Final Exam ECON3715/4715 – Labour Economics
Autumn 2023

This exam has 5 questions, with 13 sub-questions. Each sub-question counts equally. When answering the questions on the exam you should be brief and to the point! Make sure to write clearly. Difficult to decipher answers will not be counted!

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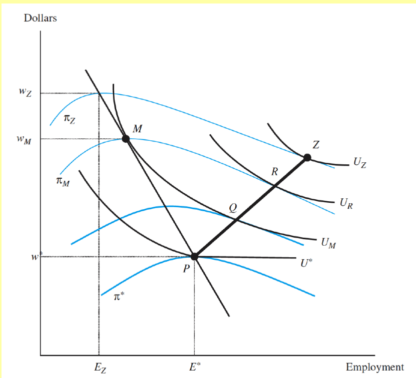
1. Indicate whether you think the statement is true or false and explain why. You do not get any points if you only state whether the statement is true or false.

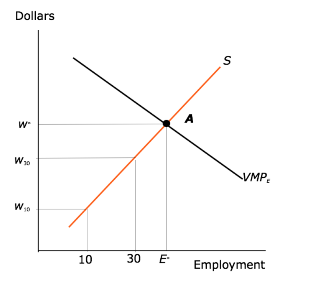
   (a) Under efficient bargaining between a firm and a labor union, the wage can be set above the value of marginal product of labor for the marginal worker.

   True. With efficient bargaining, the firm and the union may decide to allocate above to the right of the firm's labor demand curve along the contract curve (see figure below). This would imply that the wage offered to the marginal worker actually exceeds the value generated by this worker (i.e., $w > VMPL$).



   (b) In a perfectly discriminatory monopsony, the wage offered to a worker increases with the value of marginal product of labor generated by the worker.

False. The wage offered to a worker in a perfectly discriminatory monopsony equals his/her reservation wage. As the firm has downward sloping demand curve (diminishing return to scale), the there is actually a negative relationship between VMPL and the wage offered to each additional worker.



A perfectly discriminating monopsonist faces an upward-sloping labor supply curve and can hire different workers at different wages. Therefore the labor supply curve gives the marginal cost of hiring. Profit maximization occurs at point *A*. The monopsonist hires the same number of workers as a competitive market, but each worker is paid his or her reservation wage.

(c) In a firm with efficiency wages, an optimal wage must satisfy the condition that the elasticity of worker's effort with respect to the wage is equal to zero.

False. In a firm with efficiency wages, the optimal wage satisfies the condition that the elasticity of effort with respect to the wage is unity (not zero). The students may show the derivation/formula for an efficiency wage as follows, but are not required to show this. They should provide an intuition.

A profit-maximizing firm solves the following problem:

$$\max_{w,L} \ \pi = F(E(w) \cdot L) - w \cdot L$$

The FOCs are as follows:

$$\begin{aligned} \frac{\partial \pi}{\partial L} &= F' \cdot E(w) - w = 0 && \longrightarrow \quad F' = \frac{w}{E(w)} \\ \frac{\partial \pi}{\partial w} &= F' \cdot E'(w) \cdot L - L = 0 && \longrightarrow \quad F' = \frac{1}{E'(w)} \end{aligned}$$

which gives:

$$E'(w) \cdot \frac{w}{E(w)} = 1 \qquad \textit{and} \qquad F' \cdot E(w) = w$$

Each firm hires labor up to the point where its marginal product equals this efficiency wage

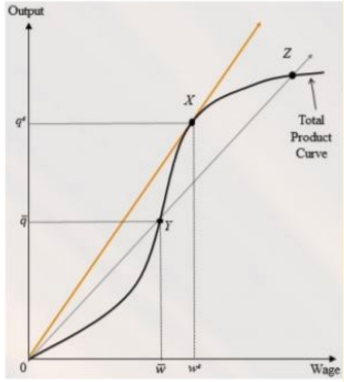
$$F' \cdot E(w^e) = w^e$$

The optimal wage satisfies the condition that the elasticity of effort with respect to the wage is unity.

$$E'(w^e) \cdot \frac{w^e}{E(w^e)} = 1$$

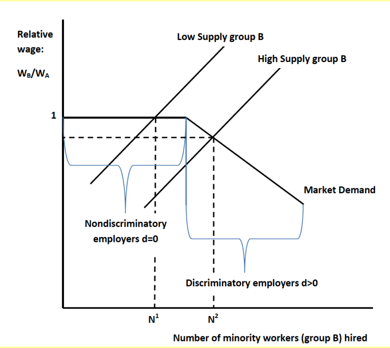
That is the wage rate that minimizes wage costs per efficiency unit of labor.

Students may also show and discuss this graphically as follows.



(d) Under employer taste-based discrimination, the equilibrium wage of a negatively discriminated worker is always below the wage of a non-discriminated worker.

False. As can be seen in the graph, employer taste based discrimination can result in a wage gap between minority workers (B) and workers from the majority group (A), but this need not always be the case. It depends on the number of (non)discriminatory employers and on the labor supply of minority workers. If at $w_A = w_B$ the demand for minority workers is equal to the supply of minority workers there will not be a wage differential because all the minority workers will be hired by nondiscriminatory employers. If, however, the demand for minority workers is less than the supply of minority workers there will be a wage differential.



2. This question is about incentive pay. The output is given by $y = \mu + \varepsilon$, where μ is the worker's effort and random shock $\varepsilon \sim N(0, \sigma^2)$. The firm offers the worker a wage contract $w = s + by$, where s is a base salary and b is a piece rate. The worker is risk neutral and maximizes utility $U = w - c(\mu)$, where $c(\mu)$ is the cost of effort with $c'(\mu) > 0$ and $c''(\mu) > 0$. The firm determines (s, b) by maximizing expected profit $E[py - w]$ subject to worker's participation constraint $E[w - c(\mu)] \geq \bar{U}$, where p is fixed price per output unit, and $\bar{U}$ is worker's outside option. Worker decides whether to accept or reject the contract and an effort level if the contract is accepted.

(a) Show that first-order conditions to the firm's problem imply $(p - c'(\mu)) \frac{d\mu}{db} = 0$. Interpret this equation and explain why the optimal piece rate b^* is efficient.

First, let's derive the worker's first-order condition. The worker maximizes expected utility wrt effort:

$$\max_{\mu} E\left[U\right] = E\left[s + by - c(\mu)\right] = s + b\mu - c(\mu)$$

This gives the FOC: $c'(\mu^*) = b \iff \mu^*(b) = c'^{-1}(b)$.
Then, let's consider the firm's problem. The firm maximizes expected profit wrt (s, b) subject to the PC (which is binding):

$$\max_{s,b} E\left[\pi\right] = E\left[py - w\right] \ s.t. \ E\left[w - c(\mu)\right] \geq \bar{U}$$

$$\max_{s,b} \mathcal{L} = E\left[py - w\right] - \lambda\left(E\left[w - c(\mu)\right] - \bar{U}\right)$$

$$\max_{s,b} \mathcal{L} = p\mu - (s + b\mu) - \lambda\left((s + b\mu) - c(\mu) - \bar{U}\right)$$

We take the derivative of $\mathcal{L}$ wrt s:

$$\frac{d\mathcal{L}}{ds} = -1 - \lambda = 0 \iff \lambda = -1$$

We take the derivative of $\mathcal{L}$ wrt b, taking into account that the worker's optimal choice of effort $\mu^*(b)$ depends on b, as follows:

$$\begin{aligned}\frac{d\mathcal{L}}{db} =& p\frac{d\mu}{db} - (b\frac{d\mu}{db} + \mu) - \lambda\left((b\frac{d\mu}{db} + \mu) - c'(\mu)\frac{d\mu}{db}\right) = 0\\ \Rightarrow& p\frac{d\mu}{db} - (b\frac{d\mu}{db} + \mu) + 1\left((b\frac{d\mu}{db} + \mu) - c'(\mu)\frac{d\mu}{db}\right) = 0\\ & p\frac{d\mu}{db} - c'(\mu)\frac{d\mu}{db} = 0 \Leftrightarrow (p - c'(\mu))\frac{d\mu}{db} = 0\end{aligned}$$

We note that $\frac{d\mu^*}{db} > 0$ since both $c'(\mu) > 0$ and $c''(\mu) > 0$. Thus, at the optimal choice b^*, this condition implies $(p - c'(\mu)) = 0$ so that the worker's marginal cost of effort equals to the expected marginal value of their effort, which is equal to the unit price of output. Inserting for the worker's FOC $c'(\mu^*) = b$, this implies that $(p - b^*)\frac{d\mu^*}{db} = 0$ at the optimal choices of b^* and μ^*. The piece rate is thus efficient in the sense that the worker exerts optimal effort, and receive the expected marginal value of their effort.

(b) Assume the worker's cost function is $c(\mu) = \frac{c\mu^2}{2}$ where $c > 0$ and the worker is now risk averse and has a constant absolute risk aversion utility with expectation:

$$E\left[U\right] = E\left[-e^{-r(w - c(\mu))}\right] = -e^{-r\left[s + b\mu - \frac{rb^2\sigma^2}{2} - c(\mu)\right]}$$

where r is the worker's coefficient of absolute risk aversion. The firm's first-order conditions now imply $(p - c'(\mu))\frac{d\mu}{db} - r\sigma^2 b = 0$. Interpret this equation and discuss how this affects the worker's optimal choice of effort.

First, let's again derive the worker's first-order condition. The worker maximizes expected utility wrt effort:

$$\max_{\mu} E[U] = -\,e^{-r\left[s+b\mu-\frac{rb^2\sigma^2}{2}-c(\mu)\right]}$$

This gives the FOC: $\underbrace{re^{-r[.]}}_{>0}(b - c'(\mu^*)) = 0 \iff c'(\mu^*) = b$ as before.

Using $c(\mu) = \frac{c\mu^2}{2}$, we get $c'(\mu) = c\mu$, which together with the worker's FOC gives $\mu^* = b/c$ and $\frac{d\mu^*}{db} = \frac{1}{c}$.

Inserting $c'(\mu^*) = c\mu^* = b^*$ and $\frac{d\mu^*}{db} = \frac{1}{c}$ in the firm's FOC, we can now derive:

$$(p - c\mu^*)\frac{1}{c} - r\sigma^2 b^* = 0$$
$$\Leftrightarrow b^* = \frac{p}{1 + rc\sigma^2} < p$$

This equation shows that the principal faces a trade-off between providing incentives and insurance to the worker. Unlike in (a) where the optimality conditions imply $b^* = p$, when the worker is risk averse, the piece rate is less than p, implying that the worker is partially insured. The higher is the risk aversion r, the marginal cost of effort c and output risk σ^2, the lower is the optimal piece rate offered to the worker. Since optimal effort is $\mu^* = b/c$, we can also show that the worker exerts less effort than in (a) as follows:

$$\mu^* = \frac{b^*}{c} = \frac{p}{c(1 + rc\sigma^2)} < \frac{p}{c}.$$

3. This question is about the structure of collective bargaining.

   (a) Explain Figure 1 below that is taken from Moene, K. O. and M. Wallerstein (1997). Pay Inequality. *Journal of Labor Economics* 15(3): 403–430.

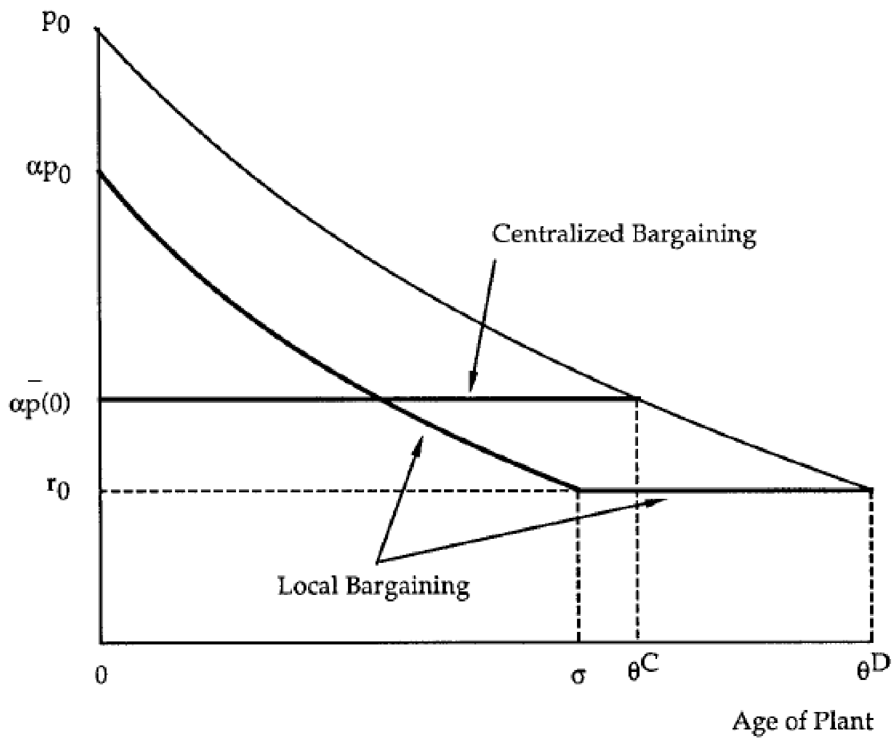


FIG. 1.—The distribution of wages across plants with local and centralized bargaining

Discussed in the notes for Lecture 5 and Seminar 3.

(b) Imagine that there has been a gradual decentralization of the bargaining structure in the economy over time, where some industries have gone from a centralized to a local bargaining, while other industries continue to have a centralized bargaining (i.e., a standard wage rate). How would you test the predictions of the Moene and Wallerstein (1997) model? Explain the data requirements and the assumptions needed in your empirical strategy.

We can test the following hypotheses (students can also mention additional model implications/hypotheses discussed in the paper):
a) Greater wage dispersion across (surviving) plants under local bargaining.
b) The number of plants in operation goes up under local bargaining: plants that remain active between θ^C and θ^D in Figure 1 above.
c) In the most productive plants (located closest to 0 along the x-axis): higher wages and lower profit under local bargaining.
d) In the least productive plants (furthest away from 0 along the x-axis): lower wages and higher profit under local bargaining.
Example of an empirical strategy (students can mention other strategies):
- A difference-in-differences type strategy across industries over time, where we control for industry fixed effects and common changes over time, and rely on within industry changes in bargaining structure.
- This avoids (potentially biased) cross-sectional comparisons.
Data requirements:
- Panel data on workers and plants (e.g., as in Dahl et al. (2013)).
- Data on the number of plants operating in each industry in each period.
- Characteristics of plant: ideally, a measure of each plant's productivity (in the model, this is proxied by plant age).
- Characteristics of workers: (average) wage level (in the simpliest version of the model, all workers are homogeneous).
Assumptions needed:
- Parallel trends in outcomes across all industries in the absence of decentralization (counterfactual is centralized bargaining in all periods).
- Absence of spillovers or labor mobility across industries.

4. This question is about a signaling model of education. Bedard, K. (2001). Human Capital versus Signaling Models: University Access and High School Dropouts. *Journal of Political Economy* 109(4): 749–775. The author presents a model with three education groups: high school dropouts, high school graduates and university graduates. Each individual is fully described by a single dimensional ability, θ. The author states that a separating equilibrium must satisfy the following conditions, where θ_h and θ_u are cutoff ability levels that are implicitly defined:

$$
\begin{aligned}
E(\theta|\theta < \theta_h) =& \phi - C_h(\theta_h) \\
E(\theta|\theta > \theta_u) =& \phi + C_u(\theta_u),
\end{aligned}
$$

where ϕ is the expected wage of high school graduates, $E(\theta|\theta < \theta_h)$ is the expected wage of high school dropouts, $E(\theta|\theta > \theta_u)$ is the expected wage of university graduates, and $C_h(.)$ and $C_u(.)$ are the costs of high school and college, respectively.

(a) Explain the concept of a separating equilibrium in the signaling model of Bedard and explain what it means that the cutoff ability levels θ_h and θ_u are implictly defined by the conditions provided above.

In a separating equilibrium, individuals choose different levels of education as the cost of obtaining education is decreasing in ability. Individuals below the cutoff θ_h will drop out of high school, individuals between θ_h and θ_u will finish high school, and unconstrained individuals with ability above θ_u will go on to earn a university degree. The cutoff points θ_h and θ_u are defined implictly by equating utilities of the marginal students:

$$\begin{aligned} w_d =& w_h - C_h(\theta_h) \\ w_h - C_h(\theta_u) =& w_u - C_u(\theta_u) - C_h(\theta_u) \end{aligned}$$

Thus, the marginal student with ability θ_h is indifferent between dropping high school with wage w_d or completing high school with wage w_h at cost $C_h(\theta_h)$, and the marginal student with ability θ_u is indifferent between completing high school with wage w_h at cost $C_h(\theta_u)$ or completing college with wage w_u at cost $C_h(\theta_u) + C_u(\theta_u)$. Using $w_d = E(\theta|\theta < \theta_h)$, $w_h = \phi(\theta)$, and $w_u = E(\theta|\theta > \theta_u)$ yields the conditions above.

(b) Imagine that the government implements a reform that leads to an easier access to university education. What will happen to the average wage of high school graduates and the share of high school graduates? Explain the mechanisms.

The students may refer to or show a version of Figure 1 in Bedard (2001). As previously high-ability individuals will now leave for university, the wage of high school graduates will fall. This reduces the incentives of those close to θ_h and θ_u to finish high school. This implies that the cutoffs shift inwards and the high school graduate share therefore falls.
Mathematically, the average wage of high school graduates ϕ is a weighted mean of those that would go to high school regardless (i.e., those with $\theta_h \leq \theta < \theta_u$ which is the share $[F(\theta_u) - F(\theta_h)]$) and the constrained individuals who finish high school but would graduate from university if they were unrestricted (i.e., $\theta \geq \theta_u$ with a share $(1-p)[1 - F(\theta_u)]$). NOTE: The students are NOT required to remember or provide the formula for $\phi = \frac{[F(\theta_u)-F(\theta_h)]E(\theta|\theta_h \leq \theta < \theta_u) + (1-p)[1-F(\theta_u)]E[\theta|\theta \geq \theta_u]}{[F(\theta_u)-F(\theta_h)]+(1-p)[1-F(\theta_u)]}$. But they should be able to provide an intuition, e.g., using arguments as in Figure 1.

5. This question is about discrimination. Bartoš et. al. (2016). Attention Discrimi-

nation: Theory and Field Experiments with Monitoring Information Acquisition. *American Economic Review* 106(6): 1437–1475.

(a) Assume that, due to information asymmetry, profit-maximizing employers discriminate workers on an observable characteristic. Can this kind of discrimination persist in a competitive market with free entry and exit?

> This kind of discrimination is statistical discrimination. This kind of discrimination can persist in the long run if the information asymmetry persists.

(b) Assume that there are two groups of workers, A and B, where the average productivity of group B is higher than that of group A and the productivity distributions across groups are otherwise identical. Assume that the average applicant is acceptable for an employer. As in Bartoš et. al. (2016) assume that information acquisition about an applicant is equally costly for the employer independent of applicant type. Which applicant group is the employer most likely to screen? Explain the intuition.

> As the average applicant is acceptable this is what the authors refer to as a lemon-dropping market. As group A is the least productive group, employers are most likely to spend resources on group A to weed out undesirable applicants. This is the same situation as the renters market in the paper. This is because more information should be acquired when its expected benefits are higher, which is when there is a higher chance that the informed decision differs from the status quo. In this example, group A is on average less desirable and the information acquisition in group A is therefore expected to yield larger benefits for the employer.

(c) Assume that you had observational data on job applications, worker characteristics and employer hiring decisions. In order to measure discrimination, why may you still prefer to run experiments with artificial applicants?

> Applicant characteristics are likely to covary with the observable characteristic (e.g., ethnicity) on which one may expect discrimination to take place. Therefore, realized acceptance rates across workers with and without this observable characteristic may reflect other factors than discrimination based on this one characteristic (e.g., ethnicity). The artificial applications allow the researcher to isolate the effect of a single characteristic and thereby hopefully identify discrimination. This point has been repeatedly discussed in both lectures and seminars. As the student has access to the course material, the student should be able to provide a satisfactory answer.