Final Exam ECON3715/4715 – Labour Economics
Autumn 2020

This exam has 6 questions, with 14 sub-questions. Each sub-question counts equally.
When answering the questions on the exam you should be brief and to the point!
Make sure to write clearly. Difficult to decipher answers will not be counted!

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1. In this question you have to indicate whether you think the statement is true or false and explain why. You do not get any points if you only state whether the statement is true or false.

   (a) If the price of a substitutable input factor j rises, the demand curve for input i shifts down.

   > False. If the price of an input factor j rises, then the firm will increase its demand for another input factor i that can easily substitute factor j. Therefore, the demand curve for input i will shift up and not down.

   (b) A profit maximizing firm chooses an optimal input mix of labor and capital at given prices. If the firm minimized costs, holding the output fixed at the optimal level, it may not necessarily choose the same mix of labor and capital.

   > False. At the cost-miniziming solution, the slope of the isocost equals the slope of the isoquant, which implies that the marginal rate of technical substitution equals the ratio of input prices. At the profit-maximizing solution, the value of marginal product of each input factor equals the factor price. This means that profit-maximization implies cost-minimization. And if the firm produces output at the optimal level and minimizes its costs (which is assumed here), then the optimal mix of labor and capital *must be* identical to the optimal input that follows from profit maximization (formally, follows from duality theorem*). However, at other choices of output, the input mix acheived by cost-minimization may not necessarily be identical to the input mix under profit-maximization, so cost-minimization does not imply profit-maximization.

*The students are not required to show this. But must be familiar with the implications of profit maximization for the allocation under cost minimization.

(c) Any allocation on an efficient contract curve satisfies Pareto efficiency, but not necessarily allocative efficiency.

True. Allocations on a *strongly* efficienct contract curve, however, must satisfy both Pareto efficiency and allocative efficiency.

2. Inequality has increased in the US in recent decades. In this question you are asked to discuss possible explanations for this increase.

(a) There has been an increase in the supply of educated workers in the US. At the same time there has been an increase in the educational wage gap between educated and uneducated workers. In a simple demand and supply framework discuss how skill-biased technological change can rationalize these two findings. Has technological change always been skill-biased? Discuss.

If supply of educated workers rise, then the relative wage should fall. However, this has not been the case in the US. To rationalize that the education wage gap has increased, the relative demand for educated workers must have increased even more. This is interpreted as skill-biased technological change. Skill-bias is interpreted as new technology being a complement to skills. Technology is not necessarily skill-biased. Early industrial revolution largely replaced skilled workers with a combination of machines and low skilled workers.

(b) Not all countries have experienced the same rise in inequality as the US, despite experiencing similar technological development. Discuss possible reasons.

If technology is easily transferable between countries, then for a given relative supply of educated workers, inequality should also increase in other countries. We have not seen equal developments in other countries, which means that countries then must differ either in the relative supply of educated workers or have different wage setting institutions. A third issue is measurement. If educational systems differ across countries we might not be comparing the correct educational levels.

3. This question is about the human capital model. Assume that an individual has to decide on how many years of schooling to take. The individual has the following earnings function:

$$Y = exp(aS - \frac{1}{2}bS^2) \tag{1}$$

where Y is earnings and S is years of schooling. Assume that a and b are such that the earnings function monotonically increase in S and is concave in appropriate intervals of S.

(a) Assume that the individual faces a discount rate of r. What is the optimal level of schooling?

The marginal return is given as $MRR = \frac{Y'}{Y} = a - bS$.
Setting MRR equal to the discount rate r, we get:
$MRR = r \Leftrightarrow a - bS = r \Leftrightarrow S = \frac{a-r}{b}$

(b) Assume that you as a researcher observe two individuals, Anna and Benjamin, who have identical earnings functions but differ in their discount rate. You observe the educational choices and earnings of Anna and Benjamin but you do *not* observe their earnings function. How would you estimate the slope of earnings function in this simple case with two individuals? What is the formula for the estimated marginal return for Benjamin using observed outcomes? Will this estimate be biased? Discuss why/why not.

Because Anna and Benjamin share the same earnings function and only differ in their discount rate we can use their educational choices to estimate the slope of the earnings function.
The slope of the earnings function is given as:

$$\hat{\beta} = \frac{Y_{Anna} - Y_{Benjamin}}{S_{Anna} - S_{Benjamin}}$$

The estimated marginal rate of is

$$M\hat{R}R = \frac{\hat{\beta}}{Y_{Benjamin}} = \frac{Y_{Anna} - Y_{Benjamin}}{S_{Anna} - S_{Benjamin}} \frac{1}{Y_{Benjamin}}$$

When individuals have the same earnings function but different discount rates we can trace out the earnings function. If they had had different earnings function our estimate of the MRR would be biased as we would compare people who would differ in their counterfactual outcomes.

4. This question is about: Fehr, E. and L. Goette. (2007). Do Workers Work More if Wages are High? Evidence from a Randomized Field Experiment. *American Economic Review* 97(1): 298-317. The authors consider the following lifetime utility maximization problem.

$$\max U_0 = \sum_{t=0}^{T} \delta^t u(c_t, e_t) \text{ subject to } \sum_{t=0}^{T} \frac{(\hat{w}_t e_t + y_t - \hat{p}_t c_t)}{(1+r)^t} = 0, \tag{2}$$

where u is strictly concave and twice differentiable in c and e, c_t is consumption in period t, e_t is labor supply in period t, $\hat{p}_t$ is the price of consumption good, $\hat{w}_t$ is the wage, δ is the discount rate, and r is the interest rate.

(a) At the optimal path, (2) can be equivalently represented as:

$$\max\ v(e_t) = \lambda w_t e_t - g(e_t), \tag{3}$$

where $w_t := \frac{\hat{w}_t}{\delta^t(1+r)^t}$, $g_e^{'} > 0$, $g_e^{''} > 0$ and λ is the lifetime marginal utility of income. Interpret (3) and discuss the implications of an anticipated temporary wage increase.

Since λ is the lifetime marginal utility of income, one can interpret $\lambda w_t e_t$ as the discounted utility of income arising from effort in period t, while $g(e_t)$ is the discounted disutility of effort. In other words, in (3) individuals choose effort as to maximize the *net* discounted utility of effort. Workers who choose effort according to (3) respond to an anticipated temporary increase in w_t with a higher effort e_t. A rise in w_t increases the marginal discounted utility arising from effort, λw_t, which increases the optimal effort level in period t.

(b) The authors also consider a model with nonseparable utility:

$$\max\ v(e_t, e_{t-1}) = \lambda w_t e_t - g(e_t(1 + \alpha e_{t-1})), \tag{4}$$

where e_{t-1} is labor supply in period $t-1$. Interpret (4) and discuss why it is reasonable to assume $\alpha > 0$ in the context of this paper.

In (4), the discounted disutility of effort in period t, $g(e_t(1 + \alpha e_{t-1}))$, depends on effort exerted in period *t-1*. The marginal disutility of effort in period t is:

$$\begin{aligned} \frac{dg(e_t(1+\alpha e_{t-1}))}{de_t} &= g'(e_t(1 + \alpha e_{t-1}))(1 + \alpha e_{t-1}) \\ &= g'(e_t(1 + \alpha e_{t-1})) + \alpha e_{t-1} g'(e_t(1 + \alpha e_{t-1})) \end{aligned}$$

If $\alpha > 0$, the marginal disutility of effort in period t is higher compared to when $\alpha = 0$. And, the marginal disutility of effort in period t is higher the more effort was exerted in period *t-1*. In the context of this paper, the more a bicycle messenger worker has worked yesterday, the higher marginal cost of effort today as long as $\alpha > 0$. This could be reasonable as bicycle messengers may face fatigue today if they had exerting more effort yesterday.

(c) Imagine that there are only two future time periods (period 1 and period 2). Assuming that workers have nonseparable utility (4) and that they don't work in period 0 ($e_0 = 0$), we can express the two-period utility as $U = v(e_1, 0) + v(e_2, e_1)$, when we ignore discounting. Workers can receive either a high wage (w^H) or a low wage (w^L), and for a given worker, the wage rate is constant over time. Derive the first-order conditions and discuss workers' labor supply choices.

Using (4) and $U = v(e_1, 0) + v(e_2, e_1)$, we can write the two-period utility as:

$$U = v(e_1, 0) + v(e_2, e_1) = \lambda w e_1 - g(e_1) + \lambda w e_2 - g(e_2(1 + \alpha e_1))$$

The first-order conditions are as follows:

$$\frac{dU}{de_1} = \lambda w - g'(e_1) - \alpha e_2 g'(e_2(1 + \alpha e_1)) = 0$$

$$\frac{dU}{de_2} = \lambda w - g'(e_2(1 + \alpha e_1))(1 + \alpha e_1) = 0$$

If the wage rate is sufficiently high (w^H), then workers may choose to supply labor in both period 1 and period 2. The optimal effort choices would then be given by the first-order conditions (I)-(II) below. From (II), we see that an increase in e_1 causes a higher disutility of labor in period 2, which lowers the optimal choice of e_2. And from (I), we see that workers take this across-period 'effort externality' into account already when they decide on e_1, which means that the overall marginal disutility of e_1 is higher if e_2 is positive compared to when it is zero.

$$\lambda w^H = g'(e_1^{**}) + \alpha e_2^{**} g'(e_2^{**}(1 + \alpha e_1^{**})) \qquad (I)$$

$$\lambda w^H = g'(e_2^{**}(1 + \alpha e_1^{**}))(1 + \alpha e_1^{**}) \qquad (II)$$

If the wage rate is sufficiently low (w^L), then workers may choose not supply in period 2 ($e_2^* = 0$). The first-order conditions would then be:

$$\lambda w^L = g'(e_1^*) \qquad (I')$$

$$\lambda w^H < g'(0)(1 + \alpha e_1^*) \qquad (II')$$

Note that if $\alpha = 0$ (as in (3)), then $e_1^{**} > e_1^*$ since $w^H > w^L$. If $\alpha > 0$, however, $e_1^{**} \gtreqless e_1^*$. Messengers who work more periods when the wage is high *may* rationally decide to reduce the effort per shift, i.e, $e_1^{**} < e_1^*$. It is also possible that works exert more effort in both periods when the wage is high, i.e., $e_1^{**} > e_1^*$ (and by construction, also $e_2^{**} > e_2^* = 0$).

5. This question is about: Staiger, D.O., J. Spetz, and C.S. Phibbs (2010). Is There Monopsony in the Labor Market? Evidence from a Natural Experiment. *Journal of Labor Economics* 28(2): 211-236. The authors consider the Salop model of

competition around a circle, where the labor supply facing hospital i is as follows:

$$L_i = \alpha + \frac{1}{\tau}\left(w_i - \frac{w_{i-1} + w_{i+1}}{2}\right), i = 1, ..., N, \tag{5}$$

where w_i is the wage at hospital i, and w_{i-1} and w_{i+1} are wages at the two closest neighboring hospital.

(a) Suppose that the hospital sets wages to maximize profits $R(L_i) - L_i w_i$ subject to (5), where $R(L_i)$ is the revenue function and $R_L^{'} > 0$. Derive hospital i's first-order condition and interpret the expression.

> The firm's optimization problem is:
>
> $$\max\ R(L_i) - L_i w_i$$
>
> $$= R\left(\alpha + \frac{1}{\tau}\left(w_i - \frac{w_{i-1} + w_{i+1}}{2}\right)\right) - w_i\left(\alpha + \frac{1}{\tau}\left(w_i - \frac{w_{i-1} + w_{i+1}}{2}\right)\right)$$
>
> The first-order condition wrt w_i is:
>
> $$R_L^{'}\frac{1}{\tau} - L_i - \frac{1}{\tau}w_i = 0$$
>
> $$w_i = R_L^{'} - \tau L_i$$
>
> This expression shows that the monopsony wage is set below marginal revenue product ($R_L^{'}$), and the size of the wage markdown depends on the slope of the firm's labor supply equation (τ). The less elastic is the labor supply (higher τ), the larger is the wage markdown, i.e., the higher is monopsony power.

(b) Suppose $R(L_i) = L\beta$. Imagine that the distance between a hospital and its closest competitors increases, i.e., there is a higher α. How does this change monopsony power?

> Using the first-order condition derived above, we get:
>
> $$w_i = R_L^{'} - \tau L_i = \beta - \tau\left(\alpha + \frac{1}{\tau}\left(w_i - \frac{w_{i-1} + w_{i+1}}{2}\right)\right)$$
>
> $$= \beta - \tau\left(\alpha + \frac{1}{\tau}\left(w_i - \frac{w_{i-1} + w_{i+1}}{2}\right)\right)$$

And, in a symmetric equilibrium, $w_i = w_{i-1} = w_{i+1} := w_e$, so we get:

$$w_e = \beta - \left(\tau\alpha + \left(w_e - \frac{w_e + w_e}{2}\right)\right) = \beta - \tau\alpha$$

An increase in α thus increases the wage markdown for all hospitals, and thereby also increases the monopsony power held by each hospital.

6. This question is about: Parey, M., Ruhose, J, Waldinger, F., and N. Netz. (2017). The Selection of High-Skilled Emigrants. Review of Economics and Statistics 99(5): 776–792. The authors estimate the following model:

$$\log w_{0i} = X_i\beta_0 + \varepsilon_{0i} \tag{6}$$

where w_{0i} are wages in Germany and X_i are individual characteristics.

(a) The authors estimate (6) and compute $\hat{\theta}_{0i} = X_i\hat{\beta}_0$. Why do the authors compute $\hat{\theta}_{0i}$? What is the interpretation of $\hat{\theta}_{0i}$, and how does this differ from θ_0 for movers and stayers, respectively?

$\hat{\theta}_{0i}$ are the predicted earnings for individual i in the home country. The estimates are predicted for both movers and stayers using the parameters estimated for stayers. The predicted earnings at home are counterfactual for movers. The authors, therefore, need to impute the earnings that movers would have had, had they stayed in Germany. For making comparisons the predicted earnings are also used for stayers.

(b) Interpret Figure 2 panel (a) in Parey et. al (2017). What does it tell us about sorting of migrants and are the results in line with the prediction in the Roy-model? Discuss.

The figure shows the cumulative distribution function (CDF) of $\hat{\theta}_{0i}$ for stayers and for mover to more and less equal countries. If inequality stems from higher returns to skills and applicable skills in the two countries are positively correlated the Roy-model would predict that movers to more equal countries should be negatively selected. This implies that the CDF should be to the left of the stayers. Similarly, the model predicts that movers to more unequal countries should be positively selected. That implies that the CDF of movers to more unequal countries should be to the right of stayers. Both predictions are carried out in the data.

Figure 2.—Predicted Earnings of Migrants and Nonmigrants

(a) CDF for Three and Five Groups of Countries