

Problem 1: Procurement auctions (30%)

Consider a procurement auction in which the procurer requests bids to produce some units of a good. The buyer is using a sealed-bid second price auction. There are five participants in the procurement auction. All of them share the same cost structure $c_i(x) = F_i + a_i x + b_i x^2$, but with different parameters (F_i, b_i, c_i) . In particular, the five bidders have the following parameters

Bidder	F_i	a_i	b_i
1	0	100	7
2	0	200	7
3	0	300	7
4	40 000	1 000	3
5	50 000	2 000	3

a)

Describe a weakly dominant strategy in a sealed-bid second-price auction and explain why it is weakly dominant. Use the information provided above to derive the explicit bidding function for the participants in this case. Explain your answer.

b)

Assume that the procurer asks for bids for 100 units to be produced, and that the auction participants all follow their (weakly) dominant strategy. Which bidder wins the auction, what is his winning bid per unit, what does he end up getting paid per unit (the equilibrium price) and what is his total surplus?

c) Assume instead that the procurer asks for bids for 1 000 units to be produced, and that the auction participants all follow their dominant strategy. Which bidder wins the auction, what is his winning bid per unit, what does he end up getting paid per unit (the equilibrium price) and what is his total surplus?

Problem 2: Mergers and merger control (30%)

Consider two firms who compete in prices selling differentiated goods. They both share the same marginal cost c . They face the following demand system

$$q_1 = a_1 - bp_1 + dp_2$$

$$q_2 = a_2 - bp_2 + dp_1.$$

We assume that $a_i > 0, b > 0$ and $-b < d < b$.

a) Assume that the firms maximize profits by simultaneously setting prices. Derive the best response curves of the two firms.

b) Are the prices strategic substitutes or complements? How does that depend on the parameter d ?

c) Derive the price equilibrium.

Hint: Check that if you substitute $a_1 = a_2 = a$, you get

$$p_1^D = p_2^D = \frac{a}{2b-d} + \frac{bc}{2b-d}$$

d) The diversion ratio from good 1 to good 2 is defined as $DR_{12} = \frac{\frac{\partial q_2}{\partial p_1}}{\frac{\partial q_1}{\partial p_1}}$.

We say that a merger leads to upward pricing pressure for firm i if $UPP_i = (p_j - c_j)DR_{ij} - e_i > 0$, evaluated at the pre-merger prices, where e_i is the reduction in marginal cost for firm i as a result of the merger. Finally, define the gross upward pricing pressure index (GUPPI) as

$$GUPPI_i = \frac{p_j - c}{p_i} DR_{ij} = \left(\frac{p_j - c}{p_j} \right) \frac{p_j}{p_i} DR_{ij} = M_j \frac{p_j}{p_i} DR_{ij}$$

Derive the expressions for the diversion ratios, the upward pricing pressures and the gross upward pricing pressure indices, $DR_{12}, DR_{21}, UPP_1, UPP_2, GUPPI_1$ and $GUPPI_2$, in our linear model. In Table 1 we provide two numerical cases to consider. Calculate and state the values for the diversion ratios, the UPPs and the GUPPIs for each of the two cases (you can use a calculator or a spreadsheet to do this).

Table 1: Parameters for Case 1 and Case 2

	a_1	a_2	b	d	c	e_1	e_2
Case 1	1	3/2	1	0	1/2	1/2	0
Case 2	4/3	1/3	4/3	2/3	0	0	0

e) Explain what the diversion ratio measures. In which of the cases are the two firms the closest competitors as measured by the diversion ratios? How does that relate to the parameters b and d , and the extent to which the two goods are substitutes in demand?

f) If the two firms merge, we assume that the merged firm will control both p_1 and p_2 and is assumed to maximize the joint profit of the merged firm. That is, we assume that the merged firm solves

$$\max_{p_1, p_2} \Pi^M = (p_1 - c + e_1)q_1(p_1, p_2) + (p_2 - c + e_2)q_2(p_1, p_2)$$

Consider a competition authority that is tasked with screening mergers, in order to maximize total social welfare (the sum of producer and consumer surplus). What is the potential tradeoff that this agency faces in our model? Is the tradeoff equally present in both Case 1 and Case 2?

- g) In which case will the merger lead to the biggest increase in the price? In which of the two cases should a competition authority that only cares about consumer surplus allow the merger? Explain! (Hint: It is not necessary to solve the post-merger problem in order to answer this question.)
- h) Explain what the UPP and the GUPPI try to predict. Will either of them be good indicators for the competitive effects of the proposed merger? Why/why not?

Problem 3: The Bertrand outcome in real life (20%)

The Bertrand price competition model predicts that with two firms, the equilibrium price will be equal to the marginal cost. Do you expect this to be the case in many real-world industries? Explain why/why not!

Problem 4: Downstream oligopoly (20%)

Suppose a monopolist supplier serves two horizontally differentiated retailers, who incur a marginal cost of c in addition to what they buy wholesale from the supplier. Suppose the supplier can use two-part tariffs (a per-unit price w and a fixed transfer F) and that the timing is such that the retailers first enter into an agreement with the supplier and then compete in prices in the consumer market.

- a) Suppose the supplier can make take-it or leave-it offers. Can the supplier obtain the same profit level as if he owned the two retailers? (If needed, assume that the contracts are observable and cannot be renegotiated.)
- b) Suppose now that the retailers are even closer competitors than what you assumed in problem a). How will the supplier optimally choose to adjust the terms of the contracts compared to a)? Explain!
- c) Suppose now that firm 1 has a lower marginal cost than firm 2, $c_1 < c_2$. If the supplier can price discriminate between the two retailers, which firm will get the lower per-unit wholesale price? Why? Explain!