Problem 1: Procurement auctions (30%)

Consider a procurement auction in which the procurer requests bids to produce some units of a good. The buyer is using a sealed-bid second price auction. There are five participants in the procurement auction. All of them share the same cost structure $c_i(x) = F_i + a_i x + b_i x^2$, but with different parameters (F_i, b_i, c_i). In particular, the five bidders have the following parameters

Bidder	F_i	a_i	b_i
1	0	100	7
2	0	200	7
3	0	300	7
4	40 000	1 000	3
5	50 000	2 000	3

a)

Describe a weakly dominant strategy in a sealed-bid second-price auction and explain why it is weakly dominant. Use the information provided above to derive the explicit bidding function for the participants in this case. Explain your answer.

Answer: The weakly dominant strategy is that each player bids his true valuation. In this case, the valuation is the price at which the bidder gets zero profits. He is willing to take the contract at any price greater than or equal to that. Zero profit means that total costs are equal to total revenues, or that the per-unit price is equal to the average cost $\frac{c_i(x)}{x} = \frac{F_i}{x} + a_i + b_i x$*. We require an explanation. (There are also other NEs which we haven't discussed in class.)*

b)

Assume that the procurer asks for bids for 100 units to be produced, and that the auction participants all follow their (weakly) dominant strategy. Which bidder wins the auction, what is his winning bid per unit, what does he end up getting paid per unit (the equilibrium price) and what is his total surplus?

Answer:

Bidder	F_i	a_i	b_i	AC_{100}	AC_{1000}
1	0	100	7	800	7 100
2	0	200	7	900	7 200
3	0	300	7	1 000	7 300
4	40 000	1 000	3	1 700	4 040
5	50 000	2 000	3	2 800	5 050

Bidder 1 wins, bids 800 per unit and gets paid 900. His surplus is 10 000 (100 per unit times 100 units).

c) Assume instead that the procurer asks for bids for 1 000 units to be produced, and that the auction participants all follow their dominant strategy. Which bidder wins the auction, what is his winning bid per unit, what does he end up getting paid per unit (the equilibrium price) and what is his total surplus?

Answer:

Bidder	F_i	a_i	b_i	AC_{100}	AC_{1000}
1	0	100	7	800	7 100
2	0	200	7	900	7 200
3	0	300	7	1 000	7 300
4	40 000	1 000	3	1 700	4 040
5	50 000	2 000	3	2 800	5 050

*Bidder 4 is the winner, bids 4 040 per unit and gets paid 5 050 per unit. His surplus is (5 050 – 4 040) * 1 000 = 1 010 000.*

Problem 2: Mergers and merger control (30%)

Consider two firms who compete in prices selling differentiated goods. They both share the same marginal cost c. They face the following demand system

$$q_1 = a_1 - bp_1 + dp_2$$

$$q_2 = a_2 - bp_2 + dp_1.$$

We assume that $a_i > 0, b > 0 \; and - b < d < b$.

a) Assume that the firms maximize profits by simultaneously setting prices. Derive the best response curves of the two firms.

Firm i solves

$$\max_{p_1}(p_i - c)\left(a_i - bp_i + dp_j\right)$$

$$FOC: 0 = \; a_i - bp_i + dp_j - b(p_i - c)$$

$$p_i = \frac{a_i + bc}{2b} + \frac{d}{2b}p_j$$

b) Are the prices strategic substitutes or complements? How does that depend on the parameter d?

Differentiate p_i with respect to p_j to obtain $\frac{d}{2b}$. The best response curves are upward(downward)-sloping if $d > 0$ $(d < 0)$. If $d > 0$ the goods are substitutes in demand and the prices are strategic complements, and vice versa.

c) Derive the price equilibrium.

Hint: Check that if you substitute $a_1 = a_2 = a$, you get

$$p_1^D = p_2^D \; = \frac{a}{2b - d} + \frac{bc}{2b - d}$$

Answer:

Substitution in the best responses gives you

$$p_i = \frac{a_i + bc}{2b} + \frac{d}{2b}\left(\frac{a_j + bc}{2b} + \frac{d}{2b}p_i\right)$$

Which solves to

$$p_i = \frac{2ba_i + da_j}{(2b + d)(2b - d)} + \frac{bc}{2b - d}$$

d) The diversion ratio from good 1 to good 2 is defined as $DR_{12} = \frac{\frac{\partial q_2}{\partial p_1}}{-\frac{\partial q_1}{\partial p_1}}$.

We say that a merger leads to upward pricing pressure for firm i if $UPP_i = (p_j - c_j)DR_{ij} - e_i > 0$, evaluated at the pre-merger prices, where e_i is the reduction in marginal cost for firm i as a result of the merger. Finally, define the gross upward pricing pressure index (GUPPI) as

$$GUPPI_i = \frac{p_j - c}{p_i} DR_{ij} = \left(\frac{p_j - \mathrm{c}}{p_j}\right)\frac{p_j}{p_\mathrm{i}} DR_{ij} = M_j \frac{p_j}{p_i} DR_{\mathrm{ij}}$$

Derive the expressions for the diversion ratios, the upward pricing pressures and the gross upward pricing pressure indices, $DR_{12}, DR_{21}, UPP_1, UPP_2, GUPPI_1$ and $GUPPI_2$, in our linear model. In Table 1 we provide two numerical cases to consider. Calculate and state the values for the diversion ratios, the UPPs and the GUPPIs for each of the two cases (you can use a calculator or a spreadsheet to do this).

Table 1: Parameters for Case 1 and Case 2

	a_1	a_2	b	d	c	e_1	e_2
Case 1	1	3/2	1	0	1/2	1/2	0
Case 2	4/3	1/3	4/3	2/3	0	0	0

Answer:

We get that $DR_{12} = DR_{21} = \frac{\frac{\partial q_2}{\partial p_1}}{-\frac{\partial q_1}{\partial p_1}} = \frac{d}{b}$. *The rest are best left as is, without further substitution.*

Values	a1	a2	b	d	c	e1	e2	p1	p2	q1	q2	M1	M2	DR1	DR2	UPP1	UPP2	GUPPI1	GUPPI2
Case 1	1	1,5	1	0	0,5	0,5	0	0,75	1	0,25	0,5	0,333	0,5	0	0	-0,5	0	0	0
Case 2	1,333	0,333	1,333	0,667	0	0	0	0,567	0,267	0,756	0,356	1	1	0,5	0,5	0,133	0,283	0,235	1,063

e) Explain what the diversion ratio measures. In which of the cases are the two firms the closest competitors as measured by the diversion ratios? How does that relate to the parameters b and d, and the extent to which the two goods are substitutes in demand?

The firms are closer competitors when the diversion ratio is higher, as this means that more consumers will have the other firm as their second choice, and more of the lost sales due to a higher price in one firm, ends up as recovered sales in the other firm. If so, the firms are the closest competitors in Case 2, as ½ >0. This corresponds to the case where d is closer to b, so the goods are substitutes, while for $d = 0$, the goods are totally independent in demand. This therefore matches our intuition. All candidates should be expected to see that the two firms are closer competitors in Case 2 than in Case 1, as $d = 0$ in Case 1.

f) If the two firms merge, we assume that the merged firm will control both p_1 and p_2 and is assumed to maximize the joint profit of the merged firm. That is, we assume that the merged firm solves

$$\max_{p_1,p_2} \Pi^M = (p_1 - c + e_1)q_1(p_1, p_2) + (p_2 - c + e_2)q_2(p_1, p_2)$$

Consider a competition authority that is tasked with screening mergers, in order to maximize total social welfare (the sum of producer and consumer surplus). What is the potential tradeoff that this agency faces in our model? Is the tradeoff equally present in both Case 1 and Case 2?

The so-called Williamson-tradeoff: The tradeoff between increased production efficiency (through e_i) and an increased price for the consumers, through increased market power. This tradeoff is only present in Case 1 (where firm 1 gets an increased efficiency). In Case 2 there are no efficiency gains from the merger (and the merger should be blocked).

g) In which case will the merger lead to the biggest increase in the price? In which of the two cases should a competition authority that only cares about consumer surplus allow the merger? Explain! (Hint: It is not necessary to solve the post-merger problem in order to answer this question.)

Since there are no quality improvements of any kind, the price is all we need to evaluate the consumer surplus. In Case 2, we merge two close substitutes and there are no efficiency gains, so both prices will go up. In Case 1 the goods are independent in demand – hence no diversion – and there are efficiency gains. Hence no prices go up, and at least one will go down following the merger. It is therefore in case 2 that the price increase is the biggest. As both prices go up in Case 2, a competition authority that only cares about consumer welfare should reject the proposed merger in that case. In Case 1, prices go down on average, so that should be accepted.

h) Explain what the UPP and the GUPPI try to predict. Will either of them be good indicators for the competitive effects of the proposed merger? Why/why not?

The UPP/GUPPI measure the unilateral incentive for the merged firm to raise its price following the merger. The GUPPI only takes into account the fact that after the merger, some of the consumers that will be lost when you increase the price, now will be recaptured by the other firm instead. The UPP in addition captures the cost savings due to synergies from the merger. Therefore, the UPP is a more complete measure. If the competition authority wants to base its decision on the GUPPI, it must remember that it neglects the efficiency effect, and

it has to trust that the merging parties correctly argue for cost savings when they are relevant. If not, the authority could be too restrictive in its practice.

Problem 3: The Bertrand outcome in real life (20%)

The Bertrand price competition model predicts that with two firms, the equilibrium price will be equal to the marginal cost. Do you expect this to be the case in many real-world industries? Explain why/why not!

Answer:
The simplest Bertrand model is based on a lot of unrealistic assumptions:

- *Perfect substitutes*
- *Unlimited (and exogenous) capacities*
- *No cost uncertainties*
- *Perfectly informed consumers*

In any real-world industry we would expect many of these to fail. In class we struggled to come up with proper examples of perfectly homogeneous goods (gasoline and electricity are the two canonical examples), capacities are very often deliberately chosen by the firms, there are always some uncertainties regarding the costs and consumers do have to spend resources getting informed in the market. If any one of these conditions fail to be met, our theory predicts prices above marginal cost.

The candidates should explain how the real world can be expected to differ from the simplest assumptions and why that leads to $p \neq mc$.

Problem 4: Downstream oligopoly (20%)

Suppose a monopolist supplier serves two horizontally differentiated retailers, who incur a marginal cost of c in addition to what they buy wholesale from the supplier. Suppose the supplier can use two-part tariffs (a per-unit price w and a fixed transfer F) and that the timing is such that the retailers first enter into an agreement with the supplier and then compete in prices in the consumer market.

a) Suppose the supplier can make take-it or leave-it offers. Can the supplier obtain the same profit level as if he owned the two retailers? (If needed, assume that the contracts are observable and cannot be renegotiated.)
b) Suppose now that the retailers are even closer competitors than what you assumed in problem a). How will the supplier optimally choose to adjust the terms of the contracts compared to a)? Explain!
c) Suppose now that firm 1 has a lower marginal cost than firm 2, $c_1 < c_2$. If the supplier can price discriminate between the two retailers, which firm will get the lower per-unit wholesale price? Why? Explain!

Answers:

a) Yes, he can. He should set w to internalize the competition downstream (i.e. such that the firms themselves choose to set the monopoly price), and then use the fixed term to extract all the surplus.

b) *When they become closer competitors, they will compete their margins down (i.e. price closer to marginal cost – which is w). The supplier should therefore raise w to get the price closer to the monopoly price again.*

c) *If he can price discriminate, he would like the most efficient – and therefore the most profitable – retailer to have the largest market share. He can achieve that by offering firm 1 a lower per-unit wholesale price. He of course extracts all of the surplus by increasing the fixed fee* F_1.