Seminar 1, question 7

In (a) you should set up the problem to find the compensated demand for leisure (labour). In (d) I ask you to use a Taylor approximation to find an expression of the relative excess burden. I do not ask you directly or explicitly, but you could figure it out since we spent some time on linear approximation of the EB in the lectures.

Solution An individual derives utility by consuming commodities (c) and leisure (l); $u(c, l)$. The wage rate is w, the price of goods is 1, time endowment for leisure and work (L) is T and y is non-work income. The primal problem is to maximize $u(c, l)$ subject to $c = w(T-l)+y$. The dual problem is to minimize $y = c+wl-wT$ subject to $u(c, l) = U$. Substituting the optimal choice of leisure (labour) into the minimand gives the expenditure function $E(w, U)$.

(a) The compensated demand for leisure is denoted $l^c(w, U)$ and we find it by taking the derivative of the expenditure function with respect to w (we use the envelope thm and obtain $\frac{\partial E}{\partial w} = l^c - T = -L^c(w, U)$. Which is pretty obvious; if a person works 8 hours and the wage increase by one unit the exogenous income can be reduced with 8 NOK if the wage increses with 1 NOK.

(b) The government imposes a tax on (the fixed) wage. Denote the initial wage w^0, the after tax wage is $(1-t)w^0 = w^1$. Suppose the consumer obtains utility U^1 after the tax is imposed. The equivalent variation is given by $EV = E(w^1, U^1) - E(w^0, U^1)$. Since the derivative of the expenditure function is equal to $-L^c(w, U^1)$ we can express the

$$EV = -\int_{w^1}^{w^0} L^c(w, U^1) dw \tag{1}$$

(c) Draw (we did it on the black board)

(d) We do not know the compensated labour supply function (can it be backward bending?) But we can find a linear approximation of the compensated labour supply around the after tax wage w_1.

$$L^c(w, U^1) \approx L^c(w^1, U^1) + \frac{L^c(w^1, U^1)}{dw}\Big|_{w=w^1}(w - w^1)$$

Plugging this approximation into (1) we obtain

$$EV \approx -\left(\int_{w^1}^{w^0} L^c(w^1, U^1) + \frac{L^c(w^1, U^1)}{dw}\Big|_{w=w^1}(w - w^1)\right) dw$$

solving the integral (remember that both $L^c(w^1, U^1)$ and $\frac{L^c(w^1, U^1)}{dw}\big|_{w=w^1}$ are constants) we obtain

$$EV \approx -L^c(w^1, U^1)(w^0 - w^1) - \frac{L^c(w^1, U^1)}{dw}\Big|_{w=w^1} \frac{(w^0 - w^1)^2}{2}$$

using the fact that $w^0 - w^1 = tw^0$ we can write

$$EV \approx -L^c(w^1, U^1)tw^0 - \frac{L^c(w^1, U^1)}{dw}|_{w=w^1} \frac{(tw^0)^2}{2}$$

We can see that the $EV < 0$ - to keep the consumers on the same utility level the government can take money away from the consumers if they remove the tax.

We know that the excess burden is equal to the the absolute value of EV minus the tax income given the compensated labour supply.

$$EB \approx L^c(w^1, U^1)tw^0 + \frac{L^c(w^1, U^1)}{dw}|_{w=w^1} \frac{(tw^0)^2}{2} - L^c(w^1, U^1)tw^0$$

The first and the last term cancel (explain why). If we express EB as a percentage of tax revenue we get

$$\frac{EB}{L^c(w^1, U^1)tw^0} \approx \frac{1}{2} \frac{\frac{L^c(w^1, U^1)}{dw}|_{w=w^1} (tw^0)^2}{L^c(w^1, U^1)tw^0} = \frac{1}{2} \frac{L^c(w^1, U^1)}{dw}|_{w=w^1} \frac{w^1}{L^c(w^1, U^1)tw^0} \frac{(tw^0)^2}{w^1}$$

Rearranging and using the formula for the compensated elasticity we obtain

$$\frac{EB}{L^c(w^1, U^1)tw^0} \approx \frac{1}{2} \varepsilon_L^c \frac{t}{1-t}$$

Hence of the compensated elasticity is 0.6 and the tax rate is 0.4 the excess burden is 20% of the revenue collected.