## Seminar 2 (Monday 14.03.2016)

- 1. In the lectures we analyzed a model of optimal commodity taxes when all households are equal
  - (a) Why is it inconsequent to assume away lump sum taxes in this model.
  - (b) Assume there are H different households in the economy and N+1 different commodities. The government can tax N commodities and is searching for a tax vector  $\boldsymbol{\tau}$  that maximizes  $W\left(V^1(\mathbf{q}),...V^H(\mathbf{q})\right)$  ( $\mathbf{q}$  is N+1-dimensional consumer price vector) subject to the constraint that  $\sum_{i=1}^N \tau_i x_i = R$ . Characterize the structure of optimal commodity taxes.
  - (c) Explain how an economy with heterogeneous households affect the optimal commodity tax formula.
- 2. The economy made up of individuals with identical preferences defined over consumption c and labor l. The utility function takes the simple form:

$$u(c,l) = c - \frac{1}{(1+k)}l^{1+k}$$

where k > 0 is a given fixed parameter. Individuals have different productivity or wage rates. An individual with wage rate w supplying labor l, earns z = wl and consumes c = z - T(z) where  $T(\cdot)$  is the income tax. Suppose there is a distribution of skills w with density f(w) > 0 over  $[0, \infty)$ . The total population is normalized to one so  $\int_0^\infty f(w) = 1$ 

- (a) Consider a linear income tax system  $T(z) = -R + \tau \cdot z$  where R > 0 is the lump sum transfer and  $\tau$  is a flat tax rate. Solve for the optimal labor supply choice l as a function of R and the net-of-tax wage rate  $w \cdot (1 \tau)$ . Derive the uncompensated and compensated elasticities of labor supply as a function of k. Find the income effect on labour supply (income elasticity is  $\eta$ ).
- (b) Suppose taxes collected are all rebated through the demo-grant so that  $R = \tau Z$  where Z is average earnings. Solve for the Rawlsian optimal tax rate  $\tau$  (i.e., the tax rate that maximizes the utility of the worst-off individual). Solve for the utilitarian optimal tax rate  $\tau$  (i.e., the tax rate that maximizes the sum of utilities). In both cases, explain the intuition behind your results.
- (c) Redo (a)-(b) using instead utility function

$$u(c, l) = log(c) - l$$

(it is not always possible to derive exact analytical expressions with this functional form, in that case, just give implicit formulas with economic explanation - do not get stuck here, try and then move on).

- (d) Return to utility function  $u(c,l)=c-\frac{1}{(k+1)}l^{1+k}$ . Assume the government imposes the following two-bracket income tax:  $T(z)=-R+\tau_1\cdot z$  if  $z\leq \bar{z}$  and  $T(z)=-R+\tau_2\cdot z$  if  $z>\bar{z}$  Assume that  $0<\tau_1<\tau_2$  plot the budget constraint on a diagram in (l,c).
- (e) Solve for the optimal labor l and earnings z = wl choice for an individual with wage w. Show that there are three cases depending on whether the individual is in the bottom bracket, the top bracket, or earn exactly  $\bar{z}$ .
- (f) Explain how the amount of bunching observed at  $\bar{z}$  is related to the elasticity of labor supply.
- (g) Let there now be 3 types of individuals: disabled individuals unable to work  $w_0=0$ , low skilled individuals with wage rate  $w_1$ , and skilled individuals with wage rate  $w_2$ ;  $w_1< w_2$ . We assume that the fractions of disabled, low skilled, and high skilled in the population are  $\lambda_0, \lambda_1, \lambda_2$  (and that  $\lambda_0 + \lambda_1 + \lambda_2 = 1$ ). For simplicity, we assume that, in all the cases we consider, low skilled workers are always in the bottom bracket and that high skilled workers are always in the top bracket. Taking  $R, \tau_1$ , and  $\bar{z}$  as fixed, compute the tax rate  $\tau_2^*$  that maximizes taxes collected from the high skilled. Express  $\tau_2^*$  as a function of  $k, z_1$  and  $\bar{z}$ .
- (h) Taking R, and  $\bar{z}$  as fixed, and assume  $\tau = \tau_2^*$ , compute the tax rate  $\tau_1^*$  that maximizes total taxes collected. Express  $\tau_1^*$  as a function of  $k, z_1, \lambda_1$ , and  $\lambda_2$ , and  $\bar{z}$ . Explain intuitively why  $\tau_2^* < \tau^* < \tau_1^*$  (where  $\tau^*$  is from question (b)).
- (i) Suppose now that disabled workers face a cost of work q that is distributed according to a cumulated distribution P(q) with density p(q). When a disabled person pays the work cost q, she becomes like a low skilled worker with wage rate  $w_1$  and utility function

$$u(c,l) = c - \frac{1}{(k+1)}l^{1+k}$$

Compute the fraction of disabled workers who work as a function of  $w_1$ ,  $\tau_1$ , and the distribution  $P(\cdot)$ . Under this scenario, how does the tax rate  $\tau_1$  maximizing tax revenue compares with  $\tau_1^*$  from (h) which was derived assuming no disabled person could work (explain the economic intuitions if you cannot do the full math).

3. Find data for the proportion of the labour force that do not work. Has this propor-

tion decreased over the last two decades? There is now an increasing use - not only in Norway but in all welfare states - of mandatory activation of benefits recipients. How can this policy - mandatory activation (participate in a program or benefits are reduced) be defended.