

I From the textbook

Solve exercises 1 and 2 from Ch. 2 in Persson and Tabellini (pp. 41f)

II Redistribution with deadweight losses

Solve the model in Persson and Tabellini Ch. 6.1 with the function $V(x) = \ln x$.

III The Downs model

There is a continuum of voters characterized by a parameter α which is uniformly distributed on $[0, 1]$, and a policy p is to be determined. A voter of type α has a utility function

$$u(p; \alpha) = -(p - \alpha)^2$$

- a) What is the preferred policy of a voter of type α ? Does she have single peaked preferences?
- b) If the policy p is determined by direct democracy with an open agenda, what outcome should we expect?
- c) Assume now that there are two parties A and B who both propose platforms p_A and p_B . Then voters vote for their preferred party. If one party gets the majority, it wins with certainty, if both get the same number of votes, the outcome is determined by "flipping a coin", i.e. both win with probability $1/2$.

Let x_A and x_B be the share of votes received by party A and B . Explain why it is natural that party A 's utility function is

$$v_A(x_A, x_B) = \begin{cases} 0 & \text{if } x_A < x_B \\ 1/2 & \text{if } x_A = x_B \\ 1 & \text{if } x_A > x_B \end{cases} .$$

Try to depict what this function looks like.

- d) As the parties cannot choose x_A and x_B themselves, it is more convenient to express it as a function of their platforms. Given optimal voter behaviour and platforms p_A and p_B , derive the two utility functions $u_A(p_A, p_B)$ and $u_B(p_A, p_B)$.

e) Find the Nash equilibrium when both parties have the utility function you derived above and choose platforms simultaneously.

f) Assume now that there are 3 parties A, B, C where party A has preferences

$$v_A(x_A, x_B, x_C) = \begin{cases} 0 & \text{if } x_A < x_B \text{ or } x_A < x_C \\ 1/3 & \text{if } x_A = x_B = x_C \\ 1/2 & \text{if } x_A = x_B > x_C \text{ or } x_A = x_C > x_B \\ 1 & \text{if } x_A > x_B \text{ and } x_A > x_C \end{cases},$$

and similarly for parties B and C . Interpret these preferences. Show that $p_A = p_B = p_C$ is no longer a Nash equilibrium.

g) Show that there is a Nash equilibrium where $p_A < p_B = p_C$ and A wins for sure.