# ECON4622 – Public Economics II
# First lecture by DL

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1 October 2014

# Two lectures on business and capital taxation under uncertainty

Today (hopefully)

- Varian (1996) (or later)
- Lund (1993)
- Fane (1987)

Next Wednesday, 8 October

- Sørensen (2005a) (in *International Tax and Public Finance*)
- Sørensen (2005b) (in *Nationaløkonomisk Tidsskrift*)
- Lund (2002a) (in *Energy Journal*)

On Wednesday 15 October, topic will be taxation of natural resources

Seminar Monday 20 October looks at a problem set related to all three lectures

In addition to slides, diagrams will be drawn on the blackboard

# Background; importance of diversification

- Recommended background ECON4200 (microeconomics), ECON4620 (public economics)
- Point of departure: Theory of decisions under uncertainty
  - Individuals' maximization of expected utility (– known from ECON4200)
  - Markets under uncertainty (– students will have different backgrounds here)
- Start with Varian (1996), the Domar-Musgrave (1944) effect
- Then introduction to markets under uncertainty, Lund (1993)
- "Uncertain investments under limited diversification"
- Optimal decisions will depend on diversification possibilities
- Thus, effects of taxation on decisions also depend on diversification
- Two extreme cases:
  - Varian (1996), no diversification, taxes may encourage investment
  - Fane (1987), full diversification, taxes may be neutral
- Sørensen (2005b) explicitly models different of degrees of diversification

## The Domar-Musgrave effect: Tax encourages risky investment

- Domar and Musgrave wrote in 1944, before expected utility had been invented
- Show that under some circumstances, tax may encourage risky investment
- More easily shown using expected utility maximization
- Varian, *Intermediate Microeconomics,* does this in simplest possible model
- 2 states of the world (good, bad), 2 assets (safe, risky); no interest on safe
- Here: generalize somewhat; many states $i = 1, \ldots, S$; interest rate $r_0$
- Like in Varian: given wealth $w$ to invest for consumption in the (only) future period
- Let $C_i =$ consumption in state $i$, which has probability $\pi_i \in (0, 1)$, $\sum \pi_i = 1$
- Invest $x$ in risky asset, earning a rate of return of $r_i$ in state $i$; this gives

$$C_i = (w - x)(1 + r_0) + x(1 + r_i) = w(1 + r_0) + x(r_i - r_0)$$

  and the agent wants to maximize

$$E[u(C)] = \sum_{i=1}^{S} \pi_i u[w(1 + r_0) + x(r_i - r_0)]$$

- Only $x$ is endogenous, to be chosen, with first-order condition

$$\frac{\partial E[u(C)]}{\partial x} = \sum_{i=1}^{S} \pi_i u'[w(1 + r_0) + x(r_i - r_0)](r_i - r_0) = 0 \qquad (1)$$

## Domar-Musgrave, contd.

- Assume the first-order condition has unique solution, a maximum
- (See Varian (appendix to ch. 12) for second-order condition and corner solutions)
- Consider now the introduction of a tax with rate $\tau$ on the excess return $r_i - r_0$
- Deduction for risk free return like in ACE tax or Norwegian shareholder allowance
- Assume full loss offset, i.e., $r_i - r_0 < 0$ implies refund or otherwise deduction
- After-tax income gives consumption in state $i$,

$$C_i = (w - x)(1 + r_0) + x(1 + r_i) - \tau x(r_i - r_0) = w(1 + r_0) + x(r_i - r_0)(1 - \tau)$$

- The first-order condition in this case:

$$\frac{\partial E[u(C)]}{\partial x} = \sum_{i=1}^{S} \pi_i u'[w(1 + r_0) + x(r_i - r_0)(1 - \tau)](r_i - r_0)(1 - \tau) = 0 \qquad (2)$$

- The final factor $(1 - \tau)$ is the same for all terms in the sum, and cancels out
- Like Varian, define $x^*$ as the optimum when $\tau = 0$, from (1), and let $\hat{x} \equiv \frac{x^*}{1-\tau}$
- $\hat{x}$ satisfies (2), since $(1 - \tau)$ cancels out inside [ ], and $x^*$ satisfies (1)
- Conclude that a higher tax rate implies a higher optimal risky investment
- Intuition: Tax system takes same fraction of good and bad outcomes
- To get same risk exposure when taxed, agent will invest more in risky asset

# Meaning and importance of "limited diversification"

- In Domar-Musgrave model: Only one risky asset, no diversification
- Diversification means to invest in different types of assets; portfolio
  - Helpful if rates of return of different types are not perfectly correlated
  - Reduces the variance of the rate of return on the total portfolio
  - Expected rate of return is not reduced similarly; a weighted average
- In general, more diversification is better
  - Would prefer possibility to invest in all assets, globally
  - Reduced variance for each new asset type that is included in portfolio
  - But undesirable if expected rate of return is (very) low
- Each investment decision will depend on all sources of future income
  - Will evaluate how new investment contributes to total portfolio
  - "Portfolio" includes future labor income and other non-marketable assets
- Lack of diversification means too large or too small holdings of some assets
- Several reasons why some people or countries (or firms?) are poorly diversified
  - (Claims to) own future labor income cannot be sold (– slavery prohibited)
  - Countries typically do not sell natural resources before they are developed/extracted
  - Shares in small firms are typically not widely traded, not internationally
- Will look at models with various degrees of diversification

# Model of limited diversification, no taxes

- Model in Lund (1993) with risk averse agent maximizing expected utility
- Some sources of future income are exogenously given (e.g., labor income)
- In addition to these, may invest in financial assets for future consumption
- 2 periods, $t = 0$ (now) and $t = 1$ (future); $n$ risky financial assets, 1 risk free asset
- Consumption in both periods; maximization of time additive $u(C_0) + \theta E[u(C_1)]$
- Investments in risk free asset and at least one risky asset are chosen optimally
- Will not go into details on solution, straightforward; first-order conditions imply

$$R_0 \equiv 1 + r_0 = \frac{u'(C_0)}{\theta E[u'(C_1)]}, \tag{8}$$

where $r_0$ = rate of return on risk free asset; $C_t$ = consumption at time $t$; moreover

$$E(R_j) = R_0 - \frac{\text{cov}[u'(C_1), R_j]}{E[u'(C_1)]}, \tag{11}$$

for risky asset $j$ chosen optimally (define $R_j \equiv \frac{P_{j1}}{P_{j0}}$; $P_{j1}$ incl. dividends, if any)

- (8) known from standard consumer theory; (11) similar, but adds risk adjustment
- Covariance is typically negative, thus adding to required $E(R_j)$:
  - For risk averse agent, $u'' < 0$; high values of $C_1$ coincide with low $u'(C_1)$
  - Typically rates of return are positively correlated, also with consumption
- Interpret: Require higher $E(R_j)$ when $R_j$ has high covariance with $C_1$
- High covariance means that $R_j$ contributes much to the variance of $C_1$

## Model, contd.: Simplification and aggregation

- Invoke one of the following two simplifying assumptions:
  - The $u$ function is a quadratic function, or
  - the returns have a (joint) normal distribution
- Each (separately) implies agents only care about $E(C)$ and $\text{var}(C)$
- This allows an aggregation across all agents in the economy, giving

$$E(R_j) - R_0 = \frac{Y_m \,\text{cov}(R_j, R_m) + \text{cov}(R_j, Y_x)}{Y_m \,\text{var}(R_m) + \text{cov}(R_m, Y_x)}[E(R_m) - R_0] \tag{26}$$

  for an asset $j$ which is chosen optimally by all agents, where
  - $Y_m$ = total investment of all agents in assets they all can choose optimally
  - $R_m$ = return on that total investment, "return on market portfolio"
  - $Y_x$ = total income at $t = 1$ of all agents from other sources
  - ($Y_m, Y_x$ are called $V_m, V_x$ in Lund (1993); we need $V$ for something else)
- Interpretation: The expected excess return $E(R_j) - R_0$ is explained by
  - The expected excess return on the market portfolio, $E(R_m) - R_0$
  - The covariance between $R_j$ and the return $R_m$ on the market portfolio
  - The covariance between $R_j$ and the other income sources, $Y_x$
- A higher expected return is required if $R_j$ contributes much to variance
- On previous page, this was the variance of $C_1$ for each agent
- But $E(R_j)$ is the same for all; aggregation means total $R_m Y_m + Y_x$ matters
- Also, the simplification means that individual $u$ functions do not matter

# Digression: calculations of covariances, etc.

- Short "mathematical appendix" to remind you of some rules of calculation
- $E$ denotes expectation, cov denotes covariance
- When $X, Z, X_1$, and $X_2$ are stochastic variables, and $\alpha, \beta$ are constants:
- $E(\alpha X + \beta Z) = \alpha E(X) + \beta E(Z)$
- If $X, Z$ are stochastically independent, then $E(XZ) = E(X)E(Z)$
- If not, the difference is known as the covariance,

$$\text{cov}(X, Z) = E\{[X - E(X)][Z - E(Z)]\} = E(XZ) - E(X)E(Z)$$

We find $\text{cov}(\alpha X_1 + \beta X_2, Z) = E(\alpha X_1 Z + \beta X_2 Z) - [\alpha E(X_1) + \beta E(X_2)]E(Z)$

$$= \alpha E(X_1 Z) + \beta E(X_2 Z) - \alpha E(X_1)E(Z) - \beta E(X_2)E(Z) = \alpha \,\text{cov}(X_1, Z) + \beta \,\text{cov}(X_2, Z)$$

- You can now derive $E[XE(Z)] = E(X)E(Z)$, and some formulas in these notes, e.g.,

$$E(R_j) = E(P_{j1} \cdot \frac{1}{P_{j0}}) = \frac{1}{P_{j0}} E(P_{j1}), \quad \text{and}$$

$$\text{cov}(R_j, R_m) = \frac{1}{P_{j0}} \text{cov}(P_{j1}, R_m)$$

since $E(Z)$, $P_{j0}$, and $\frac{1}{P_{j0}}$ are not stochastic variables

# The Capital Asset Pricing Model (CAPM); valuation functions

- The most commonly used model in financial economics (cf. ECON4510)
- Based on either one of the simplifying assumptions on top of p. 8
- CAPM is the case without $Y_x$; no agents have exogenous income sources
- The remaining equation for expected excess return on asset $j$ is then

$$\text{CAPM:} \quad E(R_j) - R_0 = \frac{\text{cov}(R_j, R_m)}{\text{var}(R_m)}[E(R_m) - R_0] \tag{27}$$

- (Optimistic interpretation:) $R_m$ is observable; (27) can be implemented, tested
- For our purposes, most important feature is the *valuation function:*
- First, from (26): Since $R_j \equiv P_{j1}/P_{j0}$, we can solve for $P_{j0}$
- Define $\lambda_{Y_m} = [E(R_m) - R_0]/[Y_m \text{var}(R_m) + \text{cov}(R_m, Y_x)]$, aggregate, no $j$
- In (26), multiply both sides by $P_{j0}$, rearrange, find *valuation function* (ver. (26)):

$$P_{j0} = V_{(26)}(P_{j1}) \equiv \frac{1}{R_0} \left\{ E(P_{j1}) - \lambda_{Y_m} \left[ Y_m \text{cov}(P_{j1}, R_m) + \text{cov}(P_{j1}, Y_x) \right] \right\}$$

- Interpretation: Valuation in market at $t = 0$ of claim to receiving $P_{j1}$ at $t = 1$
- Present value of $\{\ \}$ expression, which is $E(P_{j1})$ minus risk adjustment
- Higher risk adjustment the higher is covariance of $P_{j1}$ with $Y_m R_m + Y_x$

# CAPM, valuation functions, contd.; value additivity

- Consider now the CAPM, (27), instead of (26)
- Define $\lambda = [E(R_m) - R_0]/\operatorname{var}(R_m)$ (to replace $\lambda_{Y_m}$); find CAPM valuation function

$$P_{j0} = V_{(27)}(P_{j1}) \equiv \frac{1}{R_0}\left[E(P_{j1}) - \lambda \operatorname{cov}(P_{j1}, R_m)\right]$$

- Typically, rate of return of a stock index is used as approximation for $R_m$
- Both valuation functions, $V_{(26)}$ and $V_{(27)}$, have the following properties:
  - If $a, b$ are constants, and $X, Y$ are risky cash flows next period,

$$V(aX + bY) = aV(X) + bV(Y)$$

  - If $Z$ is a deterministic cash flow next period, the usual present value formula,

$$V(Z) = E(Z)/R_0 = Z/R_0$$

- The first of these is known as *value additivity*
- Necessary condition for equilibrium in market for financial assets
- If values are not additive in this way, extra value can be created in markets
- Market participants could then buy and combine existing assets, sell at higher price
- Or, vice versa, buy and split existing assets, sell at higher price
- One implication: No need for firms to diversify, leave to shareholders

# Neutral taxation of firms

- Section 2 of Fane (1987) on neutral taxation of firms; skip section 3
- *Neutral* here means a tax that does not influence decisions
- Firms are assumed to maximize their market value for shareholders
- Shares in the firm are supposed to be traded in stock market
- Valuation function in market has property *value additivity*
- Two versions of $V()$ in lecture so far; third version in Fane, eq. (1)
- Need to extend valuation function to many periods with uncertainty
- Let $V_{t_1,t_2}(X)$ denote valuation at time $t_1$ of cash flow $X$ at time $t_2$
- Consider firm with cash flow $\mathbf{X} = (X_1, X_2, \ldots, X_T)$ in periods $1, 2, \ldots, T$
- Let $V_t(\mathbf{X})$ denote time $t$ valuation of vector starting at $t + 1$; then

$$V_0(\mathbf{X}) = \sum_{t=1}^{T} V_{0,t}(X_t)$$

- Introduce now a *pure cash flow tax* on the firm
  - Proportional tax on real (non-financial) cash flows
  - Full, immediate loss offset: Refund in years with negative cash flow
  - Tax similar to equity participation by government (without voting rights)
  - Pure cash flow tax known as Brown tax, suggested by Brown (1948)

# Neutral taxation of firms: Brown tax

- Tax base each period will be $X_t$, whether positive or negative
- With tax rate $\tau \in [0,1)$, valuation of vector of after-tax cash flows is

$$V_0((1-\tau)\mathbf{X}) = \sum_{t=1}^{T} V_{0,t}((1-\tau)X_t) = (1-\tau)V_0(\mathbf{X})$$

- Neutrality follows from this:
  - Each project can be valued separately, due to value additivity
  - Project with cash flow vector $\mathbf{X}$ will be accepted if value $> 0$
  - After-tax value will be positive if and only if taxed value is positive
  - A pure cash flow tax will thus not influence decision on project
  - Clearly, cash flow tax at rate $\tau$ reduces value compared with no tax
  - But in this theory of the firm, there are no income effects; decision unchanged
- No Domar-Musgrave effect here; tax does not encourage investment
- Domar-Musgrave effect appeared because tax system took part of risk
- But in this case also, tax system takes part of risk
- Difference lies in decision criteria: diversification or not
- Domar-Musgrave effect when individual is not diversified, no risk market
- Here, gains from diversification are taken already; value additivity holds

# Neutral taxation of firms: desirable?

- Neutrality seems to be a desirable property
  - Welfare theorems say market equilibrium is Pareto optimal; don't interfere
  - Can also have good reasons to interfere (externalities, distribution)
  - But interference should be targeted and not unnecessarily distortive
- So, why is the Brown tax (or something similar) not introduced everywhere?
- Some possible objections
  - Brown tax would pay a share of any project, also those with negative value
  - Government are perhaps not willing to take so much risk
  - Ex post negative value is quite common, bad luck, e.g., low output prices
  - Even projects with ex ante negative value may be started if additional gain
  - E.g., firm can try out new technology; get additional gains for other projects
  - Pure hobbies might also be organized as firms and receive "tax support"
- Tax base for Brown tax is rent, not normal return to capital
- A project that earns just the normal return, has zero net value
  - If cash flows are known with certainty and return is risk free rate
  - Under uncertainty, use "zero value" to *define* normal return
- Most actual tax systems will tax normal return also
- In ordinary industries with competition, rents are not large
- Where project values are large, entry of competitors will reduce them
- Natural resources may give large rents if not paid for up front

# Modified cash flow tax (MCFT)

- In practice, attempts to approximate pure cash flow taxes
- Instead of refunds when yearly cash flow is negative: Loss carry-forward
  - But deduction next year (or later) has lower present value
  - May compensate by allowing carry-forward with interest
- Fane: Carry-forward with accumulation at *risk-free* interest rate
- Contrasts with recommendations from previous articles (cf. top of p. 99)
  - Some previous writers suggest to use risk-adjusted discount rate
  - Suggest the risk-adjustment that would be used for project as a whole
- But sufficient to accumulate interest so firm is indifferent between receiving refund and receiving postponed deduction
- Important whether government promises effective deduction eventually
- If firm is sure to receive effective deduction, there is no risk
- Accumulation at risk free interest rate is sufficient
- This is the case that Fane considers theoretically
- Worries whether firm may go bankrupt in future; deduction "lost"
- Similarly, could worry whether government will change taxes
- In practice, governments seldom promise effective deductions/refunds
- See Lund (2014) in *FinanzArchiv* for more on this
- Calculates risk of tax deductions separately, in stylized model

# More details on method for proving neutrality

- In discussion so far:
  - Have assumed all prices, interest rates etc. are exogenous
  - In fact, changing the tax system may change these
  - In small, open economy, "exogenous" is not a bad assumption
  - Have also assumed known, constant risk free interest rate
- In that case, easy to show neutrality also under postponed deduction
- Assume some negative cash flow $I$ deducted at $t_2$, not at earlier $t_1$ when it accrues
- Use value additivity and simple discounting of risk free cash flows:

$$\left[\sum_{t=1}^{T} V_{0,t}((1-\tau)X_t)\right] + \frac{\tau I}{(1+r)^{t_1}} - \frac{\tau I(1+r)^{t_2-t_1}}{(1+r)^{t_2}} = (1-\tau)V_0(\mathbf{X})$$

- The first term, the sum in square brackets, is the after-tax cash flows as if the tax is not "modified," so all negative cash flows are deducted immediately, with payout of negative taxes if necessary
- The second term is negative ($I < 0$), and subtracts the tax refund for $I$ which is part of the first term (not given under MCFT)
- The third term ($> 0$ since $I < 0$) adds this back in a later period (MCFT)
- The second and third (postponement) terms sum to zero; neutrality
- Condition is that accumulation interest rate equals discount rate

# Briefly on Fane's method; non-constant interest rate

- In Fane, multiperiod uncertainty described by a number of future states of the world
- Tree structure; possible states at $t+1$ contingent on outcomes at $\ldots, t-2, t-1, t$
- As seen from time $t$, several states ($a = 1, 2, \ldots, a_{\max}$) are possible for time $t+1$
- Depending on which of these states are realized, there will be a new set of states ($b = 1, 2, \ldots, b_{\max}$) possible for time $t+2$
- Depending on which of these states are realized, there will be a new set of states ($c = 1, 2, \ldots, c_{\max}$) possible for time $t+3$, etc.
- Assumes markets for state-contingent claims (ECON4240, ECON4510, ECON5200)
- Each period, market prices are determined for claims to next period's states
- $\Pi(c, b, a)$ is value in $t+2$ of state-contingent security which pays \$1 in state $c$ at $t+3$, given that states $b, a$ occurred at $t+2, t+1$, resp.
- Instead of writing cash flows as random variable $X_{t+s}$, they are now specified as functions of those states that can occur at $t+s$
- Value at time $t$ of claim to \$1 in state $c$ at time $t+3$ may depend on realizations of states in meantime, $t+1, t+2$
- State contingent risk free interest rate defined by $1/[1 + r^*(a, t+1)] = \sum_b \Pi(b, a)$
- Modified cash flow tax is neutral, with accumulation at such state contingent rates

# Practical applications of modified cash flow taxes

- In many texts, "cash flow taxes" refer to modified, not pure cash flow taxes
  - "Cash flow taxes" allow immediate deduction of all costs; they tax rent
  - Corporate income taxes require depreciation; they tax normal return plus rent
- Popular concept since Meade (1978), important tax report to the British government
- Widespread in taxation of natural resource rent (oil, gas, coal, minerals)
  - A variant of MCFT proposed by Garnault and Clunies Ross (1975)
  - Called Resource Rent Tax (RRT); included also an extension with progressivity
  - No good answer to what interest rate should be used for carry-forward
  - Ignored problem that firm may end up never being able to deduct
  - Criticism on this basis in Dowell, Ball and Bowers, Mayo (see ref.s p. 98)
  - More on natural resource taxation in lecture Wednesday 15 October
- But similar ideas have been influential in ordinary taxation of firms
- Boadway and Bruce (1984): modification can include depreciation deductions
- Tax works like MCFT when (present value of deductions for any cost) = cost
- Costs deducted immediately, or loss carry-forward with interest, or depreciation
- With depreciation deductions, deduct also interest on non-depreciated capital
- "Allowance for corporate capital" (ACC) in addition to depreciation deductions
  - (Related also to "Allowance for corporate equity" (ACE), no details here)
- Fane (1987) was the first to extend Boadway and Bruce (1984) to uncertainty
- More details in Bond and Devereux (1995), also with uncertainty