ECON4622 – Public Economics II
Second lecture by DL

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Sørensen (2005a) in *International Tax and Public Finance*

- Neutrality of Norwegian Shareholder income tax, introduced 2006
- Suggested by public ("Skauge") commission with Sørensen as member
- Much of Sørensen's (2005a) article is covered in ECON4620
  - Spring 2014: Lectures 5 March (end of) and 12 March (beginning of)
- Here, cover section 4.3: Neutrality under uncertainty
- Define $V_t()$ as value at time zero of claim to cash flow t periods later
- (Use tilde, ˜, over variables that are stochastic:)
- Model of firm with one project lasting three periods
  - Period 0: Investment K_0, partly financed by debt B
  - Period 1: Project revenue $\tilde{R}_1$; also promised full repayment of debt (but risky)
    - ★ If $\tilde{R}_1 < (1+r)B$: bankruptcy, creditors take over, then sell firm to new owners
  - Period 2: Project revenue $\tilde{R}_2$, salvage value $\tilde{K}_2$, define $\tilde{X}_2 \equiv \tilde{R}_2 + \tilde{K}_2$
- Promised interest rate r is determined to reflect project risk, not risk free
- If new owners in period 1, price they pay is market determined
- Define indicator function for bankruptcy, $\tilde{b} = 1$ if $\tilde{R}_1 < (1+r)B$; $\tilde{b} = 0$ otherwise
- Without taxes, project value to shareholders (seen from time 0) is

$$NPV^* = -(K_0 - B) + V_1[(1-\tilde{b})(\tilde{R}_1 - (1+r)B)] + V_2[(1-\tilde{b})(\tilde{X}_2)]$$

Sørensen (2005a), contd., who earn positive net value?

- Method to arrive at neutrality result: Assume original owners get all rent
  - Creditors get no rent, (promised) r just high enough to compensate for risk
  - (Will solve for this r)
  - Thus net value of loan is zero (both to creditor and debtor)
- Likewise, in case new owners come in after bankruptcy:
  - In case bankruptcy, new owners pay fair market value for firm, thus get no rent
  - (Will solve for this value)
  - Thus net value of their transactions in period 1 and 2 is zero
- Common reason for zero values in both situations: Same V function for all
  - Must also assume tax system is unchanged; if not, new owners may gain or lose
- Only opportunity for anyone to earn rent (strictly positive net value) in model:
  - Original owners (period 0) have access to (real) investment opportunity
  - A real investment opportunity may (or may not) give positive net value
  - All firms look for positive net values, but competition will often restrict this
  - Competition may lead to lower output prices or higher factor prices
  - We can assume some opportunities with positive net value exist, sometimes
- Consider first the value of debt; r will be such that the net value is zero:

$$-B + V_1[(1-\tilde{b})(1+r)B + \tilde{b}\tilde{R}_1] + V_1[\tilde{b}V_2^1(\tilde{X}_2)] = 0$$

  where $V_2^1()$ is the valuation in period 1 of a cash flow in period 2
- Last term reflects assumption that, if bankruptcy, new owners pay fair price

Sørensen (2005a), contd., removing loan from NPV expression

- With assumption that net value of loan is zero, Sørensen shows that

$$NPV^* = -K_0 + V_1(\tilde{R}_1) + V_2(\tilde{X}_2)$$

  as if there had been no loan at all
  - Maybe surprising, given bankruptcy possibility
  - But makes sense, when r is set high enough to compensate
- Some remarks on how to arrive at the NPV^* result above
- Need to substitute $V_2(\tilde{X}_2) = V_1[V_2^1(\tilde{X}_2)]$, which is OK, since
  - At $t = 0$, owners know they will be indifferent at $t = 1$ between receiving the amount $V_2^1(\tilde{X}_2)$ and receiving, at $t = 1$, a claim to receiving $\tilde{X}_2$ at $t = 2$
  - Thus they are indifferent to switching between these also at $t = 0$
- Apart from this, Sørensen only needs value additivity to arrive at the result
  - E.g., $V_1[(1 - \tilde{b})(1 + r)B + \tilde{b}\tilde{R}_1] = V_1(1)(1 + r)B - V_1(\tilde{b})(1 + r)B + V_1(\tilde{b}\tilde{R}_1)$
  - Observe: $V_1(1) = 1/(1 + i_1)$, where i_1 is the risk free rate, not equal to r
  - Observe also: Value additivity does not allow simplification of $V_1(\tilde{b}\tilde{R}_1)$
- Leave to you to show that the NPV^* expression simplifies as shown above

Sørensen (2005a), contd., introducing tax

- Shareholder income tax, deduction for risk free interest on investment
- Tax at rate τ; symmetric, i.e., full and immediate loss offset
- Consider first tax payment by new shareholders in case of bankruptcy,

$$\tilde{T}_2^n = \tau[\tilde{X}_2 - (1 + i_2)V_2^1(\tilde{X}_2)]$$

- Will show that the valuation $V_2^1(\tilde{T}_2^n)$ of this payment is zero
- (Related to discussion in Lund (2002a), p. 45)
- Valuation at $t = 1$ of receiving any amount $M(1 + i_2)$ at $t = 2$ for sure, with i_2 being risk free interest rate between $t = 1$ and $t = 2$, is equal to M
- Both factors $1 + i_2$ and $V_2^1(\tilde{X}_2)$ are known at $t = 1$
- Thus, valuation of deduction is $V_2^1[(1 + i_2)V_2^1(\tilde{X}_2)] = V_2^1(\tilde{X}_2)$
- (If confused, remember that $V_2^1(1) = 1/(1 + i_2)$)
- The result that $V_2^1(\tilde{T}_2^n) = 0$ rests heavily on full loss offset
- $\tilde{T}_2^n$ must be positive for some $\tilde{X}$ outcomes, negative for others

Sørensen (2005a), contd., valuation of tax payment

- Remains to determine valuation of taxes paid by original owners at $t = 1, 2$

$$\tilde{T}_1 = \tau(1 - \tilde{b})[\tilde{R}_1 - (1 + r)B - i_1(K_0 - B)] - \tau\tilde{b}(1 + i_1)(K_0 - B)$$

$$\tilde{T}_2 = \tau(1 - \tilde{b})[\tilde{X}_2 - (1 + \tilde{i}_2)(K_0 - B)]$$

- A few points to notice here:
  - Risk free interest rate between $t = 1$ and $t = 2$ is $\tilde{i}_2$, uncertain as seen from $t = 0$
  - The rate-of-return allowances are $i_1(K_0 - B)$ and $\tilde{i}_2(K_0 - B)$ at $t = 1, 2$
  - When owner's position is closed down ($t = 2$, but $t = 1$ if bankrupt), there is also a tax deduction for the (nominal) original value of investment at $t = 0$, $K_0 - B$
- With expressions above, after-tax value of project to shareholders is

$$NPV = -(K_0 - B) + V_1[(1 - \tilde{b})(\tilde{R} - (1 + r)B)] + V_2[(1 - \tilde{b})\tilde{X}_2] - V_1(\tilde{T}_1) - V_2(\tilde{T}_2)$$

(Equation (17) in Sørensen (2005a) has a typo, but (18) and (19) are correct)

- Substitute in the two $\tilde{T}_t$ expressions, and collect terms
  - In particular, the terms multiplying K_0 can be written as

$$-1 + \tau i_1 V_1(1 - \tilde{b}) + \tau(1 + i_1)V_1(\tilde{b}) + \tau V_2[(1 - \tilde{b})(1 + \tilde{i}_2)]$$

$$= -1 + \tau\{i_1 V_1(1 - \tilde{b} + \tilde{b}) + V_1(\tilde{b}) + V_2(1 + \tilde{i}_2) - V_2[\tilde{b}(1 + \tilde{i}_2)]\}$$

$$= -1 + \tau\{\frac{i_1}{1 + i_1} + V_1(\tilde{b}) + V_2(1 + \tilde{i}_2) - V_1(\tilde{b})\} = -1 + \tau\frac{i_1 + 1}{1 + i_1} = -1 + \tau$$

Neutrality of Shareholder income tax under uncertainty

- Collecting terms we find the after-tax value to original owners is $NPV =$

$$(1-\tau)[-(K_0-B)+V_1(\tilde{R}_1)+V_2(\tilde{X}_2)]-(1-\tau)[V_1(\tilde{b}\tilde{R}_1)-(1+r)BV_1(1-\tilde{b})-V_2(\tilde{b}\tilde{X}_2)]$$

- Expression in second [ ] is equal to B (see p. 3 above), so that

$$NPV = (1-\tau)[-K_0 + V_1(\tilde{R}_1) + V_2(\tilde{X}_2)] = (1-\tau)NPV^*$$

  where NPV^* was the value to original owners in the absence of taxation
- Conclusion: Even in complicated case (loan, possible bankruptcy, i_t nonconstant):
  - Again we have a tax system that is a proportional tax on rent
  - Projects have positive value after tax if and only if positive before tax
  - This means tax system is neutral
- Need final deduction for original equity $K_0 - B$ whether bankruptcy or not
  - Except if project life $\rightarrow \infty$ for sure
  - In that case, present value of deduction at the end will $\rightarrow 0$; becomes irrelevant

Sørensen (2005b) in *Nationaløkonomisk Tidsskrift*

- Taxation of shareholders with limited diversification
- Version of Capital Asset Pricing Model (CAPM)
  - Mean-variance preferences due to normally distributed returns
  - Explicitly specified utility function, CARA; this implies eq. (2)
  - Less general than CAPM, but allows explicit solutions, eq. (11)–(15) etc.
- Only portfolio decisions, no production decision
- Only two time periods: first period portfolio decision, then consumption
- Since model ends, wealth W in second period is equal to consumption
  - (Notation: Sørensen uses V for wealth in (2005b), but $V()$ for valuation in (2005a), which is more common, so we will use W for wealth.)
- Only three types of risky assets, number 3 traded internationally
- Risky assets 1 and 2 only traded domestically, due to, e.g., information problems, or other fixed costs of trading shares in smaller firms across borders, "home bias"
- Main point: Combine (realistic) lack of diversification with tax analysis
- Results
  - Can achieve neutrality under some conditions; in general not
  - Insurance effect of taxes encourages investment; the Domar-Musgrave effect

Solving the model of Sørensen (2005b)

- In Fane (1987) and Sørensen (2005a): Fully diversified shareholders
- But here: Cannot use the same valuation function $V()$ throughout
- Instead: Solve explicitly for required pre-tax expected rates of return
- Consider first after-tax view of each agent; second-period wealth is

$$W = [1 + v_1 R_1 + v_2 R_2 + v_3 R_3 + (1 - v_1 - v_2 - v_3) i] \, W_0$$

  where W_0 is initial wealth, v_j is fraction invested in risky asset j with *after-tax* rate of return R_j; remainder invested at risk free after-tax interest rate i

- CARA and normal distn. $\Rightarrow$ Expected utility is $E[U(W)] = E(W) - (\hat{\rho}/2)\,\text{var}(W)$;

$$E(W) = [1 + v_1 R_1^e + v_2 R_2^e + v_3 R_3^e + (1 - v_1 - v_2 - v_3) i] \, W_0$$

  where the e superscripts denote expectations, moreover

$$\text{var}(W) = \left(v_1^2 \hat{\sigma}_1^2 + v_2^2 \hat{\sigma}_2^2 + v_3^2 \hat{\sigma}_3^2 + 2 v_1 v_2 \hat{\sigma}_{12} + 2 v_1 v_3 \hat{\sigma}_{13} + 2 v_2 v_3 \hat{\sigma}_{23} \right) W_0^2$$

  where $\hat{\sigma}_j^2 = \text{var}(R_j), \hat{\sigma}_{ij} = \text{cov}(R_i, R_j)$, and $\hat{\rho} =$ coefficient of absolute risk aversion

- Parameters $R_j^e, \hat{\sigma}_j, \hat{\sigma}_{ij}, i$, and initial wealth W_0 are exogenous to the agent

Solving the model of Sørensen (2005b), contd.

- Agents may have different W_0 and $\hat{\rho}$; each maximizes $\max_{v_1,v_2,v_3} E[U(W)]$ as defined on previous page, with first-order conditions $\partial E[U(W)]/\partial v_j = 0$ for $j = 1, 2, 3$, i.e.,

$$R_1^e - i - \frac{\hat{\rho}}{2} W_0 (2v_1\hat{\sigma}_1^2 + 2v_2\hat{\sigma}_{12} + 2v_3\hat{\sigma}_{13}) = 0$$

$$R_2^e - i - \frac{\hat{\rho}}{2} W_0 (2v_2\hat{\sigma}_2^2 + 2v_1\hat{\sigma}_{12} + 2v_3\hat{\sigma}_{23}) = 0$$

$$R_3^e - i - \frac{\hat{\rho}}{2} W_0 (2v_3\hat{\sigma}_3^2 + 2v_1\hat{\sigma}_{13} + 2v_2\hat{\sigma}_{23}) = 0$$

- Sørensen introduces W_0-based relative risk aversion coefficient $\rho = \hat{\rho} W_0$
- Three equations in v_1, v_2, v_3; Sørensen assumes unique solution; maximum
  - In fact, second-order conditions and uniqueness are likely to be OK
  - More problematic: No discussion whether $v_j < 0$ can be part of solution
- Sørensen introduces alternative tax systems into model
- Solution above, based on after-tax rates of return, is valid throughout
  - In section 3: Tax on full return to equity
  - In section 4: Tax on equity premium
  - Will skip third alternative analysis in section 5
  - But notice the remarks at the end of that section: "…stimulate risk taking …" "…may well be socially desirable …" when diversification is limited

Tax on full return to equity

- Symmetric tax τ on full return to shares, $R_j = (1-\tau)r_j$; r_j is pre-tax
- Implies $R_j^e = (1-\tau)r_j^e, \hat{\sigma}_j^2 = (1-\tau)^2\sigma_j, \hat{\sigma}_{ij} = (1-\tau)^2\sigma_{ij}$; σ_j and σ_{ij} are pre-tax
- "Symmetric" means full loss offset; if $r_j < 0$, there is effective deduction
- First-order conditions can now be written in pre-tax terms, as

$$r_1^e = \frac{i}{1-\tau} + \rho(1-\tau)(v_1\sigma_1^2 + v_2\sigma_{12} + v_3\sigma_{13})$$

$$r_2^e = \frac{i}{1-\tau} + \rho(1-\tau)(v_2\sigma_2^2 + v_1\sigma_{12} + v_3\sigma_{23})$$

$$r_3^e = \frac{i}{1-\tau} + \rho(1-\tau)(v_3\sigma_3^2 + v_1\sigma_{13} + v_2\sigma_{23})$$

- Under the assumption that i and r_3^e are exogenous, given from world markets, solve the last of these equations for $\rho(1-\tau)$; insert in other two equations

Tax on full return to equity, contd.

- Similar to CAPM equations, with r_3 taking role as market rate of return:

$$r_1^e = \frac{i}{1-\tau} + \beta_1 \left(r_3^e - \frac{i}{1-\tau} \right) = (1-\beta_1)\frac{i}{1-\tau} + \beta_1 r_3^e$$

$$r_2^e = \frac{i}{1-\tau} + \beta_2 \left(r_3^e - \frac{i}{1-\tau} \right) = (1-\beta_2)\frac{i}{1-\tau} + \beta_2 r_3^e$$

where the two β_j (for $j = 1, 2$) are defined as

$$\beta_1 = \frac{v_2\sigma_2^2 + v_1\sigma_{12} + v_3\sigma_{23}}{v_3\sigma_3^2 + v_1\sigma_{13} + v_2\sigma_{23}}, \quad \beta_2 = \frac{v_1\sigma_1^2 + v_2\sigma_{12} + v_3\sigma_{13}}{v_3\sigma_3^2 + v_1\sigma_{13} + v_2\sigma_{23}}$$

- Equations determine r_1^e and r_2^e in equilibrium
- Consider in particular case with well-diversified shareholders, $v_1 \to 0, v_2 \to 0$
- In the limit, the beta expressions are reduced to $\beta_j = \sigma_{j3}/\sigma_3^2$ for $j = 1, 2$
- These betas may be less than or greater than unity, with implications:
  - If $\beta_j < 1$, the required r_j^e is increasing in tax rate τ
  - If $\beta_j > 1$, the required r_j^e is decreasing in tax rate τ
- The latter result is related to Domar-Musgrave
  - Domestic shares with high β_j contribute much to portfolio risk
  - Tax contributes to reducing this risk, thus lower required return

Tax on equity premium

- Alternative tax system: Only excess over interest rate is taxed
- Again, assume symmetric tax, full loss offset

$$R_j = r_j - \tau(r_j - i)$$

- Different equations for expected rates of return; for asset 1:

$$r_1^e = i + \rho(1-\tau)(v_1\sigma_1^2 + v_2\sigma_{12} + v_3\sigma_{13}) \quad \text{and similar for } r_2^e, r_3^e$$

- Assume again i, r_3^e are exogenous; find endogenous required returns

$$r_1^e = i + \beta_1(r_3^e - i) \quad \text{and} \quad r_2^e = i + \beta_2(r_3^e - i)$$

  with β_1 and β_2 defined as before
- Tax rate cancels out; tax only influences returns via v_1, v_2, v_3
- In limit when $v_1 \to 0, v_2 \to 0$, then again, $\beta_j = \sigma_{j3}/\sigma_3^2$ for $j = 1, 2$
- So, in limit, with well diversified shareholders, tax rate has no impact
- Confirms Fane (1987): when shareholders are well diversified, tax on rent is neutral
- Summing up:
  - Taxes will affect risk taking except perhaps in limit, $v_1 \to 0, v_2 \to 0$
  - A tax on the full return may encourage investment with high betas
  - A tax on the excess return may be neutral when investors are diversified

Digression: Sørensen (2005b) varying the language

- In scientific articles, a precise and consistent language is more important than a colorful, varied language
- If the same phenomenon is mentioned more than once, there are good reasons to use the same words

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uncertain after-tax rate of return R_1; a fraction v_2 of his wealth in unquoted shares of type 2 with a risky net rate of return R_2, and a fraction v_3 of his wealth in quoted shares generating an uncertain after-tax return R_3. The remaining fraction of initial wealth is

- In this context:
  - "uncertain" means the same as "risky"
  - "after-tax" means the same as "net"
  - "rate of return" means the same as "return"
- But beware: In other contexts, these have different meanings

Lund (2002a) in *Energy Journal*

- Topic: Suggested reforms of petroleum taxation in Norway and Denmark
  - Public commissions appointed in 1999 and 2001, respectively
  - Explicit aim in both countries to get closer to neutral tax systems
  - Important parts of recommendations later became actual policies
- Main recommendation in both countries
  - Reduce investment-related deduction, "uplift" $\approx$ rate-of-return allowance
  - But allow loss carry-forward with interest and/or refund of negative tax
  - Thus making value of investment-related deductions equal for all companies in all circumstances, and equal to the investment (in present value terms)
- Several issues in implementing theory of neutral taxation
- Oil companies disagreed over valuation model (by Fane, Sørensen, etc.)
  - Claim(ed) they cannot distinguish cash flow elements with different risks
  - Potentially also a problem outside of resource sectors
  - ACE or ACC systems for CIT similarly allow interest accumulation
  - Neutrality depends on firms' decision making in line with theory
- Also different interpretations of relevant risk free rate
  - To make shareholders indifferent to postponed deductions:
  - Do they need accumulation at $i(1-\tau)$ or i?

Short summary of Lund (2002a)

- Both Norway and Denmark combine(d) corporate income tax (CIT) and rent tax
- CIT distortionary when projects are equity financed, tax wedge in required return
  - Except in closed economy when all alternative returns are taxed similarly
- Authorities did not want rent taxes to create additional distortions
- "Neutrality" relative to a situation with only CIT
- Before 2001, Denmark's rent tax had very high investment-related deductions
- Collected almost no revenue
- Norway's rent tax collected lots of revenue; not so clearly too generous
- Before 2000, economists often recommended using a risk-adjusted interest rate
  - for allowance for corporate capital, ACC ($\approx$ capital return allowance, CRA)
  - for loss carry-forward
- But based on Fane and others: New recommendation: Use risk free rate
- But only if tax deductions can be made (almost) risk free
- Various suggestions to make deductions close to risk free
  - Loss carry-forward with interest
  - Allow sale of unused deductions to other companies
  - Refund (payout) of negative taxes, always or in some circumstances
- Starting 2006, Norway introduced refund of negative petroleum taxes
  - during exploration phase if companies had no other income
  - at final close-down of a company's activities in sector

Example of use of risk free interest rate, non-intuitive?

- Lund (2002a) table 1 shows income tax w/ACC; also cash flow tax for comparison
  - For both taxes assume full, immediate loss offset (refund or other income)
  - Tax rate 50% (– could be anything); nominal risk free rate of 5% (for ACC)
- Example with 3 time periods, invest at $t = 0$, then two production periods
- Before-tax cash flows $X_0, \tilde{X}_1, \tilde{X}_2$ have zero net value by assumption
  - Expected rate of return $\approx$ 8 percent; exceeds 5 percent; zero value due to risk

Year	Before tax		Cash flow tax ($\tau = 0.5$)	Income tax ($\tau = 0.5$) w/linear depreciation schedule and 5 percent ACC			
	Cash flows	Expected cash flows	Expected c.f. after tax	Deduc -tions	Expected tax base	Exp. tax	Exp. c.f. after tax
	(ii)	(iii)	(iv)= 0.5(iii)	(v)	(vi)= (iii)-(v)	(vii)= 0.5(vi)	(viii)=(iii)-(vii)
0	X_0	-100	-50				-100
1	$\tilde{X}_1$	56	23	55	1	0.5	55.5
2	$\tilde{X}_2$	56	23	52.5	3.5	1.75	54.25

- Column (v): depreciation (50) and ACC = 5% of remaining capital value
- Seen from $t = 0$, present value of column (v), at 5% discount rate, is 100
- If realized outcomes are as expected, strictly positive tax is paid
- But tax is nevertheless neutral ex ante, a modified cash flow tax

Example is seemingly paradoxical, but not really

- Example on previous page seems paradoxical
  - By assumption, before-tax cash flow has zero net value
  - Expected tax payments at $t = 0, 1, 2$ are $0, 0.5, 1.75$, respectively
  - But after-tax cash flow also has zero net value
- Why is after-tax value zero? Modified cash flow tax, two parts:
  - Proportional tax on revenues at $t = 1, 2$
  - Deduction for investment, either at $t = 0$, or later, but with same PV
- Know the cash flow tax is neutral, thus also modified c.f. tax
- Cannot tell effect of tax by considering expected outcomes only
- Must take risk characteristics of cash flow elements into account