Will China Escape the Middle-Income Trap? A Politico-Economic Theory of Growth and State Capitalism

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Abstract

Is China's rapid growth sustainable with the current institutions? If not, will the slow-down of growth trigger political changes? This paper proposes a theory of politico-economic transition of China to answer these questions. In oligarchy, a political elite extracts surplus from the state sector and taxes the private sector. However, to maintain the power, the elite needs political support from a sufficient number of citizens. A "divide-and-rule" strategy is implemented to guarantee such support: the elite gives state workers high wages and turns them into supporters, at the cost of the private workers. The elite also distorts the capital allocation in favor of the state sector to maintain enough state workers. The consequences are: in the short term, the private sector low wage helps both private firms and aggregate output to grow rapidly. In the long term, the capital market distortion slows down the growth. The theory suggests that the economy develops along an endogenous three-stage transition: *rapid growth* is followed by *state capitalism*, and then the economy may follow one of the two paths in the third stage, *middle-income trap* or *sustained growth*, depending on whether democratization occurs. The theory is consistent with salient aspects of China's recent development and gives predictions on China's future political and economic development. It also offers explanations for some general questions in development, e.g., the challenges to sustain growth and the growth pattern after democratization.

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1 Introduction

China has by now been growing at a stellar rate for more than three decades. While this is generally acknowledged to be a great historical achievement, there is major controversy on how far in time and scope the Chinese success story can go. The optimists argue that China can provide a new model for growth as an alternative to the liberal democracy growth model known as the Washington Consensus. For example, in a debate hosted by The Economist (see also Musacchio and Lazzarini (2012)), Aldo Musacchio argues that China's hybrid form of capitalism can become a new growth model for the 21st century. In his view, such a model offers attractive features including mitigating recessions, focusing on long-term investment, and producing world champion companies. These features make him optimistic about the sustainability of China's future growth, and even the possibility that China could become a role model for other developing and emerging countries. In contrast, pessimists predict that China's growth will soon slow down. For example, Acemoglu and Robinson (2012) argue that China's current institution is not compatible with innovation and sustainable long-run growth, for the following reason. The extractive institution can lead to rapid growth in the early stage, when economic growth is in line with the interest of the ruling elite. However, in the long-run, the elite fears losing its economic rent to new technology or even losing its political power to groups rising from the growth and does not adopt economic arrangements favoring growth. China's growth process driven by catch-up may continue for a while, given the current institution, it will come to a halt as soon as China reaches the living standards of a middle-income country.

The pessimistic perspective of Acemoglu and Robinson raises a number of questions. Will slowing growth, which they predict, trigger changes in the political system, with unsatisfied citizens outing the oligarchy, and in turn allowing growth to resume under a more democratic system? Or, alternately, will the oligarchy be able to retain sufficient support even in a low-growth economy? Modernization theory suggests that democratization is likely to occur. But, then, one can argue that it may have been right for China to adopt its hybrid form of state capitalism to achieve high economic growth in the catch-up stage, and then switch to liberal democracy when state capitalism runs out of steam. The Chinese model, in other words, could be a model of transition, albeit not a mode of long-

run growth for mature economies. This view, however, may well be overly optimistic: at the time of transition, the political elite could be unwilling to give up state capitalism, and might seek to maintain political power and control of economic resources, as we see in countries like Venezuela. In the language of Acemoglu et al. (2006), state capitalism may be appropriate to promote growth at an early stage of development, but may be impossible to reform when it becomes a barrier to further economic growth.

To answer the above questions - first, whether China's growth can continue, and second, whether changes in political system will occur - this paper proposes a theory of politico-economic transition. A two-sector dynamic general-equilibrium model is built and calibrated to China's economy. The theory is consistent with salient aspects of China's recent developments, including: rapid growth with low wages, large state investments, financial restriction on private firms, the support of the middle-class for the government, and so on.

In this theory, a political elite runs the government and is able to extract surplus from state firms and tax the private sector at an exogenous rate. However, it faces a political constraint, that is, support from a sufficient number of citizens. I assume that the government can use the following policy tools to maintain the support: regulating the state sector wage, and controlling capital allocations in the state and the private sector. [1] How does the elite use these tools? First, to buy support from state workers, it sets the state wage sufficiently high - higher than the income that a worker expects to in democracy. Therefore a dual labor market is created. State workers receive high wages and in turn support the elite. Private worker wages are reduced due to the general equilibrium effect, as follows. Facing high wages, state firms hire less than they could if wages were determined by the market. More workers are pushed to the private sector, and private sector wages are reduced. This "divide-and-rule" strategy gains support from state workers at the cost of private workers. Second, to keep enough supporters in the state sector, the elite needs to balance capital in the state and the private sector. When private firms hold little capital, it is cheap and easy to meet the political constraint, because workers' expected wages and incomes in democracy are low, and also because private firms hire few workers and the number of workers in the state sector is larger than the number of supporters needed. To extract more tax from the private sector, the elite encourages its growth and helps it to build capital. Once private entrepreneurs get rich and private firms hold more capital, a trade-off emerges: a larger private sector contributes more tax, but it

[1] This means that the government can only adopt clientelism to gain the support, as in Robinson et al. (2013). Other tools, for example, using direct lump-sum transfer to buy the support, are assumed away, following Acemoglu (2003). More discussions on this are in section 3.

also increases the cost of maintaining supporters, because it increases the wage and also competes for labor. Then the elite chooses to financially repress the private sector, i.e., to limit its borrowing from banks. This restrains the growth of the private sector capital and relative size.

Because the government policy for the capital market changes as the private sector grows, the economy's growth patterns look different in different stages of development. More specifically, the economy develops along a three-stage transition as follows. The first stage is *rapid growth*, during which the GDP share of the private sector grows rapidly, triggering reallocation and high productivity growth. Private firms benefit from low wages in the private sector, which are induced by the policy. The government supports privatization because it increases tax revenue. However, as privatization continues and the state employment share declines to the critical level, the economy enters the second stage, *state capitalism*. In this stage, the elite over-invests in the state sector to keep the state employment sufficiently large. The government also imposes gradually increasing financial restriction to limit the growth of private firms. Growth continues to be fast due to large state investment, which overcomes the slowing down effect of the financial restriction on private firms.

As the private sector capital keeps growing (largely through self financing), two possible outcomes emerge. The first is *middle-income trap*[2]: over-investment of the state and financial restriction of private firms continue, while the efficiency loss also grows, due to decreasing return to capital and the capital market distortion. Eventually, growth stops and the output level converges to a level lower than in democracy. This happens in the case that the cost of retaining the regime is low, e.g., when the number of supporters needed is small. The other possible outcome is *sustained growth*, following democratization. In this case, the cost of maintaining enough supporters in the state sector is high. As the private sector capital grows, the elite finds it too costly to continue investing in the less efficient state sector, and therefore chooses to democratize. Financial distortion between the state and private firms disappear and the economy keeps growing in democracy.

The first two stages in the theory are consistent with salient facts in the recent development in China. First, low private sector wages helps private firms and the economy grow rapidly. Between 1995 and 2007, the private employment share increased from 40% to 80% (see more details in section 2). This era of fast privatization implies large efficiency gain and *rapid growth*, as in the first stage of the theory. However, the employment

[2]This term is used to describe the phenomenon that a country grows rapidly out of poverty and attains a middle-level income but then fails to keep growing and become rich over a long period, in contrast to *sustained growth*, which describes the case that a country continues to grow fast from a poor country to a rich one. More details are discussed in the literature review.

share of private sector subsequently stopped growing. Private firms face tighter financial constraints, and around 60% of investment and the majority of bank loans are allocated to less productive state firms.[3] This capital market misallocation in favor of state firms implies that the economy is entering the *state capitalism* stage. The second fact is that the middle-class, consisting largely of state sector workers and private entrepreneurs, are the beneficiaries and supporters of the regime. The reason is that state workers receive high wages, and entrepreneurs benefit from the cheap and abundant labor in the private sector. Chen and Lu (2011) and Tsai (2007) document that the Chinese middle-class, including state employees and private entrepreneurs are "achieving their material interests without pursuing any real freedom". This phenomenon will be discussed in great details in section 2. Moreover, the theory is also consistent with and useful for understanding the following facts: high capital labor ratio in the state sector; low and decreasing state sector capital return; high and non-decreasing private sector capital return.

The third stage of the transition in the theory provides an answer to questions about China's future political and economic developments. The model in this paper, calibrated to China's economy, predicts that the economy will enter the *middle-income trap*. It is because the government is both economically and politically powerful. In other words, the government's cost of retaining enough supporters in the state sector is low. First, the government controls abundant financial resource through the banking sector and holds abundant financial assets, including the huge foreign reserve. It is capable of keeping investment in the state sector high and maintaining the current level state employment. Second, the current state employment share is not very large but has been sufficient to provide the support that the government needs and keep the political system relatively stable.[4] In other words, the government is politically powerful and a relatively small supporter base is sufficient. Given these conditions, support for the regime will continue, and policy distortions will persist, which will eventually slow down the growth before China converges to rich countries.

Is China doomed to fall into the *middle-income trap*? Are there possibilities to redirect China towards the other development path, i.e. *sustained growth*? Many economists have proposed reform plans to sustain growth, including political reform, financial reform and state sector reform. However, an often neglected but important question is that, does the government want to implement those reforms? Many reforms which are beneficial for growth can be harmful for the elite's interests. With the aid of the model, I can study

[3]See Brandt and Zhu (2010) on the investment in state sector. Hsieh and Klenow (2009) estimate that the total factor productivity (TFP) of state firms is 42% lower than the TFP of domestic private firms.

[4]The state employment share is about 20% in the urban area, according to the statistical yearbook 2014.

consequences of these reforms, including whether a reform can lead to sustained growth, its effect on long-run growth, and how a reform affects the elite's interest. The last is important in determining whether a reform is likely to implemented, and therefore this analysis is useful for predicting China's future policies and directions of reforms.

This paper is related to three strands of literature. The first is on China's economic growth with structural transition. Song et al. (2011) construct a two-sector growth model to study how the capital and labor reallocation from the state to the private sector leads to rapid growth. Brandt and Zhu (2000, 2010) document the contribution of private firms to growth and the consequence of the government's strategy of maintaining state sector employment. Cheremukhin et al. (2015) study China's structural changes and the evolution of wedges in the labor and capital markets and their contribution to growth from 1953. These studies capture key features of China's economic growth, including the capital and labor market frictions. However, an important question is not answered yet: why do large labor and capital market frictions persist and how will they evolve in the future? Political constraints can be the root of these frictions, including the financial friction in Song et al. (2011) and the state employment constraint in Brandt and Zhu (2010). This paper provides the micro-foundation for the endogenous evolution of capital and labor market frictions. In contrast to the conventional wisdom which expects these frictions to gradually decline as China's labor market and financial market become more mature, this paper predicts that they will be persistent and will even increase within the current political regime.

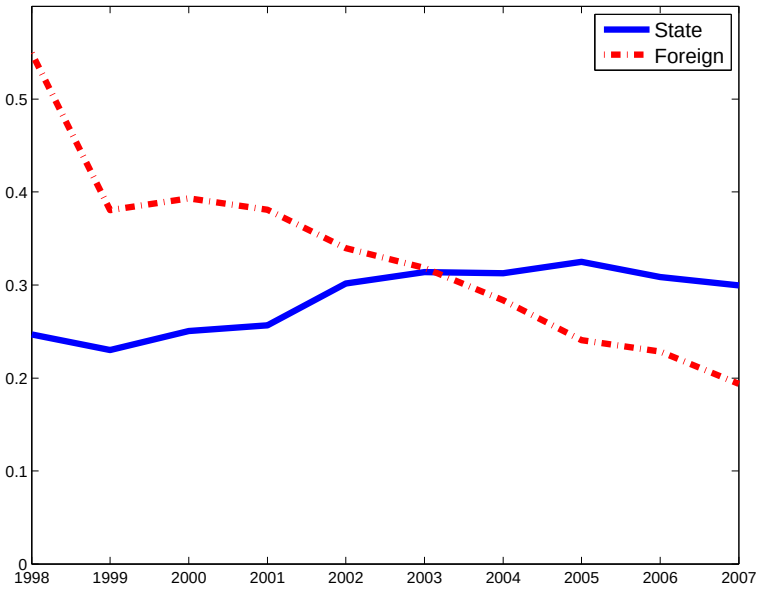
Second, the theory contributes to the study of *middle-income trap*, i.e., the phenomenon that some middle-income countries, which have rapidly grown out of poverty, experience slow growth and get stuck at the middle-income level for very long time, e.g., Argentina and Indonesia. In contrast, some others continue to grow fast and converge to rich countries, e.g., Korea. The empirical studies on the middle-income trap have been developing and the understandings on this phenomenon have been improving. The literature starts from (incorrectly) claiming that when countries reach the middle-income level, defined as an income range in *absolute* term, e.g. \$2,418 to \$15,220 in 2005 PPP, growth becomes more difficult and slower, and they are likely to stuck in this income range. See, e.g., Gill and Kharas (2007). Later researchers, e.g., Bulman et al. (2014) and Han and Wei (2015), study cross country income panel data and reject the initial claim. They show that on average MICs do not grow slower compared to other countries, and in the long-run, none of them will stay below certain absolution income level. Their results suggest the following. First, in some MICs, it is not the absolute income but the income *relative* to the U.S. may stuck in a moderate range. Han and Wei (2015) show that while in the long-run

all MICs will reach income levels above $15,220, the relative income to the U.S. of many MICs will stay between the range of 16% to 36% and 36% to 75%, corresponding to the lower-middle-income and the upper-middle-income level in relative term. Second, the middle-income trap is a *conditional* phenomenon. Conditional on policies and fundamentals, some MICs can avoid the trap and converge to the U.S., while others fail to do so. Han and Wei (2015) identify important policies and macroeconomic conditions for the growth of MICs, e.g., openness. MICs which fail to provide the right policies and conditions, are likely to experience slow growth and get stuck in that group for very long. Why do some MICs successfully adopt policies to escape the middle-income trap while others fail to do so? The discussion has been heated, but there is a lack of theoretical frameworks to guide the discussion. In this paper I try to provide a tractable framework. A growth model is built to study how and why an economy initially grows rapidly fails to sustain the growth. The model is also used to analyze policies and conditions that determine the development path of a MIC, and political and economic reforms that help to avoid the middle-income trap.

The third strand of literature is on the relation between political development and economic development. Acemoglu and Robinson (2012) explain how political institutions affect economic performance in the long run. They argue that the *extractive* political institution is detrimental to long-run growth. On how economic development affects political development, the modernization theory, originated from Lipset (1959a) suggests that the economic development will ultimately lead to political modernization, i.e., liberal democracy. This paper's contribution to this strand of literature is two-folds. First, it combines both sides of the relations between political and economic developments and studies their interactions. This is done by incorporating political economy into a growth model. Second, the theory distinguishes the short-term effect of political institutions on economic development from the long-term effect, i.e., institutions that help rapid growth in the catch-up stage can be detrimental to growth in the long run.

The rest of the paper is organized as follows. Section 2 shows important empirical facts on China's political-economic development that motivate the theory. Section 3 discusses a two-sector dynamic growth model with the three-stage political-economic transition. The first two stages explain important phenomena and puzzles in China's recent development, while the third stage predicts future politico-economic trend. In section 4, extensions and robustness of the model are discussed. Section 5 concludes.

Figure 1: State Sector Wage Premium



Source: Ge and Yang (2014).

2 Empirical Facts on China's Recent Development

In this section, the following key facts in China's recent development that motivate the theory are documented: (1) large wage gap between the state and the private sector; (2) low support for democracy from the middle-class; (3) the slow-down and stop of privatization; and (4) financial market wedges between the state and the private firms.

2.1 Large State-Private Wage Gap

China's state workers have been enjoying a wage premium of around 20% to 30%, as documented by Ge and Yang (2014). Their finding is based on a Mincer regression controlling for observable characteristics - age, education, industry, region and so on, and their result on the state wage premium is reproduced as the blue line in figure 1, showing a persistent labor market friction between the state and the private sector. In contrast, the foreign firm wage premium, shown as the red dashed line, has been declining, implying that the labor market has become more efficient, at least in the private sector. To compare with other countries, the wage premium of state workers in Canada, Germany and the U.S. are estimated to be lower than 5% or insignificant after the 1990s. See Melly (2002), Mueller (1998) and Poterba and Rueben (1994).

2.2 Middle-class Support for the Regime

Given that the state workers earn high wages, it is not surprising to see that they are more supportive of the current political system compared to non-state workers, as documented by Chen and Lu (2011). The authors use survey data of 2810 individuals, collected in three Chinese cities in late 2006 and 2007 to estimate how the individual's political opinions depend on his or her characteristics, especially the social group identity. They find that state sector workers and the middle class are less supportive for democratic values. For example, only 24.9% of the middle class support multi-party competition, while 38.7% of the lower-class do. Only 22.9% of the middle class agree that demonstrations should be allowed, while this number is 35.6% for the lower class.[5] Similar patterns apply for other questions related to democratic values and institutions. To formally show the difference between the middle class and the lower class, the authors combine answers to multiple questions into one index of support for democratic values and institutions using factor analysis.[6] Then they run a regression of this index on individual characteristics, including a dummy for middle-class membership and a dummy for state employment. The coefficients of dummies for middle-class and state employment are both negative (-1.23 and -0.54) and significant at at 1% level. In contrast, party membership does not affect the political opinion too much, after controlling for other variables. The coefficient of the party membership in the regression is -0.37 and not significant at 5% level. These suggest that economic interest plays a more important role than ideology. In other words, the middle class, including many state sector workers, are more supportive for the current political system. In another paper, Tsai (2007) documents that the Chinese entrepreneurs are "achieving their material interests without pursing any real freedom," in contrast to "the business classes in historical England, France and the United States" who "have risen up against the government to defend material interests. "

In short, the Chinese middle class, consisting largely of state workers and private entrepreneurs do not support democracy, contrary to the European history and the conventional wisdom that the middle-class are the driving force for democratization and reforms

[5]The authors define class according to the employment status. Individuals with jobs which usually pay low wages are classified as the lower class, including blue-collar workers, unemployed and self-employed with very little capital. The middle class mainly consist of white-collar workers. They distinguish private entrepreneurs from the middle class, while private entrepreneurs are usually considered an important part of the middle class. The authors also report that private entrepreneurs hold similar political opinions as the middle class.

[6]The survey data contain four dimensions of questions on support of democratic values, including right consciousness, valuation of political liberty, support for participatory norm and support for competitive election. The index for support for democratic values and institutions is the constructed as the single dominant factor using factor analysis.

(see Chen and Suen (2015) for example).

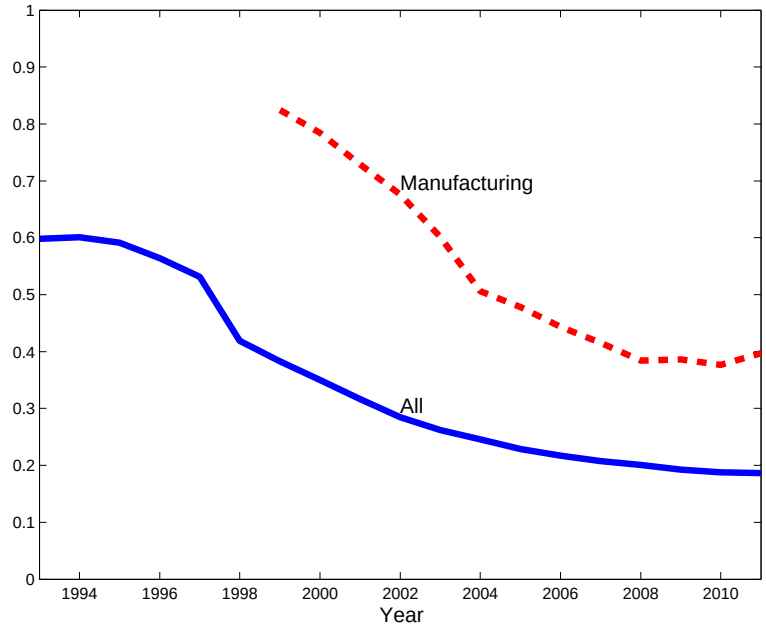
2.3 The Slow-Down and Stop of Privatization

If state workers support the government while many state firms are not productive, will the government allow the state employment to decline? The answer is mixed: initially the government allows the state employment to decline in order to improve the efficiency of the economy, but it does not allow the state employment to become too low, because state employees are an important supporter base. The decline of state employment and the privatization of state firms was very rapid for a couple of years, after the fifteenth national congress in 1997, which initiated the state firm reform. Many inefficient state firms bankrupted or got privatized, while many private firms entered the market and grew rapidly. As the blue line in figure 2 shows, the employment share of state sector in urban areas declined rapidly from 53% in 1997 to 23% in 2005. After that, the privatization slows down and the state employment share stagnated at around 20%. If we focus only on the manufacturing, mining, and construction, represented by the red line, the trend is similar while the state employment share stops declining at a higher level around 40%, and even slightly increases after 2011. This is the so-called "the state advances as the private sector retreats" phenomenon and it suggests that the privatization and the decline of state employment has come to a halt. Moreover, there is more direct evidence that the government intentionally keep the state sector alive. For example, the closing announcement of the Third Plenary Session of 18th Chinese Communist Party Central Committee in 2013 stated that "China will stick to the dominant role of public ownership, playing the leading role of the state-owned economy, while encouraging, supporting, and guiding the non-public sector."

2.4 Capital Market Wedge between the State and the Private Sector

How do state firms survive and hire a significant fraction of workers, if they are much less efficient than private firms, as documented Hsieh and Klenow (2009)? State firms survive because they are in a more advanced position in the financial market compared to private firms. It is easier and cheaper for state firms to get loans from state banks compared to private firms. Song et al. (2011) document, as shown in figure 3, that while state firms finance more than 30% of their investment through bank loans and government budget, this number is less than 10% for private firms. Their result is reproduced in figure 3. Brandt et al. (2012) estimate that the capital wedge, i.e. the ratio of costs per unit of capital

Figure 2: State Employment Share in the Urban Sector



Source: Statistical Yearbook 2012 and Storesletten and Zilibotti (2014).

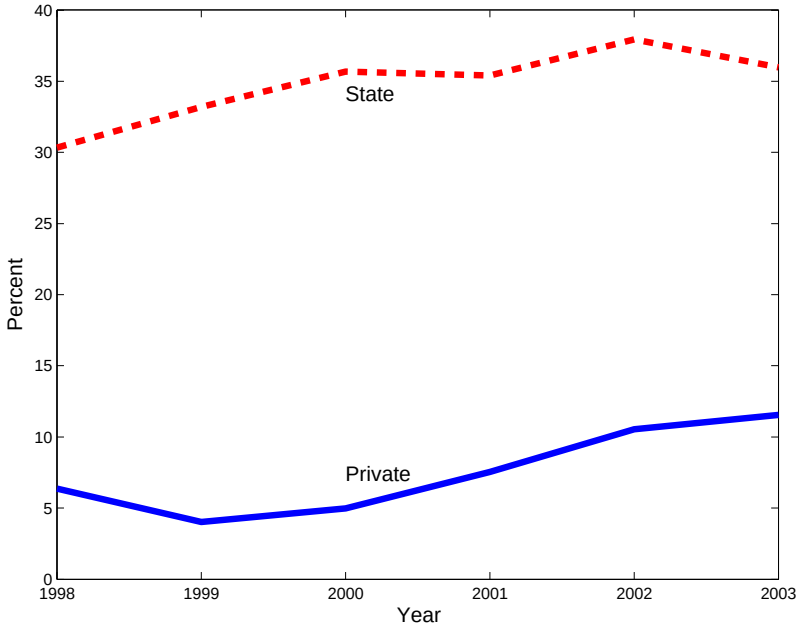
between state and private firms, has increased in all the provinces, on average from 4.2 in 1996 to 6.8 in 2007.

Is the capital wedge between state and private firms due to China's immature financial market so it will decline as the financial market develops over time? Or alternatively, is the capital wedge maintained by the government to keep the state sector large enough, and will the government strategically keep it or even increase it in the future? In the next section, we can study these questions with the help of a general equilibrium growth model with political constraints. The model is also used to explain other facts discussed in this section, including the state wage premium, middle class's support for the regime, and the decline of state employment share. Moreover, the model is used predict the future trend of these phenomena, as well as economic growth, political transition.

3 The Model

This section presents a theory of politico-economic transition to address the questions and to understand the key phenomena discussed above. I build a two-sector dynamic general equilibrium growth model which incorporates the political constraint and the political choices of agents to study the interactions between political and economic developments. I first discuss the general properties of the model and then study its implications for China

Figure 3: Share of Investment Financed by Bank Loans and Government Budget



Source: Song et al. (2011).

with the aid of a calibrated economy.

3.1 Preferences and Technology

The model economy is populated by three classes of agents: an elite (e), private entrepreneurs (p), and workers (w). Each class consists of infinitely many members. This means that a single agent is small and takes prices as given.[7] The population of workers is normalized to measure 1, and the population of elite members and private entrepreneurs are both assumed to be small and of measure 0.

There are two sectors and two types of neoclassic firms. State (S) firms produce in the state (S) sector, while private (P) firms in the private (P) sector. There are infinitely many of them, so they are price takers. They produce the same final goods using capital and labor, and they maximize profits. They are different in two aspects: owners and productivities. a S firm is owned by an elite member and its capital is financed by the elite member using her asset and bank loan, while a P firm is owned by an entrepreneur and financed by her similarly. S firms are less productive than P firms. The technology of

[7] As we will see later, only when certain class controls the government, the representative agent in that group sits in the government and makes decisions that affect prices.

S and P firms is described by the following production functions:

$$\begin{aligned} Y_S &= (z_S K_S)^{\alpha} L_S^{1-\alpha}, \\ Y_P &= K_P^{\alpha} L_P^{1-\alpha}, \end{aligned}$$

where $z_S < 1$, K_S, K_P are S and P sector capital while L_S, L_P denote for S and P sector labor, respectively.

The elite and entrepreneurs earn income from their assets and capital returns from S and P firms, respectively. The elite may also receive transfers from the government. An elite member or an entrepreneur lives for infinite periods, and maximizes her lifetime utility. The instantaneous utility is assumed to be logarithmic and the discount factor is denoted by β. Workers provide 1 unit of labor inelastically. For simplicity, workers are assumed to live hand-to-mouth and they are myopic, i.e., they consume all their income every period, and in each period they care only about current period income.[8]

Banks are owned by the state and compete with each other. They can borrow and save in the international bond market at the exogenous interest rate r. This means that if there is no distortion, the market interest rate is at r. The elite has deep pocket. There is no constraint on how much bank loans it can get and therefore how much capital it can supply to S firms. An entrepreneur faces a credit constraint: the bank loan cannot exceed $\eta - 1$ fraction of its asset. This implies that in P sector the leverage (ratio of capital over net asset) is bounded above by η. The government can influence η, with some limits. η is set by the government subject to $\eta \in \left[\underline{\eta}, \bar{\eta}\right]$. The above setting on the financial market is similar to Song et al. (2011), which also assume that the state firms have unlimited access to bank loans while private firms face financial constraints. The key difference is that I allow the S firm bank loans and the P sector financial constraint - leverage η - to be endogenously determined by the government. First, the government can order the banks to provide certain amount of loans at certain price to S firms, to control the capital in S sector. Second, the government can create barriers for loans to private entrepreneurs, and can directly give administrative instructions to banks (see Brandt and Zhu (2000)), in order to influence the leverage in P sector. $\underline{\eta}$ is the lower bound of the leverage. For example, $\underline{\eta}$ equals 1 means that the strictest policy that the government can set is to order banks not to lend to private entrepreneurs at all. Then the entrepreneurs can still finance their investment using their own assets. $\bar{\eta}$ is the highest leverage if the government doesn't restrict private sector financing at all. The upper bound for the leverage can be thought as the consequence of a moral hazard problem, as in Song et al. (2011), i.e., an entrepreneur with too

[8]Expected outcomes after relaxing this assumption are discussed in section 4.

high level loan compared to her asset chooses not to repay the loan and run away.

All above agents are infinitely small, competitive, and autonomous. Each agent - an S firm, a P firm, an elite member, an entrepreneur, or a bank - takes aggregate prices and macro policies as given, and decides on its or her action. As you will see below, the government has incentives to influence some agents' actions, but it has to do it through manipulation of prices but can not directly control the actions of the agents.

3.2 Political Systems and the Government

There are two types of political regimes: democracy and oligarchy. In either political regime, the government can collect taxes and make transfers. I make the following important assumption on the tax system.

Assumption 1. *The government can not make lump-sum transfer to the ruled group.*

As argued in Acemoglu (2003), it is crucial to assume that the government can not freely transfer to the ruled group. Otherwise, Coase theorem applies and politics doesn't matter: in all regimes, the same maximized output is achieved, while the only difference is how the ruling government uses transfer to distribute the output. See more discussions on this assumption in subsection 4.1. Similarly, for other assumptions and important settings in the rest of the paper, detailed discussions and explanations are left to subsection 4.1, and how relaxing or modifying them affect the model results are discussed in section 4.

Democracy The government is elected by a majority vote. Hence the representative worker runs the government forever, given the dominating size of workers. According to assumption 1, the government can tax the elite and entrepreneurs and can only transfer to workers. The tax rate is exogenously given as $\tau^D > 0$. [9]To simplify the expressions, I assume that the tax is on the gross capital return instead of the net capital return, which is the former deducting depreciation. In each period, the capital and the labor market are assumed to be competitive without distortions on prices or the credit constraint. This means that $\eta^D = \bar{\eta}$, and each entrepreneur decides capital supply to her P firm subject to the financial constraint $K_P^D \leq \bar{\eta} a_p^D$, while each elite member chooses capital supply to her S firm without constraint. Each worker supply labor to an S or P firm. In other words, democracy implies that in each period, the economy is in a competitive equilibrium given

[9]Alternatively, one can endogenize the tax decisions while still getting the equivalent results, as in Acemoglu (2008) and Besley and Persson (2009). Suppose that the government decides which groups to tax and tax rates, and tax payers can hide their income at the cost of τ^D fraction of the income. Then, the government sets tax rate at the highest level: τ^D which does not trigger tax hiding.

taxes on the elite and entrepreneurs. The distortion in democracy is only the capital taxes. This setting is natural and is in the spirit of Acemoglu (2008).

Oligarchy The elite controls the government, but it faces a political constraint, that is, it needs political support from a sufficiently large fraction of workers.[10] Each worker, after being employed by either an S or a P firm and observing the government policies, decides whether to support the oligarchy based on the expectation on her incomes in each regime. Oligarchy is sustained if more than $\underline{L}$ workers choose to support it. If less than $\underline{L}$ workers support oligarchy, revolution occurs, and it leads to democratization. There is no cost for revolution.[11] In oligarchy, the government collects taxes from entrepreneurs and private firm workers and then transfer to the elite. The tax rate is exogenously given as $\tau > 0$. [12]

Due to the political constraint, the elite controlled government is motivated to influence the labor and the capital market. For example, it wants to set high wages for some workers to get their support. The market is no longer in a competitive equilibrium. What the oligarchic government can control in the labor market is assumed as follows.

Assumption 2. *In oligarchy, the government can set a minimal wage in S sector - w_S, but cannot directly set the S sector employment - L_S. Given w_S, each S firm decides employment to maximize its profit.*

The assumption implies that the government can control w_S as long as it is set to be higher than the competitive equilibrium wage without distortion while it cannot at the same time decide S sector labor freely. It has to take into account the influence of w_S on L_S. For example, the government can not increase S sector wage while forcing S firms not to reduce employment, given other things, e.g., capital, the same. Also, the government is not allowed to use direct labor subsidy to offset the effect of increasing S sector wage on S sector labor. Furthermore, this assumption also implies that the government can not set the wage in P sector, namely, it can use higher wages to buy support from S sector workers

[10] Notice that in oligarchy, the government, the elite, and the representative elite in the government are the same and interchangeable.

[11] As we will see in section 4, revolution is off equilibrium path because there is no uncertainty, so its cost does not directly affect the economy. However, if may affect the equilibrium through the expectations. The consequence of adding the cost of revolution is discussed also in that section.

[12] I use τ instead of τ^O, to simplify the notation. In the rest of the paper, the superscript O for variables in oligarchy is dropped when there is no confusion. Similar to the case in democracy, tax decisions can be endogenized. The elite optimally chooses not to tax S workers to make it easier to buy their support. The elite may or may not set the tax rate on entrepreneurs to the highest possible level, depending on how much asset entrepreneurs hold. In our calibrated model, the numerical solution with endogenous tax rate decision is that the elite always optimally chooses to tax entrepreneurs and P workers at the highest rate τ, so it is equivalent to the simple setting of exogenous tax rate.

but not P sector workers. Same as in Robinson and Verdier (2013), the distribution to buy the support takes the form of public sector jobs.

The government also controls the capital market. Though state banks, it influences bank loan allocation to S sector or P sector. First, the government sets bank loans to the elite members and the S firms. This can be done through price manipulation: for example, the government can offer a special interest rate of loans to S sector through banks, and the associated amount of the special loans. If the interest rate is set properly, the elite takes the loans and invest in S firms, and find that the after-tax marginal return of S firm capital equals the loan interest. Since there is no credit constraint for the elite and its S firms, the government can allocate as much bank loans as it wants into S sector and it directly sets K_S. The difference between the market interest rate and the bank loans to S firms is paid by the government, which eventually reduces the transfer to the elite, and enters the elite's final income. S firms compete for the loans and equivalently the capital in S sector. Second, the government can influence the capital allocation to P sector through the leverage η. Then P firms compete for capital in P sector given $K_P \leq \eta a_p$.

3.3 Equilibrium and Aggregate Dynamics

Given the settings described above, the rest part of this section presents the solution of the model. The dynamic equilibrium consists of infinite periods, and each period can be separated into three stages: (1) determination of capital in S and P sectors, (2) political outcome and the equilibrium of the labor market in this period given capital allocation, and (3) decisions on consumption and saving. In the following, I first focus on stage (2) of each period and study the political and economic outcomes given capital allocation, and then I present how capital is allocated and saving is determined.

3.3.1 Equilibrium Given Capital Allocation

Democracy The labor market is competitive. Wages in S and P firms are the same and are equal to the marginal productivity of labor:

$$w^D = (1-\alpha)\left(z_S K_S\right)^{\alpha}\left(L_S^D\right)^{-\alpha} = (1-\alpha)\left(K_P\right)^{\alpha}\left(L_P^D\right)^{-\alpha}.$$

A worker's income equals the wage plus the tax collected from entrepreneurs and the elite:

$$\begin{aligned} y_w^D &= w^D + \tau^D \left(\pi_S^D + \pi_P^D \right) \\ &= \left(1 + \tau^D \frac{\alpha}{1-\alpha} \right) w^D, \end{aligned}$$

where π_S^D and π_P^D are the capital incomes of the elite and entrepreneurs from S and P sectors, respectively, and τ^D is the tax rate in democracy. Notice that to simplify the expressions, I assume that the taxes are applied to the raw capital incomes but not capital incomes net of depreciation and interest payment. The transfer to workers is $\tau^D \frac{\alpha}{1-\alpha} w^D$ simply because the tax base, capital income, is $\frac{\alpha}{1-\alpha}$ times labor income.

Oligarchy In each period, the timing of events given the capital allocation is the following:

1. The government sets S sector minimal wage.
2. S and P firms hire workers. Ex-ante identical workers are randomly selected by S firms.
3. S and P workers decide whether to support the current political system.
4. The share of supporters determines the political outcome, i.e., if oligarchy does not get enough support, revolution happens and the economy switches to the equilibrium in democracy.
5. Firms produce, labor and capital incomes are distributed.
6. The government collects taxes and makes transfer.

First, the government chooses S sector minimal wage w_S to influence the labor market outcome and the economic benefits of S and P workers. I can safely only consider the cases where $w_S \geq w^D$ so the minimal wage constraint is tight.[13] Given the minimal wage, the representative S firm chooses labor demand L_S such that wage equals marginal productivity:

$$w_S = (1-\alpha) (z_S K_S)^{\alpha} L_S^{-\alpha}. \tag{1}$$

Remember that the oligarchic government cannot use direct transfer to buy political support, so the final income of S workers is simply $y_{wS} = w_S$. A worker's political support

[13] Setting minimal wage $w_S < w^D$ is equivalent to setting $w_S = w^D$.

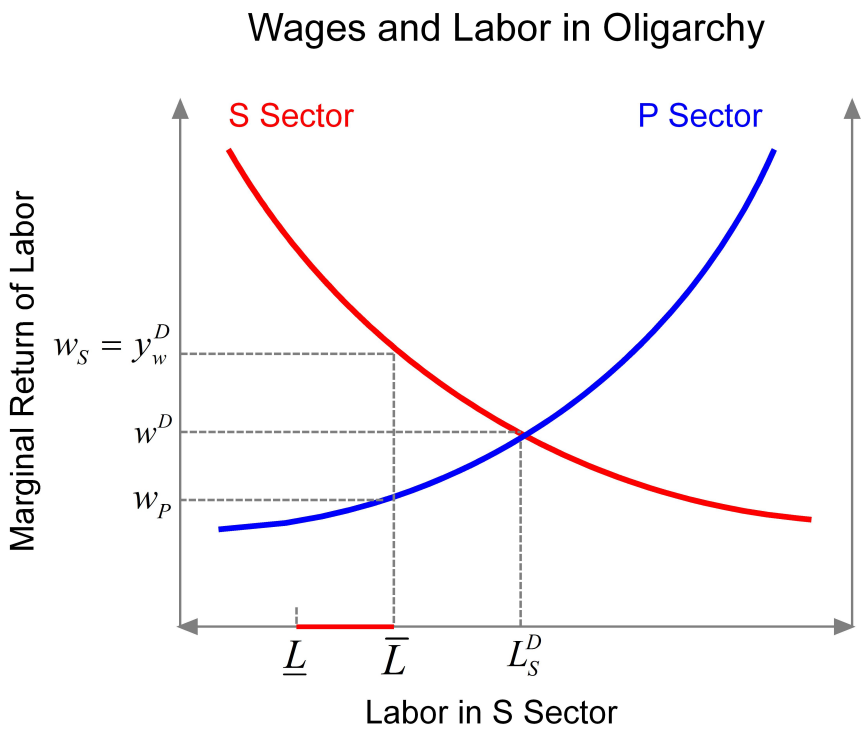


Figure 4: Labor allocation and marginal returns.

is assumed to be sincere, i.e., she supports oligarchy if and only if she expects her final income to be higher in oligarchy. Her action is as follows:

$$\max_{i \in \{D,O\}} \left\{ y_j^i \right\},$$

where $i \in \{D, O\}$ stands for the index for the regime, and j for the state or private sector. If the government wants to increase S worker income, it has to set a high S sector minimal wage, which distorts the labor market. We can see this in figure 4. Red and blue lines are the marginal productivities of labor in S and P sectors, respectively. The intersection of the two lines pins down the equilibrium in democracy: the S sector labor, wage and worker income in democracy are denoted as L_S^D, w^D and y_w^D. In oligarchy, w_S pins down S sector labor and its marginal productivity. The rest of labor is in the P sector and pins down the P sector wage w_P. Setting w_S greater or equal to y_w^D implies that the marginal productivity of S sector labor is greater or equal to y_w^D and the S firms hire less or equal to $\bar{L}$.

Observing the government policy on w_S, an S worker can determine her income in oligarchy. Given that there is complete information, she supports oligarchy if and only if her income in oligarchy is higher than in democracy, i.e., $w_S \geq y_w^D$, under the assump-

tion that she cares only about current period income. A private sector worker always gets lower income in oligarchy than in democracy and never supports oligarchy. First, high state sector wage pushes down the private sector wage through general equilibrium effect. Setting high state wage $w^S \geq w^D$ implies low state employment: $L_S \leq L_S^D$, and large size of labor in P sector: $L_P \geq L_P^D$. Then the marginal productivity and wage for P workers are low: $w_P \leq w^D < y_w^D$. Second, because the government cannot make transfers to the ruled groups, a P worker's income is equal to her after-tax wage, and therefore always lower than in democracy $y_{wP} = (1-\tau) w_P < y_w^D$.

When w_S is high enough, S workers can become supporters of oligarchy. If the number of S workers is sufficiently large, oligarchy gets enough support and is sustained. As I discussed previously, $w_S \geq y_w^D$ implies $L_S \leq \bar{L}$. Moreover, sufficiently many supporters means $L_S \geq \underline{L}$, where $\underline{L}$ is the minimal number of supporters to sustain oligarchy, exogenously given.

To summarize, the political constraint that the government faces is equivalent to two economic constraints. The first is the *high state wage constraint*, i.e. $w_S \geq y_w^D$ so that S workers support oligarchy. Then high enough state wage is equivalent to low enough state employment share $L_S \leq \bar{L}$. The second is the *minimal support constraint*, i.e.,

$$L_S \geq \underline{L}. \tag{2}$$

The government faces a critical labor market trade-off between these two political constraints: a high w_S buys S workers' political support and guarantees high state wage constraint while it implies a low level of S sector employment L_S, which may violate the minimal support constraint.

If the government can choose w_S such that the two constraints are both satisfied, oligarchy is sustained. However, it is not always true that both constraints can be satisfied at the same time. This depends on the capital allocation between S and P sectors. $\underline{L}$ is an exogenous parameter, determined by political power of workers and the elite. $\bar{L}$ is endogenously determined by y_w^D, which depends on the capital allocation K_S and K_P, as follows:

$$\begin{aligned} w_S &\geq y_w^D, \\ (1-\alpha) K_S^{\alpha} L_S^{-\alpha} &\geq \left(1 + \tau^D \frac{\alpha}{1-\alpha}\right) (1-\alpha) K_S^{\alpha} \left(L_S^D\right)^{-\alpha}, \\ L_S &\leq \nu L_S^D = \nu \frac{z K_S}{z K_S + K_P} \doteq \bar{L}, \end{aligned} \tag{3}$$

where $\nu = \left(1 + \tau^D \frac{\alpha}{1-\alpha}\right)^{-\frac{1}{\alpha}}$. If S sector capital is large enough, i.e.,

$$\frac{zK_S}{K_P} \geq \frac{\underline{L}}{\nu - \underline{L}}, \tag{4}$$

then $\bar{L} \geq \underline{L}$, and $\exists L \in [\underline{L}, \bar{L}]$, such that both the *high state wage constraint* and the *minimal support constraint* are satisfied. In other words, sustaining oligarchy requires that S sector is equipped with enough capital, relative to the P sector capital. Then the equilibrium given capital can be characterized in the proposition below.

Proposition 1 (Equilibrium given capital allocation)**.** *If there is sufficiently large capital in S sector relative to the capital in P sector- $\frac{K_S}{K_P}$, oligarchy can be sustained in the period. Both wage and capital labor ratio in S sector are higher than in P sector while capital return in S sector is lower. Capital return and entrepreneur income in P sector are higher than in democracy. If S sector capital is small, regime changes.*

In S sector, the capital labor ratio is high and capital return is low because of the high wage and low level of labor, as shown in 4. In P sector, because of the low wage and the abundant labor, capital return is high. In this case, the one period elite income net of asset return $(1 + r) a_e$, is the transfer from the government, which includes S sector profit and P sector tax income:

$$y_e \quad = \quad \pi_S - (r + \delta) K_S + \tau w_P L_P + \tau \pi_P, \tag{5}$$

where $\pi_S = \alpha (zK_S)^{\alpha} L_S^{1-\alpha}$ and $\pi_P = \alpha (K_P)^{\alpha} L_P^{1-\alpha}$ are capital incomes of S and P firms, respectively. An entrepreneur's income from capital, net of asset return $(1 + r) a_p$, is

$$y_p = (1 - \bar{\tau}) \pi_P - (r + \delta) K_P. \tag{6}$$

One can write down the elite or an entrepreneur's income in a different way and explicitly take into account the cost of loan, e.g., $r\left(K_P - a_p\right)$. It is equivalent, after some simplification.

3.3.2 The Dynamic Equilibrium

Because of the importance of economic power - capital, the government is motivated to control capital accumulation and allocation between the state and the private sector. In the following, I present the dynamic equilibrium, including the allocation of state and private sector capital, consumption, and saving, based on the equilibrium given capital allocation discussed above. The timing is the following:

1. In the beginning of each period, the representative elite in the government decides to democratize or not. If yes, the economy switches to the equilibrium in democracy. If not, the following events happen.

2. The government allocates bank loans into S and P sector. Capital is built accordingly.

3. The events in the equilibrium given capital happen. See the timing in 3.3.1.

4. Agents save. The economy enters the next period.

Democracy Workers control the government to maximize their income. The government does not want to change the political system, and since there is no political constraint in democracy, democracy continues forever in this model. The economy in democracy is assumed to be a decentralized competitive equilibrium given taxes. Labor market and capital market are both competitive.[14] The dynamics in democracy is basically a two sector growth model in which resource are gradually reallocated from the inefficient sector to the efficient one, as in Song et al. (2011). The dynamic equilibrium in democracy is summarized in the following.

Proposition 2 (Dynamic equilibrium in democracy)**.** *In democracy, each elite member gets return on her asset at interest rate r. Her income only comes from her asset and income net of asset return is* 0*:* $y_e^D = 0$*. An entrepreneur saves* β *fraction of her total resource - asset plus asset return - at the end of each period. If* β *is large enough, entrepreneur assets increase over time. Gradually, the relative size of S sector over P sector, measured by* $\frac{K_S}{K_P}$*, decreases to* 0 *.*

The intuition for the above result is the following: efficient labor allocation implies the same wage in S and P sector. S firms compete for capital in S sector, so the after-tax capital return equals the cost of financing, i.e., the interest rate r at which elite members can borrow from banks, or equivalently, the international financial market. The capital return pins down S firm capital labor ratio and wage. P firms hire workers at the same wage rate as S firms, but they are more productive, so P firm capital return is higher. Entrepreneurs, however, face financial constraint, so if entrepreneur asset and P firm capital are small, P firms can't hire all the workers and S firms still exist . In this case, entrepreneurs get a return higher than r from their asset. If β is large enough, entrepreneurs' savings increase over time, their assets and P firm capital increase over time, and finally P firms hire all workers and S firms all exit the market. Market force is decisive in such a competitive equilibrium and the inefficient S firms gradually get replaced by P firms..

[14] In fact, the democratic government has incentives to distort the capital market - it may prefer to over-invest in S sector to inflate the wage in both S and P sector. To follow the literature, I assume democracy as a competitive equilibrium and the only distortion is the tax.

Oligarchy As discussed in the model setting, the representative elite controls the government and decides on three policies: minimal wage in S sector w_S, S sector capital K_S, and P firm leverage η. Given these policies, the choices of workers and entrepreneurs are simple. S and P workers behave as in subsection 3.3.1, i.e., support oligarchy if and only if their income is higher than in democracy. Then they consume all the income.

Each entrepreneur, being a small agent, takes the political outcome and P sector capital return as given, and maximizes lifetime utility. One may think that the entrepreneurs, as a group, may benefit from increasing their asset and the P sector to facilitate democratization. This possibility is ruled out in this model, given the assumption that each entrepreneur is infinitely small, and also collective action is not considered. Only the government takes into account how its behavior affects the aggregate economy and the political system. An entrepreneur gets bank loans, lends to P firms, consumes, and saves for the future. She solves the following problem:

$$\begin{aligned} \max_{\{K_{Pt}, c_{pt}, a_{pt+1}\}} & \sum_{t=0}^{\infty} \beta^t \log c_{pt} \\ \text{s.t.,} \; & K_{Pt} \leq \eta a_{pt}, \\ a_{pt+1} = & \, y_{pt}\left(a_{pt}, K_{Pt}, r_{Pt}\right) - c_{pt}, \end{aligned} \tag{7}$$

where y_{pt} is expressed in equation 6, which depends on a_{pt}, K_{Pt}. r_{Pt} is the capital return, after P firms maximize profits according to w_{Pt}. Notice that the representative entrepreneur's capital is the same as the aggregate capital K_{Pt}. Because r_{Pt} is taken as given, we can safely write as the representative entrepreneur chooses K_{Pt}. The solution to the entrepreneur's problem is simple, stated in the following lemma and proved in the appendix.

Lemma 1. *In oligarchy, if the P firm capital return is higher than the interest rate of bank loan, an entrepreneur's optimal choice can be separated into two steps. First, she borrows as much as possible and invests all into P firms, to maximize her current period income; then, she saves a constant fraction of this period's total wealth and consumes the rest, to maximize her lifetime utility.*

The constant saving rate property is because an entrepreneur's income in each period is proportional to her asset, and the rate of return on the asset does not depend on the size of asset but on the equilibrium price.

In each period, the government maximizes the representative elite member's lifetime utility. Here the elite, the representative elite, and the government are the same and therefore exchangeable. The dynamic problem contains two parts. First, the elite chooses to

sustain oligarchy, to democratize peacefully, or to democratize after revolution:

$$W\left(a_e, a_p\right) = \max\left\{W^O\left(a_e, a_p\right), W^D\left(a_e, a_p\right), W^R\left(a_e, a_p\right)\right\}, \tag{8}$$

where W stands for lifetime utility given elite asset a_e and entrepreneur asset a_p, while W^O, W^D and W^R are the elite's lifetime utility, in the case of sustained oligarchy, peaceful democratization, and revolution, respectively. If peaceful democratization happens, the economy ends up in the dynamic equilibrium of democracy before capital is allocated. If revolution happens, which means that the elite chooses not to democratize but it allocates capital in a way that can not sustain the regime, the economy enters the equilibrium of democracy given the capital allocation. If it decides to sustain oligarchy, in the second part, she picks government policies η, K_S, w_S, τ_p to sustain oligarchy. She also decides consumption and saving to maximize her lifetime utility.

$$\begin{aligned} W^O\left(a_e, a_p\right) &= \max_{w_S, K_S, \eta, c_e, a'_e} \log c_e + \beta W\left(a'_e, a'_p\right) \\ \text{s.t. } w_S &\geq y_w^D\left(K_S, \eta, a_p\right), \\ L_S &\geq \underline{L}, \\ a'_e &= Ra_e + y_e\left(w_S, K_S, \eta, a_p\right) - c_e, \\ a'_p &= \beta\left(Ra_p + y_p\left(w_S, K_S, \eta, a_p\right)\right), \end{aligned} \tag{9}$$

where $R = 1 + r$. From the expression of y_e in equation 5, we can see that within each period a_e only contributes to the elite's income through interest revenue and does not constraint or affect other equilibrium choices at all. It doesn't directly affect future state variables a'_p and a'_e either. The contribution of a_e is simply Ra_e in the elite's budget constraint. Its only role is consumption smoothing. Therefore the representative elite's problem, similar to an entrepreneur's problem, can be separated into two sub-problems, as the following lemma states.

Lemma 2. *In oligarchy, the representative elite's optimal choices can be separated into two sub-problems. First, maximization of the lifetime income with discounting rate $\frac{1}{1+r}$ by choosing government policies. Second, maximization of the lifetime utility using a_e to allocate lifetime income and smooth consumption.*

The second sub-problem is straight-forward and does not affect the first one or the politico-economic outcomes. The first sub-problem therefore has only one state variable,

as follows:

$$\begin{aligned} V^O\left(a_p\right) &= \max_{w_S, K_S, \eta} y_e\left(w_S, K_S, \eta, a_p\right) + \frac{1}{R} V\left(a_p'\right) \\ \text{s.t. } w_S &\geq y_w^D\left(K_S, \eta, a_p\right), \\ L_S &\geq \underline{L}, \\ a_p' &= \beta\left(Ra_p + y_p\left(w_S, K_S, \eta, a_p\right)\right), \end{aligned} \tag{10}$$

where V^O stands for the part of discounted lifetime income of the elite which is not related to its asset a_e. Continuation value $V\left(a_p'\right)$ depends on the political outcome of next period, as follows

$$V\left(a_p\right) = \max\left\{V^O\left(a_p\right), V^D\left(a_p\right), V^R\left(a_p\right)\right\},$$

where $V^D\left(a_p\right) = 0$ because the elite income in democracy is simply the asset return Ra_e, as stated in Proposition 2.

This shows that the private entrepreneur's asset is the crucial state variable for the elite in oligarchy. It affects the elite's decisions on whether to democratize, choices of government policies, and its utility. A trade-off of the entrepreneur asset emerges. A larger entrepreneur asset allows for a larger private sector, which contributes more tax to the government. However, if entrepreneur asset is very large, then the private sector capital becomes large and so does the required state sector capital to sustain oligarchy, according to proposition 1. Due to the decreasing return of capital, maintaining large state capital can be very costly for the elite, and they may even prefer democratization is to oligarchy. This trade-off captures the key properties of the dynamic programming, and leads to the following property to the elite's choices: when the entrepreneur asset and private sector capital are small enough, the elite prefers them to be larger, and it chooses oligarchy over democracy. When they are large enough, the elite prefers them to be smaller, and it chooses democratization over oligarchy. This is formally stated in the proposition below.

Proposition 3 (Elite's Problem in Oligarchy)**.** *(1) When the private sector is small, a larger private sector makes the elite better off in oligarchy. If constraint 3 - high state wage constraint - is binding, then* $\forall K_S, \exists \varepsilon_1 > 0, \forall K_P < \varepsilon_1$*, such that* $\frac{\partial y_e}{\partial K_P} > 0$*; and* $\exists \varepsilon_2 > 0, \forall K_P > \varepsilon_2, \exists K_S$*, such that* $y_e > y_e^D$*.*

(2) When the private sector is larger, a larger private sector makes the elite worse off in oligarchy. If $\exists \sigma, \forall K_P > \sigma$*, both constraint 2 and 3 - minimal support constraint and high state wage constraint - are binding, then* $\exists \varepsilon_3 > 0, \forall K_P > \varepsilon_3$*, such that* $\frac{\partial y_e}{\partial K_P} < 0$ *and* $y_e < y_e^D$*.*

(3) If K_P *is increasing in* a_p*, the above results hold when* K_P *is replaced by* a_p*.*

(4) If V_e and $\frac{\partial V_e}{\partial K_P}$ are continuous on the dimension of the inter-temporal discount parameter, i.e., $\frac{1}{1+r}$, then $\exists \varepsilon_4, \forall r > \varepsilon_4$, the above results hold when y_e is replaced by V_e.

Result (1) in Proposition 2 states that given K_S, if K_P is small, increasing K_P increases the elite's current period income, and moreover, if K_P is small, the elite can achieve higher current period income in oligarchy than in democracy by properly choosing K_S. The logic is the following: when K_P is small, its marginal return is large, because there is always a significant amount of labor pushed out of S sector by the high S sector wage. Then for the elite, a larger K_P is better, at least for its current period income. Moreover, the elite can achieve higher income in oligarchy because it is not costly to keep enough supporters in S sector. Result (2) implies that if K_P becomes very large, the cost of maintaining oligarchy increases with K_P, and dominates its benefit, both marginally and compared with in democracy. This is because a large K_P requires a large K_S to maintain oligarchy and there is decreasing return to capital. Result (3) states that if K_P is positively linked to a_p, naturally the same properties hold for a_p. The above results focus on the elite's current period income, while result (4) states that if the elite discount the future enough, these properties are still the same if we study the elite's lifetime income. The analytical results hold under certain restrictive conditions, but as we will see in the quantitative analysis, these conditions are generally satisfied.

Given this property, one can expect that dynamics might be like the following: when the entrepreneur asset is small, the elite chooses to sustain the regime and promote private sector growth; however, when the private sector gradually grows larger, the elite changes the attitude and restrains its growth, and if the private sector becomes too large, which may happen or may not depending on the model parameters, the elite prefers peaceful democratization than maintaining oligarchy at the high cost. This dynamics will be shown using quantitative analysis.

3.4 Quantitative Analysis

In this subsection, I calibrate the model to the Chinese economy and solve the model numerically. The dynamics is simulated, to understand China's development in the past and to think about its future. The targets of the calibration are the key facts in China's recent development, including the wage gap, speed of privatization, and the state employment share.

3.4.1 Calibration

The economic parameters are set as follows. First, the production function is Cobb-Douglas with the capital share $\alpha = 0.5$ (Bai et al. (2006)).[15] The depreciation rate is set as $\delta = 0.1$ (Song et al. (2011)). The state capital efficiency is half of the private capital, i.e., $z_S = 0.5$. This implies that the TFP of state firms is 71% of the TFP of private firms. This is higher than 59% estimated by Hsieh and Klenow (2009) with data before 2005, but is reasonable considering that the trend of declining TFP gap discussed in Hsieh and Song (2015). Second, the interest rate of bank saving is $r = 5\%$. Third, the discount factor of entrepreneurs, which is also the saving rate of their lifetime income, is set to $\beta = 0.9$ to match the rapid private sector employment share growth from around 40% to around 80% in 5 years, as we can see from figure 2. Finally, the tax rate is set to $\tau^D = \tau = 20\%$ to match the state-private wage gap of 30%, as in figure 1. The political parameter in this model is $\underline{L}$, the minimal support needed to sustain oligarchy. I set $\underline{L} = 20\%$, as the state employment share converges to around 20% as in figure 2.

3.4.2 Numerical Solution

In the following, I explain the properties of the numerical solution of the elite's dynamic programming problem, in three steps: (1) given K_S and K_P, the choices of other variables; (2) given K_P, the choice of K_S; and (3) the choice of η that affects K_P.

First, given K_S and K_P, we know from subsection 3.3.1 that if K_S is large enough, there is some w_S that sustains oligarchy, or equivalently, some L_S that falls into the region $[\underline{L}, \bar{L}(K_S, K_P)]$. Generally, the optimal choice of w_S is y_w^D, or equivalently, $L_S = \bar{L}$. This choice implies the least labor distortion but still satisfies high state wage constraint.[16] In other words, the elite prefers not to distort the labor market more than the necessary. Furthermore, τ_P is generally set at the highest level $\bar{\tau}$.

Second, how does the government choose K_S, given K_P? In figure 5, I use a numerical example to depict how state sector labor, political outcome, elite income, and marginal

[15] One important feature in China's recent development is that the labor share has been declining. This seems to be inconsistent with the Cobb-Douglas production function with a constant labor share. However, Song et al. (2011) show that the declining labor share can be reconciled in a two sector model with Cobb-Douglas production. They explain the decline of labor share by the labor reallocation from the state sector to the private sector, where the labor share is smaller, not because of a different production functions but the payment to the management. I follow the literature and keep the Cobb-Douglas production function setting.

[16] This is true as long as the tax rate $\bar{\tau}$ is not too high. One sufficient condition is $\bar{\tau} \leq \alpha$, which is a reasonable constraint, considering that α is estimated to be around 0.5 in China. If $\bar{\tau}$ is too large, the elite can extract more from the private sector than from the state sector, the solution may change, but this is not very reasonable.

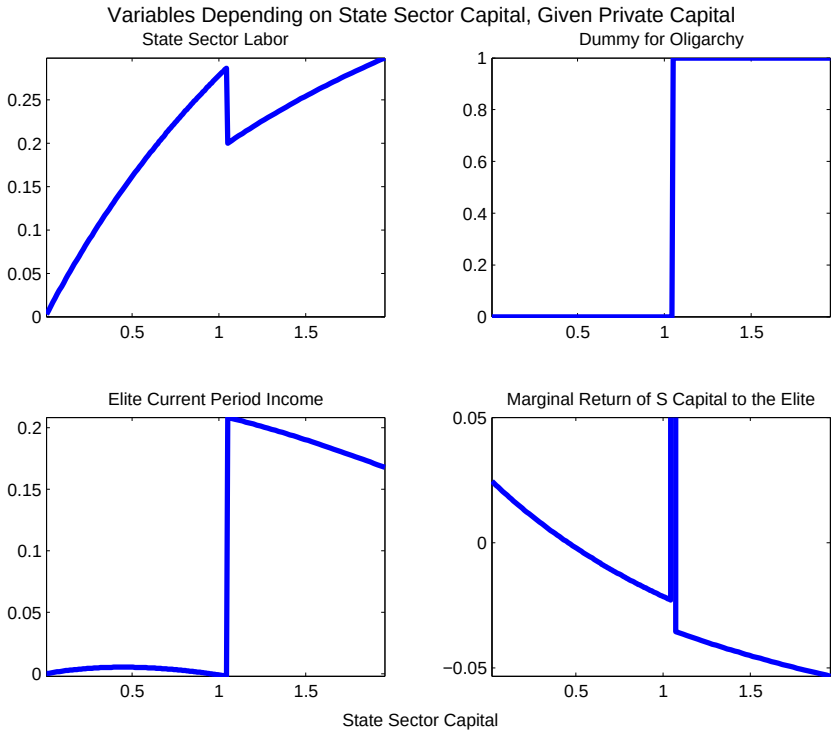


Figure 5: The outcome depending on choice of K_S

benefit of state capital for the elite depend on the choice of K_S (the x-axis). Given a K_P, if K_S reaches a certain critical level, there can be enough state workers (left-upper panel) - in this figure, $\underline{L} = 0.2$ - and oligarchy can be sustained (right-upper panel). Then there is a jump in elite income above the critical level of K_S (left-lower panel) because the elite controls the government and the tax system in oligarchy. For this reason, though the capital return goes even lower than 0 as more capital is invested in the state sector, the elite still prefers to invest until the critical level of S capital K_S (right-lower panel) to sustain oligarchy.

In the example above, given its particular level of P capital K_P, choosing S capital K_S that just sustains oligarchy gives highest current period income to the elite. But for other levels of K_P, the situation may be different. As we can see in figure 6, when K_P (the x-axis) is very small, K_S is negatively related to K_P (left-upper panel) and L_S is larger than $\underline{L}$ (right-upper panel).[17] In this region, a larger K_P, corresponds to a larger P sector labor and a smaller S sector labor, hence it is optimal for the elite to reduce investment in S sector - K_S - accordingly. However, when K_P is large enough, and S sector labor reaches the minimal level $\underline{L}$, a larger K_P implies that the government has to invest more in S sector to maintain oligarchy. We can see that a larger P sector not only increases benefit for the

[17]Figure 6 comes from the same numerical example as figure 5.

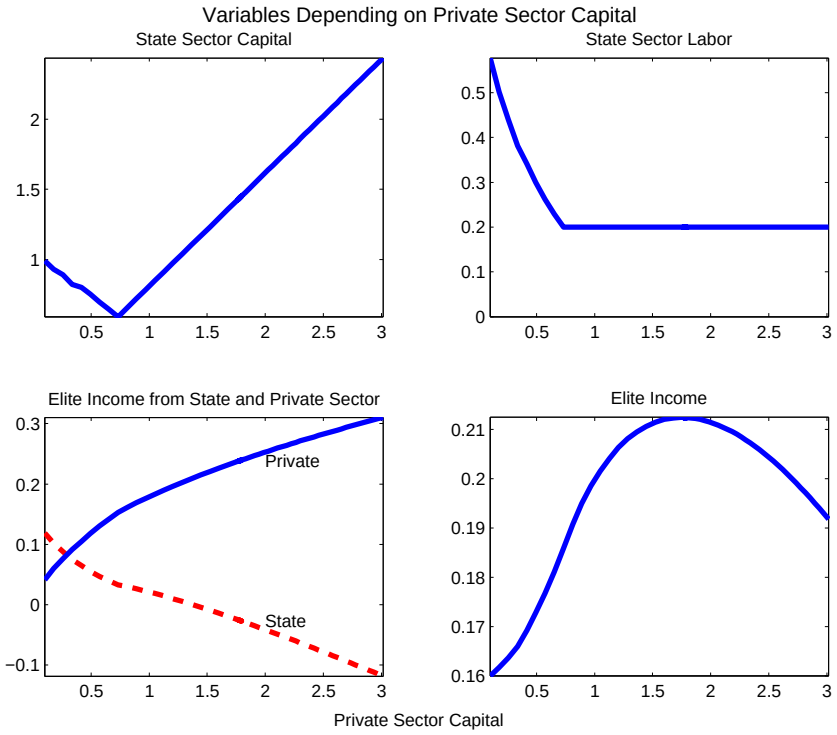


Figure 6: The outcome depending on choice of K_S

elite - tax income, but also creates higher cost - larger interest payment for K_S (left-lower panel). Due to the decreasing return to capital, there is a level of K_P that maximizes the elite income (right-lower panel).

How does the elite set K_P to be closer to the optimal level for the elite? In the third step here, I discuss the choice of η that affects entrepreneurs' borrowing ability and capital available for P firms. When the government prefers a larger K_P, it sets $\eta = \bar{\eta}$ and imposes no additional financial restriction. When it wants a smaller K_P, it sets $\eta < \bar{\eta}$, and P firms receive fewer bank loans than the maximal level. This can be seen in figure 7.[18] The x-axis is a_p. As we move a_p from very small to very large, the P firm leverage goes down gradually (left-upper panel) as the government prefers K_P neither too small or too large. The S sector capital first goes down but then goes up proportionally to the P sector capital (right-upper panel), because enough S employment share needs to be guaranteed (left-lower panel). The government's influence on K_P is limited because η is bounded by $\underline{\eta}$ and $\bar{\eta}$, so it may not be able to set K_P to its preferred level when a_p is too small or too large. This is why the elite lifetime income is maximized for an intermediate level of a_p (right-lower panel). This is the second trade-off for the elite, in addition to the first trade-off of state wage and employment. This following remark essentially restate the first part

[18]Figure 7 comes from the same numerical example as figure 5.

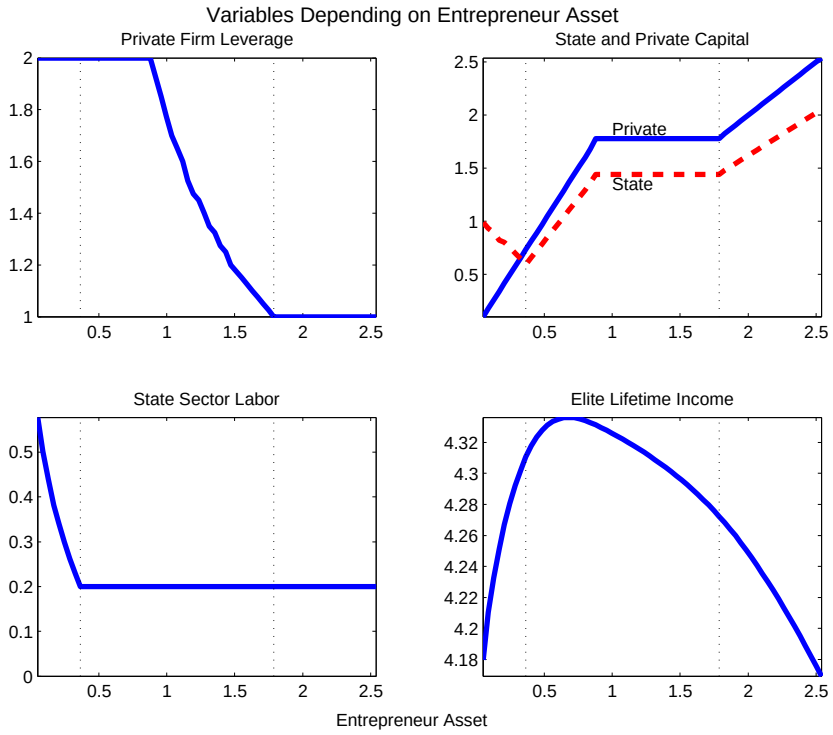


Figure 7: Equilibrium variables, depending on entrepreneurs' asset.

of proposition 2, with more details from the numerical solution.

Remark 1 (Trade-off of private sector capital). A larger K_p contributes more tax income, but it also requires larger K_S to sustain oligarchy and more interest expense. As K_P increases from a very small level, the elite's current-period income first increases and then decreases. The elite's lifetime income also follows a similar pattern. This trade-off also applies to entrepreneur asset level a_p because it determines K_P.

Under which conditions does the government choose to democratize or to sustain oligarchy? The government can invest as much as it wants in S sector to guarantee enough state employment with high wage, for any size of P sector capital. However, large investment in S sector means large cost, while the return can be small due to decreasing return to capital. If P sector capital is large enough, sustaining oligarchy gives lower lifetime income to the elite compared to democracy - the line for the elite's income in figure (7) can drop below the horizontal zero line: $V\left(a_p\right) < 0 = V^D\left(a_p\right)$ if a_p is large enough. In this case, the elite chooses to democratize.

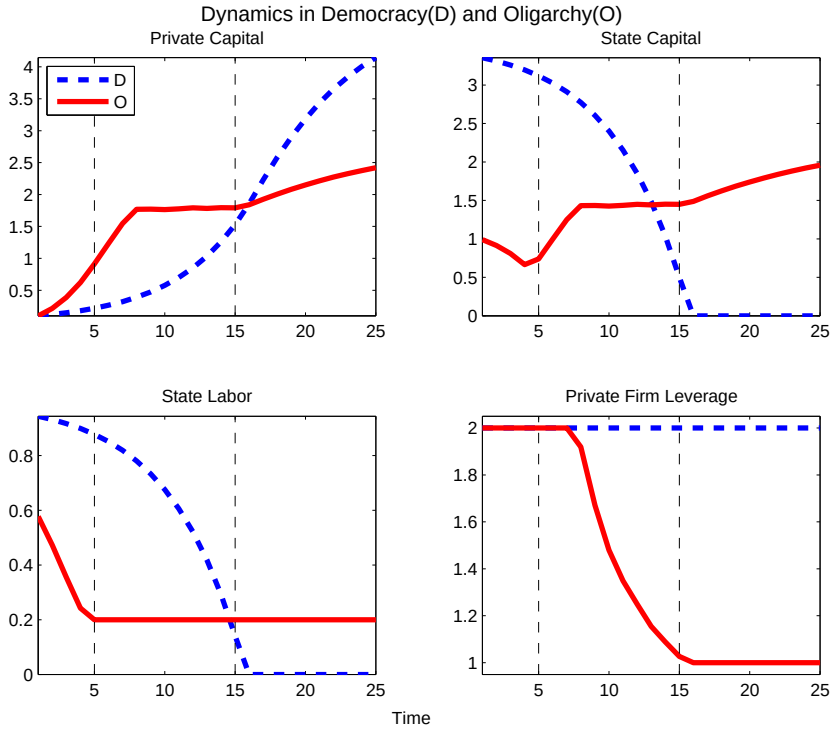


Figure 8: Dynamics in democracy (blue) and oligarchy (red) ending in middle-income trap.

3.4.3 Simulation and Dynamics

Given these parameters and the numerical solution, I can simulate the economy starting from a very small private sector: $a_e = 0.05$. Figure 8 and Figure 9 plot key variables and output during the transition in this calibration, where $\underline{L} = 0.2$. If the political system starts in democracy, the transition is the blue dashed line, while the transition in oligarchy is the red solid line. Starting in oligarchy, during the first stage, the private sector is small, and therefore not a threat to oligarchy. The elite encourages the growth of private capital to extract more tax income. So the government sets $\eta = \bar{\eta}$ to lend to private firms as much as possible (left-lower panel of figure 8). Moreover, private firms and entrepreneurs benefit from low wage and abundant labor, so private sector capital grows rapidly (left-upper panel). State employment and capital decline accordingly (right-upper panel). Because the more efficient private sector is reallocated with more capital and labor (right-lower panel), the economic growth is rapid (figure 9). For this reason, this stage is called *rapid growth*.

As the private sector grows larger and the state employment share declines to the critical level $\underline{L}$, the economy enters the second stage. The declining state employment share threats the supporter base of oligarchy. If no action is taken, the elite cannot keep

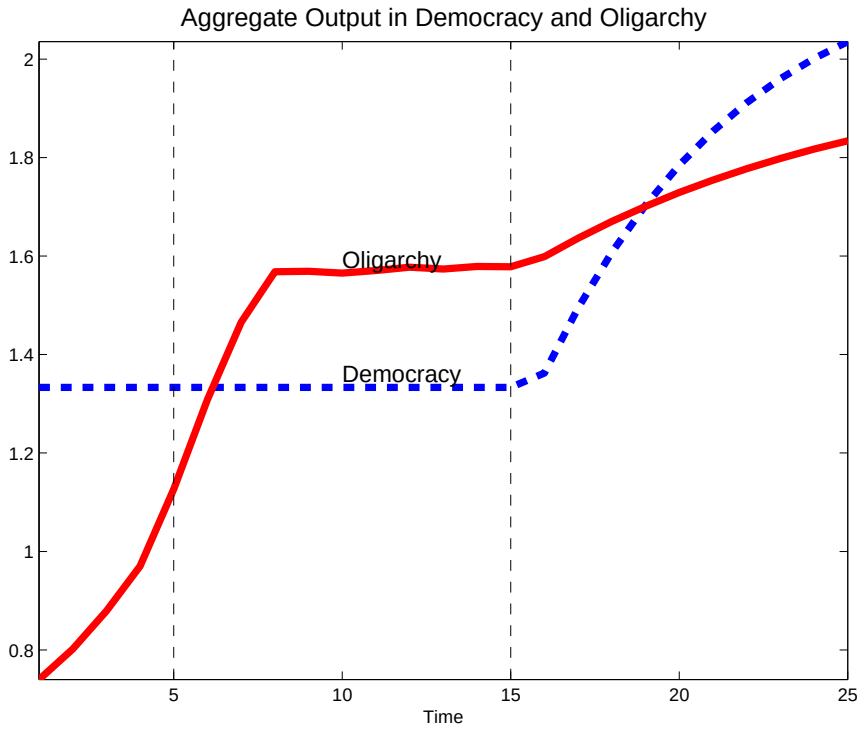


Figure 9: Output in democracy (blue) and oligarchy (red) ending in middle-income trap.

their political power any more. So it increases state investment and then restricts private firms' access to the financial market. Because of the policies in favor of state firms, the state sector keeps its relative economic power and the ability to hire $\underline{L}$ labor with high enough wage. The privatization stops, and no more labor reallocation to the more efficient private sector. However, the large investment in state sector can still keep growth high for a while. But the growth gradually slows down because the financial restriction on private firms harms the economic efficiency, as shown in the middle section of figure 9. This stage features large state investment and financial restriction on private firms, so it is a stage of *state capitalism*. Notice that though the initial output is lower in oligarchy than in democracy, due to the labor market distortion, the output can catch up with democracy in the second stage due to rapid capital accumulation and large state investment.

In the long-run, the elite finds it optimal to always sustain oligarchy. It keeps over-investing in the state sector as the private sector capital grows to it steady state level. Employment share stays at $\underline{L}$. Though the elite has to pay large investment cost, it still extracts from tax income from the private sector, so it doesn't want to democratize. The economy continues as the second stage: over-investment in state firms, financial restriction on private firms, no labor reallocation to private firms. The inefficient capital market harms growth. Furthermore, due to decreasing return to capital, growth gradually slows

down and eventually output stops growing at the middle level, which is lower than the level in democracy. So in this case, the third stage is called "*middle-income trap*".

This calibration predicts that China will stay in oligarchy and fall into the middle-income trap, given the current conditions. This is not surprising. The government is right-now strong, politically and economically, meaning that a relatively small fraction of the citizens' support is sufficient to sustain the current regime, and it has enough financial resource - for example, large foreign reserves - to build up the state sector if it needs to. After 2008 financial crisis, the Chinese government initiates the 400 billion stimulus package and bails out mostly state firms while letting many private firms die. This shows that it keeps the economy and resource allocation under control and stable, and it is able to maintain a powerful state sector to guarantee political stability, according to this theory.

Since the political power of the government, captured by $\underline{L}$ is an important parameter determining the cost of sustaining oligarchy and the decision of the elite on whether to democratize, a large $\underline{L}$ may imply a different long-run development paths. Keeping the other parameters in this calibration, if $\underline{L}$ is changed to be large enough, democratization will occur. In this case, sustaining oligarchy requires many S workers, so the elite has to invest a lot in S sector proportional to the P sector capital. As P sector capital grows larger and larger, the elite finds the cost of maintaining the state sector too large, and it is optimal to democratize for them. This development path is different in the long-run compared to the development in the calibration to China, but it is similar in the early stages: starting from small P sector, in the beginning, P sector employment share grows until it reaches the critical level for sustaining oligarchy; then the government over-invests in S sector to maintain enough supporters for oligarchy; finally the two paths differ in the long-run. This divergence of two paths is the so-called "critical juncture" of development in Acemoglu and Robinson (2012).

The dynamics with $\underline{L} = 0.4$ is simulated and showed figure 10. In this case, the elite chooses to democratize when the private sector capital reaches certain level. The cost for the elite to keep enough workers in the state sector with high wage keeps growing as the private sector capital grows. Additionally, marginal return of capital decreases, so the elite finds the cost of maintaining oligarchy dominates the income in oligarchy when private sector capital grows large enough. It chooses to democratize. As we can see in figure 10, the state capital quickly drops while the private capital soars up because the financial restriction is removed. The output, as shown in figure 11, though slightly goes down due to super rapid decline of the state sector, eventually recovers and converges to the high level in democracy. This model offers a theoretical explanation for the empirical findings on the sharp deceleration in growth following democratization and the more

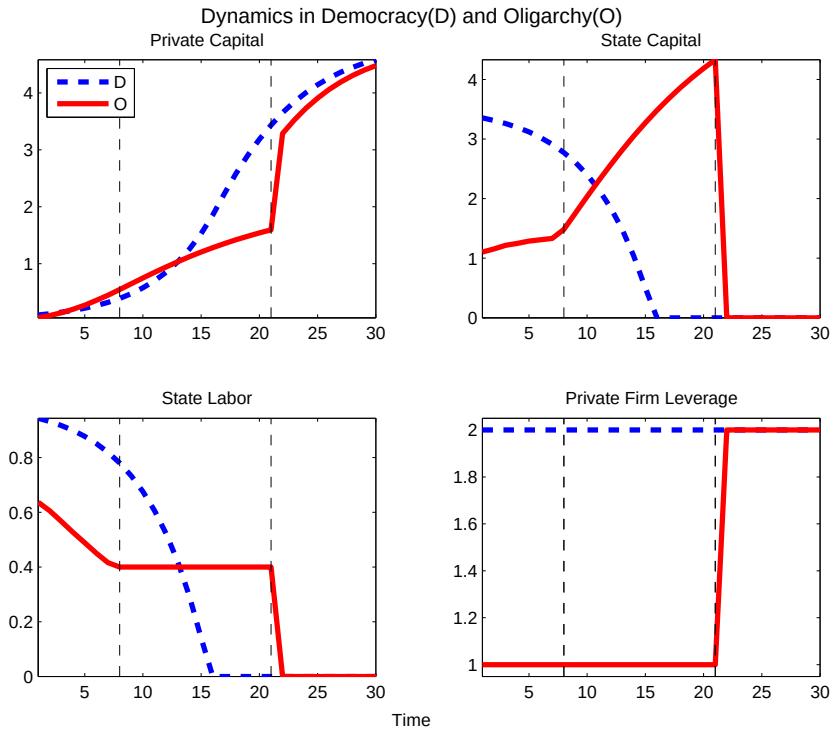


Figure 10: Dynamics in democracy (blue) and oligarchy (red) ending in sustained growth.

stable long-run growth in democracy, e.g., in Pritchett and Summers (2014).

In both cases, the transition is featured with three stages, and its properties are summarized in the following.

Remark 2 (Three stage transition). The economy, starting with a small enough private sector, develops along the following path with three stages:
Stage 1: *rapid growth*. Growth rate is high. Private sector grows rapidly, benefiting from the low wage. Moreover, the government encourages private sector growth and does not impose financial restriction: $\eta = \bar{\eta}$. Rapid privatization reallocates labor from the state to the private sector.
Stage 2: *state capitalism*. Growth continues. The government over-invest in the state sector, while restricting private firms' access to the financial market: $\eta < \bar{\eta}$. Privatization stops and the state employment share stays at the critical level $\underline{L}$.
Stage 3: Two cases.
Case 1: *middle-income trap*. Oligarchy is sustained permanently and growth slows down. State investment keeps growing at the same rate of the private sector capital, to keep state employment share at $\underline{L}$. Repression on private firm reaches the tightest level $\eta = \underline{\eta}$. This happens if $\underline{L}$ is sufficiently small.
Case 2: *sustained growth*. Democratization occurs and output growth becomes rapid again.

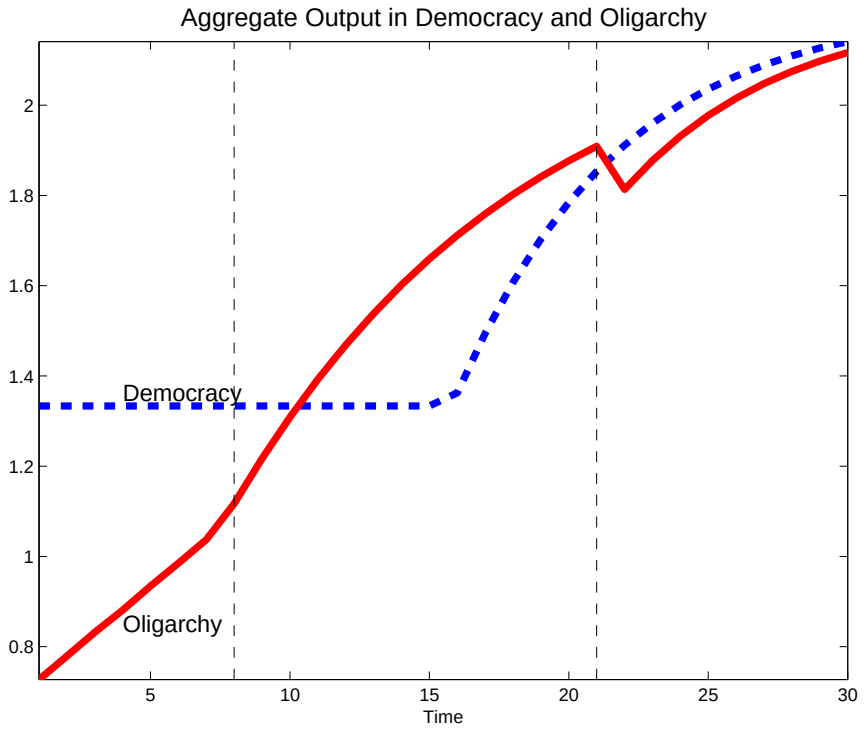


Figure 11: Output in democracy (blue) and oligarchy (red) ending in sustained growth.

Capital market and labor market distortion disappear. State sector declines while private sector grows. This happens if $\underline{L}$ is sufficiently large.

3.5 Implications of the Model

3.5.1 Static Implications of the Equilibrium Given Capital Allocation

Given capital allocation, the government creates a dual labor market: state workers get high wages and hence support the government, while private workers get low wages. This is the so-called "divide-and-rule" strategy: breaking workers into two sub-groups, and providing different economic interests to gain support from one group at the cost of the other.

This equilibrium given capital allocation is consistent with three phenomena in current China: (1) large state-private sector wage gap, (2) middle class's political support for the current regime, and (3) higher capital labor ratio and low capital return in the state sector.

The first fact is discussed in section 2, and is the immediate consequence of proposition 1. High state sector wages are necessary for getting political support from workers, and the general equilibrium effect leads to abundant and cheap labor in the private sector.

Entrepreneurs and private firms benefit from the abundant cheap labor, in the short-run. This allows potentially faster capital accumulation and growth of the private sector and the whole economy. I will discuss more on this in the dynamic model.

Second, the middle class workers - state workers in the model - are supportive to the existing political regime because of the economic benefits they receive.[19] This is consistent with the finding of Chen and Lu (2011) discussed in section 2, but contrary to the traditional wisdom that the middle class are the natural driving force of democracy, as in the European history. In the history of democratic movements in Europe, such as the Glorious Revolution and French Revolution, the middle class was against the aristocracy of the kings, whose political power relied on repression. The middle class did not rely on the state but emerged from private businesses. In contemporary China, the state sector is large and a significant fraction of the middle class have been created by and rely on the state, and in turn become supporters of the state. It is also similar in many other developing countries. This helps to understand why in some emerging markets, economic growth and the burgeoning bourgeoisie do not automatically lead to demand for democratization. For example, as reported in The Economist 2009, 95% of adult Kuwaitis work for the government, usually in white-collar civil-service jobs which are typical middle class jobs, while its private-sector middle class consists almost entirely of foreigners. The wage gap between the state and private sector is large there. These distortions keep politically important local workers in the state sector and is an efficient way to maintain oligarchy.

The third fact is well documented in the literature. Song et al. (2011) show that state sector capital labor ratio is much larger than the private sector, in every industry. Brandt and Zhu (2010) show that the capital return in the state sector is lower than 5% while the number for the private sector is above 50%. The difference in capital returns is partly due to the difference of wages and distorted labor allocations. The other reason is capital allocation, as we will see in the dynamic model below.

In a nutshell, the above analysis on the equilibrium given capital allocation is useful to illustrate the interactions of the political and economic systems in oligarchy in each period. On the one hand, the political interests, largely shape the state distortions and economic outcome. Taking into account political considerations, we can explain many economic phenomena and puzzles in China. On the other hand, economic power determines political outcome. Only when the state sector is economically powerful and equipped with enough capital, can the elite keep a large enough supporter base to sus-

[19] Entrepreneurs, as the other group of the middle class in the model, also support oligarchy. Their short-run income is higher in oligarchy, as is their lifetime income in most cases in the calibrated dynamic model.

tain oligarchy.

3.5.2 Dynamic Implications

The first two stages of the calibrated dynamic model are consistent with China's recent development. From 1997 to around 2003, it is a stage of rapid privatization, as the state employment share declines dramatically. The private sector, in terms of employment share and GDP, grows rapidly, for two reasons. First, the wage is low in the private sector. Compared to state firms which face the regulations on the wage and other payments, including pension tax, health insurance, unemployment insurance and so on, private firms pay relatively low wages, which result in high capital returns. Therefore, private firms accumulate capital rapidly and grow fast. The low wage keeps Chinese private firms competitive. It contributes a lot to the growth of export, and the growth of the economy. Second, the government encourages the private sector growth, because a larger private sector contributes more tax while it is still not too large to threat the supporter base of the government - state employment. So the government encourages various financial resource flowing into the private sector, not only bank loans but also foreign direct investment (FDI), and so on.

At around 2003, as the state employment share approaches the critical level, the privatization slows down and stops dramatically. The direct reason is that more and more investment is diverted to state firms but not private firms. State sector investment share stays at around 60% though its employment is much smaller (see Brandt and Zhu (2010)). The state over-investment retains state employment, but reduces the capital return. In the private sector, the capital return is high, not only because they are more efficient, but also because the credit constraint: private firms cannot get enough bank loans for their high return projects. In fact, the financial constraint on private firms has been getting tighter over time, signaling growing repression on them. The growing repression on private firms is formally documented as the growing state-private capital wedge in Brandt and Zhu (2010). The protection on state firms and repression on private firms have gained much attention and are called "the state advances as the private sector retreats". For example, in the passenger airline industry, by 2006, eight private carriers had grew rapidly and had challenged the three state-controlled majors, thanks to the previous government policies encouraging private investors to enter. However, afterward, the government starts supporting the state airlines and keep them alive with policies including stock purchase from the central government. The state airlines not only survived and also are able to keep their dominance. Our theory's prediction indeed explains why this is happening in the second stage "state capitalism". The elite prefers to maintain a suf-

ficiently strong state sector to guarantee the political control. This model's prediction on the capital return in the second stage is broadly consistent with the trend: a large gap between the state and private capital returns and declining state capital return. Though the capital return in state firms is so slow, the government still keep investing into them to keep them alive.

4 Discussions: Assumptions and Extensions

4.1 Assumptions

In this subsection, I explain the reasons for making the crucial assumptions, and also the logic of the important settings in the model. Some of them are specific to current China, for example, the assumption that the government can not set state firm labor. Modeling China 30 years ago or other countries requires changes on these assumptions. Some other assumptions, e.g., no transfer to the ruled, are more general phenomena which are frequently seen in other studies for other countries.

Assumption 1 is that the government can not make direct transfer to the ruled groups to buy their political support. Though direct transfer seems to be cheap and attractive, in the political economy literature it is consider as difficult to implement, for two reasons. One is the credibility and commitment problem. Acemoglu (2003) and Acemoglu and Robinson (2005) argue that even if the state promises to make a transfer to the ruled group, the latter, without political power, gets no guarantee that it will eventually receive the transfer. The promise of transfer is not credible, and transfer cannot be to used solve all the political conflicts. The other reason why direct transfer is difficult to implement in reality is the high cost due to local capture. Reinikka and Svensson (2004) document that 87% of the transfer from the central government to local schools in Uganda was not received during 1991-1995 due to local capture. This means that the cost of 1 dollar of transfer is as high as 7.7 dollars. For these two reasons, the government usually builds inefficient "white elephant" projects (see Robinson and Torvik (2005)) to guarantee the economic benefits for certain groups. In this paper, the government has to inefficiently distort the prices and allocations in the labor and capital markets to buy the support. What happens if this assumption is dropped is discussed in section 4.

The second assumption that the government can only control S sector wage but can not directly set S sector labor, and generally in this model, the government can only influence agents' decisions through manipulation of prices and policies, means that the oligarchic government is not totalitarian. Each S firm is free to make its own employment

decision. Moreover, it is responsible for the decision and pays the wage bill on its own - the government provides no labor subsidy. This is consistent with the current situation of state firms in China. After a series of state-owned enterprises (SOE) reforms, especially after the 15th Party Congress in 1997, management decisions have been gradually delegated from central and local governments to the firm level, in order to improve their efficiency. The official slogan is to turn SOEs into modern firms which "operate independently and assume sole responsibility for its profits and losses" (*zizhu jingying, zifu yingkui*). The first part of this slogan maps into the setting in this model that S firms decide employment by themselves. The second part essentially means that S firms maximize profits and there is no subsidy to firms. Of course, the government still maintain its influence over state firms, but indirectly, through the regulations on state sector wages, and the allocation of loans. Robustness of the model relaxing this assumption is in section 4.

The following settings in the model are also important and worthy of explanation and clarification. First, the logic for the minimal number of supporters $\underline{L}$ to maintain the regime is similar to Morris and Shin (2002) where the regime switches if more than a number of agents attack the regime. Alternatively, $\underline{L}$ can also be micro-founded based on Acemoglu et al. (2012). If the elite and their supporters form a coalition which has large enough political power, oligarchy is sustained. More specifically, a coalition of a set of agents holds a corresponding level of political power. If the political power of a certain coalition is large enough, it can choose the political system. In this paper, under oligarchy, the elite as the ruling group is granted political power ω_e. Each worker has political power ω_w. The aggregate political power of entrepreneurs is assumed to be 0, given its size of 0. Workers can change the political regime from oligarchy to democracy if and only if they form a coalition of size L_r whose power is larger than α, namely $\frac{\omega_w L_r}{\omega_w+\omega_e} > \alpha \Leftrightarrow L_r > \alpha\frac{\omega_w+\omega_e}{\omega_w}$, where α is exogenous. In other words, to sustain the oligarchy, there must be at least $1-\alpha\frac{\omega_w+\omega_e}{\omega_w}$ workers supporting it. This size is denoted as $\underline{L}$. Notice that $\underline{L}$ captures the relative political power of the elite compared to workers. If the elite is very powerful, it needs only a small fraction of workers as supporters to form a winning coalition. If workers are well-organized and politically motivated, $\underline{L}$ becomes large.

Second, in the model, the government can set the minimal wage in S sector but not in P sector. This is natural in China, and also in other countries. The government has better control over the state sector, so if it sets a minimal wage, state firms have to obey it. However, the government can not set a binding minimal wage in the private sector, because its ability to monitor private firms is weaker. Private firms can find ways to walk around the wage regulation if the government forces them to pay higher wages. In some countries, where the public sector is very small, clientelism is often in the form of

subsidizing a subgroup of private firms - those well-connected to the elite. This model can be easily modified to capture that.

In the rest of this section, I discuss extensions. The first extension is to consider changing the fundamental parameters of the model. The rest are thought experiments on relaxing or changing the assumptions to understand what may happen alternatively if an economy differs in some aspects.

4.2 Reforms and Comparative Statics

Is China doomed to fall into the middle-income trap? Not necessary. If the underlining conditions change, the policies and the development path can change accordingly. Mapping into the model, if the parameters such as $\underline{L}, \underline{\eta}, z_S$ change, the government policies and the dynamics, including the third stage, will change. Many policy suggestions on how to switch China's development to a more sustainable path have been made by economists and China watchers. For example, Gary Becker suggested that financial reform should be implemented, in order to allocation more resource to private firms, and rural immigrants should be given more rights. Will the government take the suggestions and implement all the policies and reforms that sustain growth? We need to notice that policies or reforms that benefit economic growth may not benefit the elite, who is very influential in the government.

Suppose the government takes a reform that gives more political rights to workers, especially the immigrant workers. This implies that the government has to buy support from a larger fraction of the population. We know that if $\underline{L}$ increases from 0.2 to 0.5 leads to democratization and sustained growth. But does the elite like that? Its income goes down to 0 if democratization occurs, so obviously this reform will encounter strong resistance from the elite.

In the above model, I assume that the government is completely under the control of the elite. Some may believe that, in some cases, some technocrats become powerful in the government, and they care only about the output growth in the long-run, but not the economic benefit of the elite. In this case, they can initiate reforms which correspond to changing the key parameters of the model, such as $\underline{L}, \underline{\eta}, z_S$. To which extend they can push the reform to depends on their political power in the government. P can be one of the key parameters $\underline{L}, \underline{\eta}, z_S$. Notice that I consider reform as changing parameters but not the endogenous policy variables such as K_S, η. This implies that technocrats get a chance to push for a big change of the society and the political and economic system, and afterwards, the government decisions will be made by the elite. The consequences of

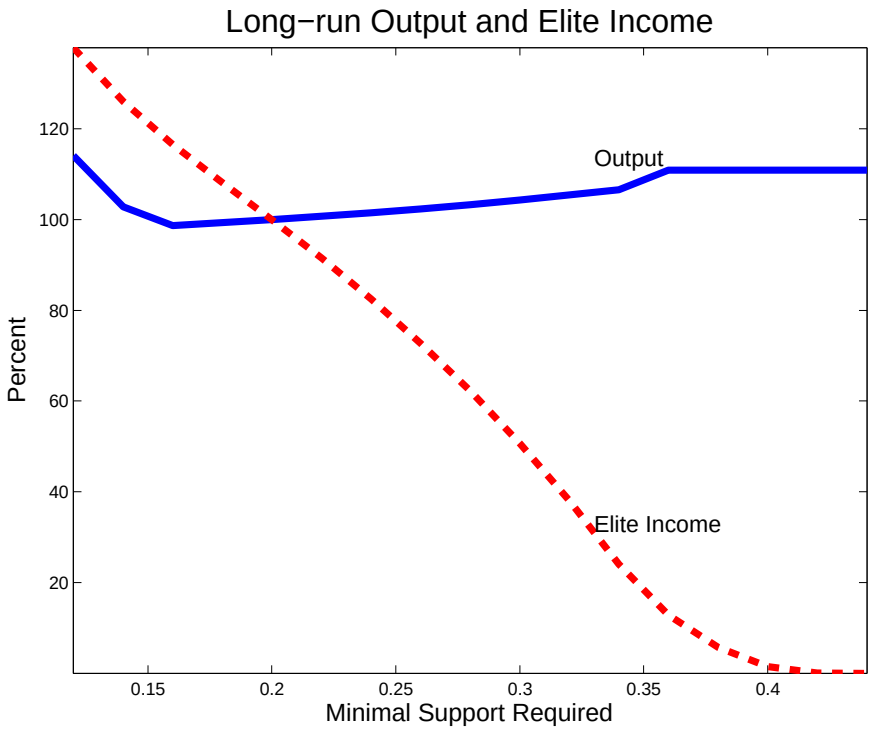


Figure 12: Elite income and long-run output depending on $\underline{L}$.

reforms are then essentially the comparative statics of the model.

Figure 12 shows how the elite's lifetime income V, and the long-run output Y_∞ which the technocrats care respond to different levels of $\underline{L}$. Technocrats would like to increase workers' political rights and increase $\underline{L}$ from the current level $\underline{L} = 0.2$, because this makes the government invest more in the state sector, or even choose to democratize. Both of them lead to larger output levels. However, the reform as the result of the bargaining can only push $\underline{L}$ to the right limited by α. If α is small, the increase of workers' political rights won't be large.

Similarly, financial reform, which reduces the financial restriction on private firms can be considered as increasing $\underline{\eta}$. It again increases output, because the private firms can grow larger, and it may even leads to democratization. But again, it harms the elite interests and is hard to be implemented.

One exception is the state firm reform. If a successful reform is taken to increase state firm productivity and reduce the TFP gap between the private and state firms, it increases the output potential. More than that, under the condition that oligarchy is sustained, a more efficient state sector implies that the government can allow the private sector to grow more without worrying about their supporter base - state workers. Less repression on private firm is needed and higher economic efficiency can be achieved. This

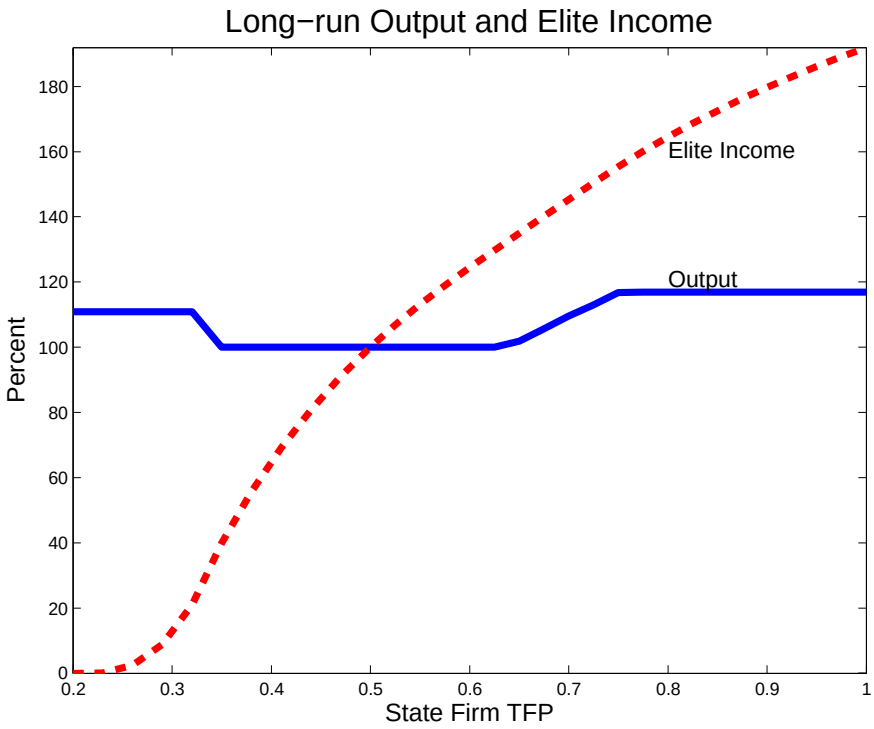


Figure 13: Elite income and long-run output depending on z_S.

reform also increases elite income because of higher total output. Figure 13 plots how the long-run efficiency, measured as the long-run output in oligarchy over democracy, can be improved by a more efficient state sector (in the region $z_S \in [0.6, 0.75]$), while the elite income always increase with that. This reform is more likely to be implemented the government. In fact, this is happening right now in China. Hsieh and Song (2015) document that the state-private TFP gap is declining. The so-called "industrial upgrading", which aims at building high-tech state firms, is at the top of the agenda for China's further economic reforms. However, it is also very difficult to completely close the gap between the state and private firms, because they are less flexible and provide less economic incentives for the managers, compared to private firms.

4.3 Political Roles of Entrepreneurs

In the benchmark model, entrepreneurs, given the small population, are assumed to have no important political power. Moreover, each entrepreneur runs a small firm and expects her behavior has no influence on the politico-economic development. What can happen if these assumptions change?

Suppose the political power of entrepreneurs is not 0 but ω_p, then the oligarchic gov-

ernment may want to not only buy the support from workers but also from entrepreneurs. However, this does not have a large effect on the equilibrium, because even in the benchmark model where the elite does not care the political support from entrepreneurs, entrepreneurs do prefer oligarchy to democracy, in almost all stages of development. As Figure 14 shows, the lifetime utility of entrepreneurs in oligarchy (red solid line) is much higher than in democracy (blue dashed line) in the early stage of development when entrepreneur asset is low. This is quite intuitive: entrepreneurs benefit from the abundant and cheap labor in the private sector due to the labor market distortion and the government imposes no extra financial constraint on them. Shouldn't entrepreneurs expect financial restriction in the future and prefer democracy? Not really. Each entrepreneur may prefer removing the restriction on her so that she can borrow more. However, entrepreneurs as a group may actually benefit from the financial restriction, because it lowers the capital stock in the private sector from the competitive equilibrium level, and makes it closer to the monopolistic level. Entrepreneurs get higher return on their asset in oligarchy. That is why, as we can see from the right part of the figure, that even when high entrepreneur asset is large, which means there is financial restriction, the entrepreneurs' lifetime utility is still higher in oligarchy. Only in a small region of entrepreneur asset, entrepreneurs' lifetime utility is lower in oligarchy than in democracy, but the difference is small. This implies that the oligarchic government gets the support from entrepreneurs almost for free, and it does not have to change the policies too much if entrepreneurs become politically powerful. This is consistent with findings from Chen and Lu (2011).

If entrepreneurs solve the collective problem and can as a group strategically decide the aggregate asset in the private sector, will they save more to promote democratization? Given that entrepreneurs generally prefer oligarchy to democracy, this is unlikely to happen.

4.4 Transfer to the Ruled

In the benchmark model, I assume away the possibility that the oligarchic government can use lump-sum transfer to buy the support, following the political economy literature and to be consistent with the observation in China. However, it is still interesting to think on what should happen if lump-sum transfer is allowed. Moreover, perhaps in some countries and certain special circumstances, lump-sum transfer can be used as a credit way to buy political support. Below I discuss three alternative settings on how transfer is made and their consequences.

First, suppose that in oligarchy transfer can be made to state workers, but its size is

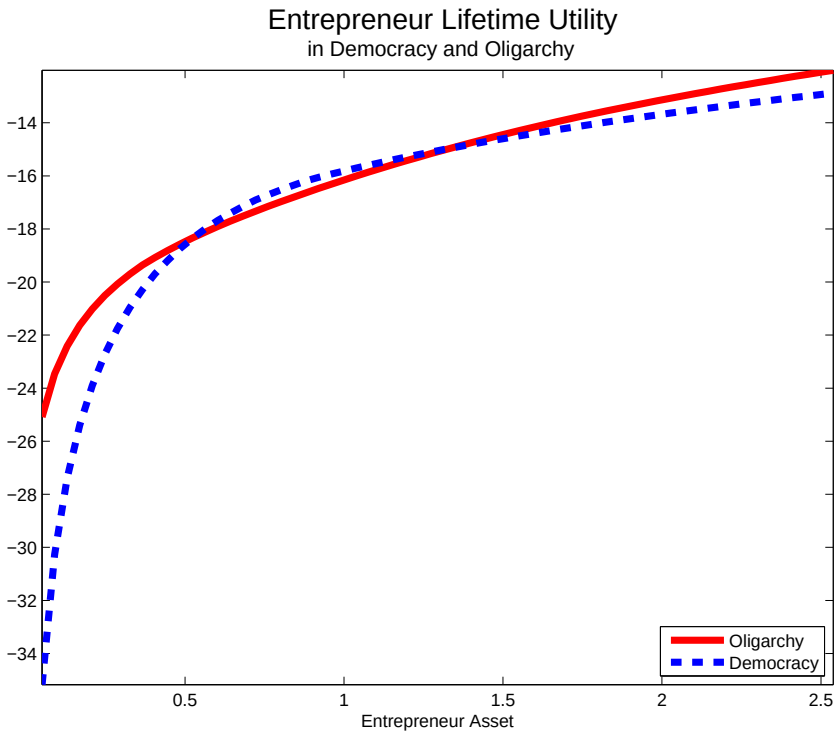


Figure 14: Entrepreneur Lifetime Utilities in Democracy and Oligarchy

constraint by an upper bound which is tight and smaller than the transfer that workers receive in democracy. Then only using the transfer is not sufficient to buy the support, so the government still chooses to distort the labor market to increase the wage of state workers. Results will be qualitatively similar to the benchmark model.

The second setting is the following: there is no constraint on the size of transfer, but it can only be made to state workers. Then the government can use transfer to get support from state workers, and it can reduce labor market distortion, though it does not have to remove the distortion. However, capital market may still be distorted because the government needs to maintain enough workers in the state sector. Moreover, a very large private sector pushes up the wage and the cost of maintaining oligarchy. Compared to the benchmark model, in this setting, the static equilibrium given capital allocation becomes less distorted, but the properties of the dynamic equilibrium should be still the same. The cost of maintaining oligarchy become smaller, which makes democratization less likely to happen, while qualitatively the dynamics should be still similar.

Third, if there is no restriction on how the transfer can be made in oligarchy, then the government can simply pick a group of private workers, pay them transfer and turn them into supporters. Then it is not necessary to keep the state sector in the long-run. One choice for the elite is to pick policies that maximize the discounted output for all

periods and use transfer to redistribute and sustain the regime. Actually, the elite can achieve higher utility than that. It can even strategically reduce the capital stock, which reduces workers' expected income after revolution, and makes it cheaper to hire labor and to buy supporters. In any case, there is no cost of maintaining oligarchy and the elite never chooses to democratize.

4.5 More Government Control

Previously, state firms are modeled as small firms which take prices and policies as given while employment decisions are made by firms to maximize their profits. This is consistent with the outcome of China's SOE reforms. Moreover, the government has no control over the wage in the private sector. This means that given capital, the government can only use state sector minimal wage to influence labor allocation and workers' income. What happen if we change these settings? I consider the following different scenarios.

First, suppose that the government can at the same time decide state employment. In this case, the government can increase public sector wage without worrying about reducing public employment. Then it is equivalent to the case that the government can pay lump-sum transfer to state workers. An interesting outcome of this setting - the opposite of the benchmark model - is that there may be "over-employment" in the state sector. Even given relative small state capital, there can be at the same time over-payment and over-employment in the state sector, while in the benchmark model, over-payment implies under-employment. In economies where state firms are under tight control of the government, this phenomenon is likely to be observed.

The second thought experiment is to allow the government to set wages of some private firms. Korea before democratization maps into this case. The large local conglomerates (chaebol) are private firms but they provide support for the government. The government's supporter base can be built on these firms instead of less efficient state firms. This of course is cheaper than building the supporter base only in the state sector. However, labor market distortions and capital market distortions are still similar to the benchmark - just replace state firms with these connected firms. Moreover, it is reasonable to think that these private firms should become less efficient over time, because they reply on the capital market advantages and are less exposed to competition. If this is the case, it is equivalent to the benchmark model.

4.6 Forward-Looking Workers

If workers are forward-looking, when they decide whether to support the regime or not, they take into account their future income. From the conclusion that in the long-run democracy brings higher output, one natural intuition is that then the forward-looking workers prefer democracy more than the myopic workers, and the oligarchic government needs to provide higher current period wage premium for them. However, it is not necessarily true. As shown in Figure 9, in the medium-run the output is higher in oligarchy than in democracy. Depending on how patient workers are, they may find the discounted future income in oligarchy more attractive and it can be cheaper to buy their current support. Of course, there can be some interest dynamics, i.e., in the first and second stage, when the growth in oligarchy is promising, state workers requires not so high wage premium; and when the growth starts to slow down, state workers become more willing to switch to democracy and they require more compensation to support the regime.

4.7 Cost of Democratization

There can be some costs associated with democratization, including revolution, and even peaceful democratization. They can be exogenous - the natural costs of changing regimes, or endogenous - created by the government which tries to prevent democratization.

First, the cost of revolution changes the workers' expected income in democracy and the wage premium, but this cost never get paid, because revolution will be off equilibrium in the quantitative analysis. The intuition is the following: even if there is no cost of revolution, for the elite, choosing policies that leads to revolution is dominated by either sustaining oligarchy when the private sector is small or peaceful democratization when the private sector is very large. The intuition for the first part is simple: oligarchy implies paying no tax and getting transfer for the elite, and when the private sector is small, oligarchy is cheap to sustain. When the private sector capital gets large and staying in oligarchy is not attractive anymore, the elite has two choices - peaceful democratization and choosing policies that lead to revolution. After peaceful democratization, all privileges disappear and the elite only gets the return r on its asset. If it chooses not to democratize, it can set the state and the private sector capital such that there are not enough supporters for oligarchy. However, in the calibrated model, before the elite gives up oligarchy, the private sector capital is already so high that then state sector capital return is already lower than r, and setting state sector capital to any positive level means lower return than saving the asset in banks and get the return r. So revolution is worse than

peaceful democratization for the elite. [20]If there is positive cost of revolution, revolution becomes even less attractive, so the result that revolution never occurs should be robust. However, the cost of revolution reduces the workers' expected income in democracy. If it is not very large, it shifts the state wage premium down without changing the economy qualitatively. If it is large enough, workers, either in the state sector or the private sector, are not willing to pay the cost to revolt. The elite maintains oligarchy forever, without paying any state wage premium. More interestingly, if the elite can pay lower state sector wages than in the private sector and force workers to stay in the state sector, it prefers to do that. Because for the elite, the marginal return of labor in the state sector is higher than the marginal return of labor in the private sector, provided that the social returns are the same. The elite extracts a fraction of marginal return of the private sector labor using taxes, while the whole marginal return of the state sector labor goes to the elite, through state firms. In such a totalitarian regime, the government keeps more workers in the state sector than the socially efficient level, at the cost of state workers. Those who can escape to the private sector are the lucky ones with higher labor productivities, though they may not get higher income, because the government is also likely to impose other costs to the private sector workers, in order to prevent the escape.

If the cost of democratization affects the cost of maintaining oligarchy as stated above, it is reasonable to consider an extension that the elite can strategically influence the expected cost of democratization to workers. The cost may be real - investing military can increase the cost of revolution, or it may simply be a bias of the workers' expectation due to incomplete information and propaganda. In either case, the oligarchic government can invest in the technology that increases the expected cost of democratization. The investment can be interpreted as control from the government, which is costly for the government but also increases the expected cost of revolution. If the cost of control increases with the cost of democratization, their dynamics should be the following: in the early state of development where maintaining oligarchy is relatively easy, the control is loose - the elite does not bother to pay high cost of control to reduce the cost of maintaining oligarchy only a little; when the private sector gets big and it becomes more costly the maintain the regime, the elite tightens the control. The political environment is expected to become tighter, same as the financial environment. This is contrary to the conventional wisdom expected the environment to be more open and freer, but consistent with empirical findings of Cantoni et al. (2014) on China's textbook reform between 2004 and 2010

[20]Notice that this is true when there is no uncertainty - the elite expects the future perfectly and plans rationally. If there are shocks, the political process may runs out of control of the elite - e.g., an unexpected loss of state capital or increase of workers' power.

that led to "more positive views of China's governance, changed views on democracy, and increased skepticism toward free markets".

5 Conclusion

This paper proposes a political-economic theory to study China's future economic and political transition and to understand China's recent development. The political constraint interacts with economic policies, which lead to a three-stage transition. The first two stages are *rapid growth* and *state capitalism*, which are consistent with several salient aspects of China's development, including (1) rapid growth with repressed wage in the private sector; (2) political support from the middle class, including state sector workers and private entrepreneurs; and (3) financial constraints on private firms and support for state firms. In the future, i.e., the third stage of development, China is likely to enter a *middle-income trap* given the current conditions, especially the economically and politically powerful state. To switch to the other development path that leads to *sustained growth*, necessary reforms have to be taken, though such reforms may face resistance from the elite.

Even though the focus of this paper is on China, it is also useful to study the development of many other emerging countries and even some developed countries with similar patterns compared to China. First, the key political constraint in the theory also exist in some other countries such as Kuwait, Korea in the 1980s, as the political elite or politicians need to buy political support from public workers or workers in industries under their control. Similar stories have occurred in these countries. Before the 1990s, the large local conglomerates (chaebol) in Korea were granted privileged access to low-cost credit. In Kuwait, the oil industry is under the control of the government, so the public sector can hire more than 90% of Kuwaiti nationals with relatively high wage while the private sector is populated with expatriates. Second, the theory is also useful to think on a question in development, namely, whether other developing countries should apply the "China model", i.e., the combination of authoritarian politics and state-guided capitalism, to promote economic growth. Some suggestions in favor of adopting this model are based on its past success, but the long-run outcome should be carefully examined and distinguished from the short-run effect. This theory provides a quantitative framework to evaluate the economic and political consequences of "China model".

Further empirical work can be done to examine the theory, especially the three-stage political-economic transition. Which conditions determine the transition to democracy and long-run growth? Is it consistent with the theory? The theory predicts that if a coun-

try can easily build a large state sector, for instance due to rich natural resource, is more likely to sustain the oligarchy, while if efficiency is very important for a country, for example because of exposure of international competition, democratization is more likely to occur. Anecdotal evidence about Gulf countries compared to export oriented economies like Taiwan seem to support the theory. Still, more systematical evidence will be useful to check and improve the theory.

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6 Appendix

6.1 Proof of Proposition 1

In the main text, I already show that given capital, high enough wage to buy support from S workers implies $L_S \leq \bar{L}$. Meanwhile, enough supporters means $L_S \geq \underline{L}$. If $\bar{L} \geq \underline{L}$, or equivalently, $\nu \frac{zK_S}{zK_S+K_P} \geq \underline{L} \Leftrightarrow \frac{K_S}{K_P} \geq \frac{z\underline{L}}{\nu-\underline{L}}$, the political constraint can be satisfied by setting w_S such that $L_S \in [\underline{L}, \bar{L}]$, and oligarchy can be sustained.

If oligarchy is sustained, given that $w_S \geq y_w^D > w_P$, shown in the main text, we know that the wage is higher in S sector than in P sector. Moreover,

$$\begin{array}{ccc} (1-\alpha)\left(z_S K_S\right)^{\alpha} L_S^{-\alpha} & > & (1-\alpha)\left(K_P\right)^{\alpha} L_P^{-\alpha} \Rightarrow \\ \frac{K_S}{L_S} & >_{\frac{1}{z_S}} & \frac{K_P}{L_P} > \frac{K_P}{L_P}, \end{array}$$

which states that the capital labor ratio is higher in S sector. Given the larger capital labor ratio in S sector, capital return is obviously lower:

$$\begin{array}{rcl} \alpha z_S^{\alpha} K_S^{\alpha-1} L_S^{1-\alpha} & < & \alpha K_S^{\alpha-1} L_S^{1-\alpha} \\ & < & \alpha K_P^{\alpha-1} L_P^{1-\alpha}. \end{array}$$

Compared to in democracy, entrepreneurs get cheap and abundant labor in P sector. As shown in the main text, $L_P > L_P^D$ and this implies higher P sector capital return

$$\alpha K_P^{\alpha-1} L_P^{1-\alpha} > \alpha K_P^{\alpha-1} \left(L_P^D\right)^{1-\alpha},$$

and higher entrepreneur income, which is simply the after-tax capital return, minus depreciation.

6.2 Proof of Proposition 2

In democracy, if S firms still exist, the return to S firm capital has to be r. If it is greater than r, S sector capital becomes positive infinity. If the return is lower than r, no loan is supplied to S sector. An elite member in democracy cannot tax others and gets no transfer, so she only replies on asset return and her income from other sources is simply 0.

An entrepreneur, supplies capital to P sector by using her own asset and borrowing from the bank, and get return. She takes the return to P sector capital - r_{Pt} - as given, in all future periods. r_{Pt} is determined by w_{Pt}, after P firms optimally choose capital labor ratio to maximize their profits in each period. Then obviously, if $r_{Pt} > r$, the representative entrepreneur is willing to borrow as much as possible and supply as much as possible capital to P sector: $K_{Pt} = \eta_t a_{pt}$, and the return to her asset is $r_{pt} = \frac{r_{Pt}\eta_t a_{pt} - r(\eta_t - 1)a_{pt}}{a_{pt}} = r + \left(r_{Pt} - r\right)\eta_t > r$. If $r_{Pt} \leq r$, the return to an entrepreneur's asset is simply r, as she can at least save in the bank and get the return r. An entrepreneur takes return to P sector capital and leverage η as given, and the return to her asset in each period r_p is also

determined. An entrepreneur standing in period 0 holding asset a_{p0} solves:

$$\max_{\{c_{pt}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log c_{pt}$$
$$\text{s.t.} \sum_{t=0}^{\infty} \frac{c_{pt}}{\prod_{t'=0}^{t} \left(1+r_{pt'}\right)} \leq \left(1+r_{p0}\right) a_{p0}.$$

Given the property of log-utility, it is easy to see that the solution of this problem is simply

$$\begin{aligned} c_{pt} &= (1-\beta)\left(1+r_{pt}\right) a_{pt}, \\ a_{pt+1} &= \beta\left(1+r_{pt}\right) a_{pt}. \end{aligned}$$

If $\beta > (1+r)$, and given that $r_{pt} \geq r$, a_{pt} keeps growing.

In democracy, if S firms still exist, competition of S sector capital supply implies

$$r_S^D = \left(1-\tau^D\right) \alpha z_S^{\alpha} K_S^{\alpha-1} \left(L_S^D\right)^{1-\alpha} - \delta = r.$$

This determines S sector capital labor ratio and wage:

$$\begin{aligned} \frac{K_S}{L_S^D} &= \left(\frac{r+\delta}{(1-\tau^D)\,\alpha z_s^{\alpha}}\right)^{\frac{1}{\alpha-1}} \Rightarrow \\ w^D &= (1-\alpha)\left(\frac{z_S K_S}{L_S^D}\right)^{\alpha}. \end{aligned}$$

The wage pins down the private sector labor, given capital:

$$\begin{aligned} w^D &= (1-\alpha)\left(\frac{K_P}{L_P^D}\right)^{\alpha} \Rightarrow \\ L_P^D &= \left(\frac{w^D}{1-\alpha}\right)^{-\frac{1}{\alpha}} K_P. \end{aligned}$$

When S firms still exist, obviously P sector capital return after paying tax and depreciation is higher than the counterpart in S sector, which is r. Then $K_P = \bar{\eta} a_p$, and if a_p keeps growing, L_P^D keeps growing and L_S^D declines gradually. When a_p reaches $\frac{1}{\bar{\eta}}\left(\frac{w^D}{1-\alpha}\right)^{\frac{1}{\alpha}}$, L_P^D reaches 1, and S sector becomes 0. Afterwards, entrepreneurs keep accumulating assets, and the economy behave like a neoclassic growth model.

6.3 Proof of Lemma 1

This is in fact already proved in the proof of proposition 2. Entrepreneurs take the return to capital in P sector and leverage η as given in both democracy and oligarchy, so their behavior are basically the same in both regimes.

6.4 Proof of Lemma 2

Denote the lifetime utility achieved by solving the two sub-problems - first maximizing lifetime income and then maximizing lifetime utility - as U^O.

First, $U^O \leq W^O$. Denote the solution for V^O as $\left\{\hat{w}_{St}, \hat{K}_{St}, \hat{\eta}_t, \hat{i}_t\right\}_{t=0}^{\infty}$, where $\hat{i}_t \in \{O, D\}$ is the choice of staying in oligarchy or not. The corresponding consumption and saving decisions obtaining U^O is denoted as $\{\hat{c}_{et}, \hat{a}_{et+1}\}_{t=0}^{\infty}$. Combine them together, the choice $\left\{\hat{w}_{St}, \hat{K}_{St}, \hat{\eta}_t, \hat{i}_t, \hat{c}_{et}, \hat{a}_{et+1}\right\}_{t=0}^{\infty}$ is a feasible choice of the original problem. This is because in each period, for any level of a_e, the choice sets of w_S, K_S, η in the original problem are the same.

Second, $U^O \geq W^O$. Denote the choice that solves the original problem and achieves W^O with stars, as $\left\{w_{St}^{\star}, K_{St}^{\star}, \eta_t^{\star}, i_t^{\star}, c_{et}^{\star}, a_{et}^{\star}\right\}_{t=0}^{\infty}$. Compare $\left\{w_{St}^{\star}, K_{St}^{\star}, \eta_t^{\star}, i_t^{\star}\right\}_{t=0}^{\infty}$ with the solution of V^O - $\left\{\hat{w}_{St}, \hat{K}_{St}, \hat{\eta}_t, \hat{i}_t\right\}$. First, $\hat{V}^O = \sum \frac{\hat{y}_{et}}{R^t} \geq V^{O\star} = \sum \frac{y_{et}^{\star}}{R^t}$. Then in the second sub-problem, choosing $\hat{c}_{e0} = c_{e0}^{\star} + \hat{V}^O - V^{O\star}$, $\{\hat{c}_{et}, \hat{a}_{et}\}_{t=1}^{\infty}$=$\{c_{et}^{\star}, a_{et}^{\star}\}_{t=1}^{\infty}$ is feasible and gives at least as high lifetime utility as $\left\{w_{St}^{\star}, K_{St}^{\star}, \eta_t^{\star}, i_t^{\star}, c_{et}^{\star}, a_{et}^{\star}\right\}_{t=0}^{\infty}$. This result is obtained because the choice set of other variables and their returns do not depend on the elite's asset at all.

To sum up, $U^O = W^O$. Solving the original lifetime utility maximization problem is the same as solving the two sub-problems.

6.5 Proof of Proposition 3

First, let us look at y_e when K_P is small, provided the conditions in the proposition. Using equation 5 and the condition that high state wage constraint is binding, i.e., $K_P < \delta$, $w_S = y_w^D$ and $L_S = \nu \frac{zK_S}{zK_S + K_P}$, we can write down y_e as

$$\begin{aligned} y_e &= \pi_e - (r+\delta) K_S + \tau Y_P \\ &= \alpha (zK_S)^{\alpha} \left(\nu \frac{zK_S}{zK_S + K_P}\right)^{1-\alpha} - (r+\delta) K_S + \tau (K_P)^{\alpha} \left(1 - \nu \frac{zK_S}{zK_S + K_P}\right)^{1-\alpha}. \end{aligned}$$

$\frac{\partial y_e}{\partial K_P}$ contains two parts: $\frac{\partial \pi_e}{\partial K_P}$ and $\frac{\partial Y_P}{\partial K_P}$. When K_P converges to 0, the first part converges to a negative but finite number:

$$\begin{aligned}\lim_{K_P\to 0}\frac{\partial \pi_e}{\partial K_P} &= \lim_{K_P\to 0}\alpha\left(\alpha-1\right)\nu^{1-\alpha}\left(zK_S\right)^{\alpha}\left(zK_S+K_P\right)^{\alpha-2}\\ &= \alpha\left(\alpha-1\right)\nu^{1-\alpha}\left(zK_S\right)^{2(\alpha-1)}.\end{aligned}$$

The second part converges to positive infinity:

$$\begin{aligned}\lim_{K_P\to 0}\frac{\partial Y_P}{\partial K_P} &= \lim_{K_P\to 0}\alpha\left(K_P\right)^{\alpha-1}\left(1-\nu\frac{zK_S}{zK_S+K_P}\right)^{1-\alpha}\\ &+ \left(1-\alpha\right)\left(K_P\right)^{\alpha}\left(1-\nu\frac{zK_S}{zK_S+K_P}\right)^{-\alpha}\left(\nu zK_S\left(zK_S+K_P\right)^{-2}\right)\\ &= +\infty+0\\ &= +\infty.\end{aligned}$$

This means that $\lim_{K_P\to 0}\frac{\partial y_e}{\partial K_P}=+\infty$ and $\exists\varepsilon,\forall K_P<\varepsilon,\frac{\partial y_e}{\partial K_P}>0$. When K_P is small, the marginal return of K_P to the elite is infinity, because the binding high state wage constraint pushes a significant amount of labor to the P sector.

The second result can be proved using the decreasing return to capital. Under the condition that $K_P>\sigma$, i.e., both minimal support constraint and high state wage constraint are binding, we have

$$\begin{aligned}L_S &= \nu\frac{zK_S}{zK_S+K_P}=\underline{L},\\ zK_S &= \frac{\underline{L}}{\nu-\underline{L}}K_P.\end{aligned}$$

This shows that if K_P becomes larger, K_S has to be proportionally larger if the elite decides to sustain oligarchy. Now the elite's income becomes:

$$y_e = \alpha\left(\frac{\underline{L}}{\nu-\underline{L}}K_P\right)^{\alpha}\left(\underline{L}\right)^{1-\alpha}-\left(r+\delta\right)\frac{\underline{L}}{z\left(\nu-\underline{L}\right)}K_P+\tau\left(K_P\right)^{\alpha}\left(1-\underline{L}\right)^{1-\alpha},$$

and

$$\begin{aligned}\lim_{K_P\to+\infty}\frac{\partial y_e}{\partial K_P} &= -\left(r+\delta\right)<0,\\ \lim_{K_P\to+\infty}y_e &= -\infty.\end{aligned}$$

The decreasing return to capital guarantees that when K_P is large enough, y_e decreases to $-\infty$. Then it is easy to find $\varepsilon > \sigma$ such that $\forall K_P > \varepsilon$, $\frac{\partial y_e}{\partial K_P} < 0$ and $y_e < 0 = y_e^D$.

Given the conditions in result (3), i.e., K_P is increasing in a_p, the sign of $\frac{\partial y_e}{\partial K_P}$ is the same as $\frac{\partial y_e}{\partial a_P}$. Moreover, one can find some ε_a such that $K_P < \varepsilon$ is equivalent to $a_p < \varepsilon_a$, and similarly for $K_P > \varepsilon$. So the properties in (1) and (2) are still valid.

To prove result (4), we can first show that when $\frac{1}{1+r} = 0$, the above properties for y_e are also true for V_e. This is obvious, because $V_e = y_e$. Under the condition that V_e and $\frac{\partial V_e}{\partial K_P}$ are continuous on $\frac{1}{1+r}$, $\exists \varepsilon, \forall \frac{1}{1+r} < \varepsilon$, these properties for V_e are still true. $\frac{1}{1+r}$ being sufficiently small is equivalent to r being sufficiently large.