Question 1 (30%), TRUE or FALSE.

a) TRUE. Candidates are expected to argue that Bertrand competition replicates the competitive prices which are always at least weakly below Cournot prices.
b) TRUE. Candidates are expected to argue that low cost firms want to mimic high cost firms to increase their profits. Additionally, candidates may mention that this happens in a pooling equilibrium but this is extra.
c) TRUE. Candidates are expected to argue that product differentiation (horizontal or vertical) gives firms market power and thus differentiated firms price above the marginal cost.
d) TRUE. Candidates are expected to argue that large enough cost savings can reduce the optimal price for the merged firm so much that the market price ends up being lower after a merger.
e) FALSE. Candidates are expected to say that excess inertia means that consumers adopt innovations too slowly. Additionally, candidates may mention that excess inertia happens if the payoff from new technology is greater in the case consumers coordinate technology adoption.
f) FALSE. The fat cat strategy is to install a large capacity which is entry deterring in the capacity-price game. The puppy dog strategy, small capacity, would be entry accommodating.
g) FALSE. Candidates are expected to argue that margin squeeze is a form of vertical foreclosure, i.e. the integrated firm sets a high upstream price and a low downstream price so that the downstream competitor's margin is squeezed.
h) TRUE. Candidates are expected to argue that when there is an upstream and a downstream monopolist, double marginalization happens. That is, the downstream monopolist puts a markup on top of the upstream price, and thus prices are higher than if there is vertical integration.

Question 2 (35%) Bertrand and Cournot competition with demand $D(p) = e^{-\alpha p}$.

a) Because products are homogenous, the demand for firm *i* is given by

$$D(p_i, p_j) = \begin{cases} e^{-\alpha p_i}\ if\ p_i < p_j \\ 0.5e^{-\alpha p_i}\ if\ p_i = p_j \\ 0\ if\ p_i > p_j \end{cases}$$

Firm i's profit maximization problem is then

$$\pi_i(p_i, p_j) = \max_{p_i}(p_i - c)D(p_i, p_j)\,.$$

The equilibrium price is $p^e = c$ so that the equilibrium amount is $Q^e = e^{-\alpha c}$. To see that this is an equilibrium, note that $p < p^e = c$ results in a negative profit, $\pi_i < 0$, and price $p > p^e$ results in a zero profit, $\pi_i = 0$. Thus, neither firm i nor firm j wants to deviate by setting a lower or a higher price.

For uniqueness of the equilibrium (for full points), note that if the equilibrium price is higher than the marginal cost, $p^e > c$, either firm has a profitable deviation such that they set the price infinitesimally below p^e $(e.g. p^e - \varepsilon)$ which results in a larger profit than price p^e because the deviating firm captures the whole demand.

b) The equilibrium price now equals the larger marginal cost, $p^e = c_1$. To see this formally, note that if $p^e > c_1$, then either firm has a profitable deviation by setting a slightly lower price (see argument above) and if $p^e < c_1$, firm 2 has a profitable deviation by setting a higher price up to c_1. Students are here free to assume that the tie braking rule is such that the lower marginal cost firm captures the whole demand. Uniqueness of this equilibrium follows from the same steps as above.

c) If the firms collude on prices, they effectively act like a monopoly and split the monopoly profits. A monopoly's profit function is $\pi^m(p) = (p - c)e^{-\alpha p}$. Taking the first order condition gives: $e^{-\alpha p}\big(1 - \alpha(p - c)\big) = 0$, which gives a price $p = (1 + \alpha c)/\alpha$.

Technical note: analyzing the first order condition is fine, because the profit function is quasiconcave (the first derivative with regards to price is monotone). Students are not expected to check this.

d) Inverting the demand function gives $Q = e^{-\alpha p} \Leftrightarrow \ln Q = -\alpha p \Leftrightarrow p(Q) = -\frac{1}{\alpha}\ln Q$ where $Q = q_1 + q_2$ is the total demand. Firm i's profit maximization problem over quantity q_1 is then: $\pi_i\big(q_i, q_j\big) = \max_{q_i}(p(Q) - c)q_i$.

e) The first order condition with regards to q_i is $p'(Q)q_i + p(Q) - c = 0$ or $-\frac{\alpha}{q_i+q_j}q_i - \frac{1}{\alpha}\ln\big(q_i + q_j\big) - c = 0 \Leftrightarrow q_i + \big(q_i + q_j\big)\ln\big(q_i + q_j\big) + \alpha c\big(q_i + q_j\big) = 0$. Firm j's first order condition is identical but we must switch the places of q_i and q_j.

Writing the first order condition as $F(q_1, q_2) = 0$ and using the implicit function theorem gives

$$\frac{dq_1}{dq_2} = -\frac{\frac{\partial F}{\partial q_2}}{\frac{\partial F}{\partial q_1}} = -\frac{\ln(q_1+q_2)+1+\alpha c}{1+\ln(q_1+q_2)+1+\alpha c} < 0.$$

Quantities are strategic substitutes: if q_2 increases q_1 decreases (another way to see this is to look at the cross derivative of the profit function.)

f) The game is identical to the quantity competition game. The easiest way to see this is to use backward induction and solve the game backwards: the problem in stage 1 is identical to the problem in d).

Question 3 (35%) steel industry and innovation.

a. The correct answer here is the replacement effect: the entrant is replacing a zero-profit firm whereas the established firms are (arguably) replacing firms already making profits.

b. The idea here was to compare a monopolist and a perfectly competitive firm to each other. While the exact details of the model the candidates use does not matter, they should argue the value of lowering the marginal cost is lower for a monopolist because of the replacement effect. Below is one way to do this.

   Suppose demand is given by downward sloping demand curve $D(p)$ and the producers have constant marginal costs. A monopolist's profit is is given by $\pi(p) = (p - c)D(p)$. Using the envelope theorem, the value of lowering the marginal cost from $\overline{c}$ to $\underline{c}$ for a monopolist is given by (from lecture notes):

$$V^m = \frac{1}{r}\int_{\underline{c}}^{\overline{c}} D(p^m(c))dc$$

   In contrast, the value of lowering of lowering the marginal cost from $\overline{c}$ to $\underline{c}$ for a zero profit firm (profit function is the same but equilibrium price after innovation is $\overline{c}$) is

$$V^c = \frac{1}{r}\int_{\underline{c}}^{\overline{c}} D(\overline{c})dc$$

   Because $p^m(c) > p^m(\underline{c}) > \overline{c}$, it then follows that $V^m < V^c$ (the latter is larger pointwise so the integral has to be larger as well).

c. Candidates should say that the efficiency effect goes the other way than the replacement effect: larger firms have larger market shares and thus capture more cost savings through lower marginal costs (innovation).

d. The grading of this problem is at the grader's discretion. The answer I was after is that steel industry is one where competition is capacity constrained and that the capacity adjusts slowly over time. Marginal costs matter for today's profits but not for today's market shares.