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Problem 7, seminar 1

Job-arrival rate with search effort: $\lambda = \underset{\substack{\uparrow \\ \text{state of} \\ \text{the economy}}}{\alpha}\,\underset{\substack{\uparrow \\ \text{search} \\ \text{effort}}}{\lambda(e)}$

First step: the reservation wage.

$$(21)\quad x = b - c(e) + \frac{\alpha \lambda(e)}{r+q} \int_x^\infty (w - x)\, dH(w)$$

Second step: optimal effort.

- optimal effort is the effort level that maximizes ~~the~~ V_u
- Since $V_u = x/r$ we find the e that maximizes (21)

$$(22)\quad \Rightarrow c'(e) = \frac{\alpha \lambda'(e)}{r+q} \int_x^\infty (w - x)\, dH(w)$$

EQs (21) and (22) jointly determine e and x

Third step: Find derivatives

Using (21), find $\frac{dx}{d\alpha}$ and $\frac{dx}{db}$.

$\frac{dx}{d\alpha}$: $$dx = dx\left[-\frac{\alpha\lambda(e)}{r+q}(1-H(x))\right] + d\alpha\frac{\lambda(e)}{r+q}\int_x^\infty (w-x)\,dH(w)$$

Solve for $\frac{dx}{d\alpha}$

$$\frac{dx}{d\alpha} = \frac{\frac{\lambda(e)}{r+q}\int_x^\infty (w-x)\,dH(w)}{1+\frac{\alpha\lambda(e)}{r+q}(1-H(x))} > 0$$

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$\frac{dx}{db}$: $$dx = dx\left[-\frac{\alpha\lambda(e)}{r+q}(1-H(x))\right] + db$$

$$\frac{dx}{db} = \frac{1}{1+\frac{\alpha\lambda(e)}{r+q}(1-H(x))}$$

$$\Rightarrow \quad 0 < \frac{dx}{db} < 1$$

Fourth step:

Then consider eq. (23) (see book)

$$(23)\quad x = b + \underbrace{\frac{\lambda(e)}{\lambda'(e)} c'(e) - c(e)}_{h(e)}$$

This equation should always hold.

Hence, if b changes, we know that x changes and to make the eq. hold, e will also have to change

Since $0 < \frac{dx}{db} < 1$ we know that $h(e)$ must ~~in~~decrease ~~to~~ if b increases.

But we do not know if $h(e)$ increase or decreases with e

Fifth step:

Find $h'(e) = \frac{(\lambda' c' + \lambda c'')\lambda' - \lambda'' \lambda c' - c'}{(\lambda')^2}$

Multiply with $(\lambda')^2$

$$h'(e)\underset{+}{(\lambda')^2} = \cancel{(\lambda')^2 c'} + \underset{+\;+\;+}{\lambda \lambda' c''} \underbrace{-\lambda''}_{+} \underset{+\;+}{\lambda c'} - \cancel{c'(\lambda')^2}$$

$$h'(e) > 0$$

Hence, when b↑, x increases but by less than b and e falls.

When $\alpha \uparrow \Rightarrow x \uparrow$ and $e \uparrow$