varepsilon^c_{11}}{\varepsilon^c_{22}+\varepsilon^c_{20}+\varepsilon^c_{11}}$, with $T_i = \frac{\tau_i}{1+\tau_i}$
(the second equality comes from the fact that compensated demand is homogenous of degree 0: $\varepsilon^c_{10}+\varepsilon^c_{11}+\varepsilon^c_{12}=0$)

Interpretation of the Ramsey model

- $\frac{T_1}{T_2} = \frac{\varepsilon_{22}^c - \varepsilon_{12}^c}{\varepsilon_{11}^c - \varepsilon_{21}^c}$: if cross derivatives among taxed good are zero we get the inverse elasticity rule, a higher tax on goods that have an inelastic compensated demand.

- $\frac{T_1}{T_2} = \frac{\varepsilon_{22}^c + \varepsilon_{20}^c + \varepsilon_{11}^c}{\varepsilon_{22}^c + \varepsilon_{10}^c + \varepsilon_{11}^c}$: since $\varepsilon_{11}^c + \varepsilon_{22}^c < 0$ we have $T_1 > T_2$ if good 1 is relatively more complementary with leisure than good 2: $\varepsilon_{20} > \varepsilon_{10}$

- A high tax on goods that are complementary with leisure will reduce demand for leisure (and households consume to much leisure at the outset since income (consumption) is taxed).

- *second best logic:* when there is a distortion in one market (leisure is not taxed) it may be beneficial to introduce additional distortions (between good 1 and 2).

- note that a uniform tax on consumption goods is optimal if all the taxed goods have the same compensated cross price elasticity with the price of leisure (Corlett, W.J., Hague, D.C. (1953). Complementarity and the Excess Burden of Taxation. Review of Economic Studies, 21(1), 21-30.)

Heterogenous households

- This rule will be modified when distributional concerns are introduced, that is, when households differ and a change in their consumption is assigned different social value.
- When the government evaluates a tax policy according to the welfare functions $W\left(V^1(\mathbf{q}), V^2(\mathbf{q})...V^H(\mathbf{q})\right)$ we should expect that the optimal tax structure will be modified by the fact that different households have different marginal utility of money $(\alpha^j \neq \alpha^k)$ and perhaps also different welfare weights $(\frac{\partial W}{\partial V^j} \neq \frac{\partial W}{\partial V^k})$.
- If a good i is consumed disproportionally much by individuals who have a high marginal utility of money (because they are poor) we expect the tax rate to be adjusted downwards compared to the optimal policy when only efficiency matters. This is exactly what will happen.

Heterogenous households

- With heterogenous households the first order condition for optimal tax structure is given by

$$\sum_h \frac{\partial W}{\partial V^h} \frac{\partial V^h}{\partial q_i} + \lambda \left[\sum_h x_i^h + \sum_j \tau_j \sum_h \frac{\partial x_j}{\partial q_i} \right] = 0 \qquad (2)$$

- Using the envelope result $\frac{\partial V^h}{\partial q_i} = -\alpha^h x_i^h$, the Slutsky equation (decomposing the price effect on demand into a substitution and income effect (z is income) and letting $\sum_h \frac{\partial h_j^h}{\partial q_i} = H_{ji}$ and $X_i = \sum_h x_i^h$ we can write (2) as $-\sum_j \tau_j H_{ji} + X_i \left[\lambda - \left(\frac{\sum_h x_i^h \left(\frac{\partial W}{\partial V^h} \alpha^h + \lambda \sum_j \tau_j \frac{\partial x_j^h}{\partial z^h} \right)}{X_i} \right) \right] = 0$

Heterogenous households

- The term $\frac{\sum_h x_i^h \left(\frac{\partial W}{\partial V^h} \alpha^h + \lambda \sum_j \tau_j \frac{\partial x_j^h}{\partial z^h} \right)}{X_i} = \beta_i$ is the social marginal welfare of income associated with good i. It is the social value of a marginal increase in income for household h :$\beta^h = \left(\frac{\partial W}{\partial V^h} \alpha^h + \lambda \sum_j \tau_j \frac{\partial x_j^h}{\partial z^h} \right)$ times the households share of the consumption of this good $\frac{x_i^h}{X_i}$.
- We can now write the first order condition as

$$-\frac{1}{x_i} \sum_j \tau_j H_{ji} = \frac{\lambda - \beta_i}{\lambda}$$

- Compared with equation (1) we can see that the right hand side is not independent of i anymore. It depends on the social marginal welfare of income associated with good i: β_i.

The relevance of the Ramsay model

It is not easy to implement an optimal differentiated consumption tax in the spirit of Ramsey since it is difficult to estimate all the parameters that are needed to design an optimal system. The model should be considered as a framework that:

1. specifies the forces that matter for designing a tax system that minimizes the efficiency loss, and
2. illustrates how the optimality conditions are altered when distributional objectives are incorporated in the model