**Suggested solution to PT 6.4.**

Watch out for typos (and thinkos)! If you find one, please notify me at `f.h.willumsen@econ.uio.no`.

## (a)

Assumptions:

- exogenous labor supply, so no incentive effects of unemployment insurance
- no credit markets

### Value functions

The Bellman equations

$$\begin{aligned} V^E &= u(c^E) + \beta\left[(1-\phi)V^E + \phi V^U\right] \\ V^U &= u(c^U) + \beta\left[(1-\vartheta)V^U + \vartheta V^E\right] \end{aligned}$$

Solve for $V^E$ and $V^U$ in terms of the primitives (using for example Cramer's rule)

$$\begin{aligned} V^E &= \frac{\left[1-\beta(1-\vartheta)\right]u(c^E) + \beta\phi u(c^U)}{(1-\beta)\left[1-\beta(1-\vartheta-\phi)\right]} \\ V^U &= \frac{\left[1-\beta(1-\phi)\right]u(c^U) + \beta\vartheta u(c^E)}{(1-\beta)\left[1-\beta(1-\vartheta-\phi)\right]} \end{aligned} \tag{1}$$

### Consumption levels

From the assumption in the problem text, we get the law of motion for the unemployment rate

$$u_t = \phi(1-u_{t-1}) + (1-\vartheta)u_{t-1}$$

Solved for steady state (i.e. $u_t = u_{t-1} = u$)

$$u = \frac{\phi}{\phi+\vartheta} \tag{2}$$

where we assume that $\vartheta > \phi$, i.e. $u < 1/2$. This means that the employed will decide on the tax rate.

The government's budget constraint has to to hold:

$$uf = \tau(1-u)l$$

Together with equation (2), this gives us an expression for the benefit level $f$:

$$f = \frac{\tau l \vartheta}{\phi}$$

Hence we have that

$$c^E = (1-\tau)l \tag{3}$$

$$c^U = f = \frac{\tau\vartheta l}{\phi} \tag{4}$$

### Optimal policy

We now assume log-utility, and maximize (1) wrt. $\tau$

$$\arg\max_{\tau} V^E = \arg\max_{\tau} \left\{ [1-\beta(1-\vartheta)] \ln[(1-\tau)l] + \beta\phi \ln\left(\frac{\tau\vartheta l}{\phi}\right) \right\}$$

FOC:

$$-\frac{1-\beta(1-\vartheta)}{1-\tau^*} + \frac{\phi\beta}{\tau^*} = 0$$

$$\iff \tau^* = \frac{\phi\beta}{1+\beta(\vartheta+\phi-1)}$$

Plug this into equations (3) and (4) to get the consumption levels

$$c^E = \left(1 - \frac{\phi\beta}{1+\beta(\vartheta+\phi-1)}\right) l \tag{5}$$

$$c^U = \frac{\vartheta}{\phi} l \frac{\phi\beta}{1+\beta(\vartheta+\phi-1)} \tag{6}$$

Some intuition: the conflict is between the employed and the unemployed. Remember that $u$ in equation (2) both measures the unemployment rate in steady state and the fraction of the lifetime each individual in the economy spends as unemployed. Hence the only reason for not giving full insurance, i.e. to have $c^E = c^U$, is that the individuals discount the future at rate $\beta$, and today and in the near future the median voter is going to be employed. This we see clearly if we set $\beta = 1$ in equations (5) and (6), since then

$$c^E = c^U = \left(\frac{\vartheta}{\phi+\vartheta} l\right)$$

If $\beta < 1$, then $c^U < c^E$.

## (b)

$$\frac{\partial\tau^*}{\partial\vartheta} = \frac{-\phi\beta^2}{[1+\beta(\vartheta+\phi-1)]^2} < 0$$

$$\frac{\partial\tau^*}{\partial\phi} = \frac{\beta[1+\beta(\vartheta-1)]}{[1+\beta(\vartheta+\phi-1)]^2} > 0$$

Intuition: see PT pp. 144. Remember that log-utility is the same as CRRA-utility with $\gamma = 1$, which is a fairly low risk aversion parameter

## (c)

The key here is to keep $u$ constant while changing $\vartheta$ and $\phi$. Differentating equation (2), we find that

$$\frac{d\phi}{d\vartheta} = \frac{u}{1-u} = \frac{\phi}{\vartheta}$$

We then find that

$$\begin{aligned}
\left.\frac{d\tau^*}{d\vartheta}\right|_{\text{u constant}} &= \frac{\partial\tau^*}{\partial\vartheta} + \frac{\partial\tau^*}{\partial\phi}\frac{d\phi}{d\vartheta} \\
&= \frac{\beta(1-\beta)(\phi/\vartheta)}{\left[1+\beta(\vartheta+\phi-1)\right]^2} > 0
\end{aligned}$$

Intuition: since the firing rate has increased, the expected time before the first unemployment spell will decrease for the decisive voter. Hence she will put more weight on the utility in this state than earlier, provided that $\beta > 0$. In the case where $\beta = 1$, i.e. in the case where we already have full insurance, the change in the optimal tax rate will be zero.