Dynamic oligopoly theory

- Dynamic price competition
- Collusion

Collusion – price coordination

Illegal in most countries
- Explicit collusion not feasible
- Legal exemptions

Recent EU cases
- Banking – approx. 1.7 billion Euros in fines (2013)
- Cathodic ray tubes – 1.5 billion Euros (2012)
- Gas – approx. 1.1 billion Euros in fines (2009)
- Car glass – approx. 1.4 billion Euros (2008)

Website: ec.europa.eu/competition/cartels/cases/cases.html

Puerto Rico, US, 2013: 5-year sentence for price-fixing

Tacit collusion

Hard to detect – not many cases.

Repeated interaction

Theory of repeated games

Deviation from an agreement to set high prices has
- a short-term gain: increased profit today
- a long-term loss: deviation by the others later on

Tacit collusion occurs when
long-term loss > short-term gain

Model

Two firms, homogeneous good, $C(q) = cq$

Prices in period t: (p_{1t}, p_{2t})

Profits in period t: $\pi^1(p_{1t}, p_{2t})$, $\pi^2(p_{1t}, p_{2t})$

History at time t: $H_t = (p_{10}, p_{20}, \ldots, p_{1,\,t-1}, p_{2,\,t-1})$

A firm's strategy is a rule that assigns a price to every possible history.

A <u>subgame-perfect equilibrium</u> is a pair of strategies that are in equilibrium after every possible history: Given one firm's strategy, for each possible history, the other firm's strategy maximizes the net present value of profits from then on.

T – number of periods

T finite: a unique equilibrium

period T: $p_{1T} = p_{2T} = c$, irrespective of H_T.

period T – 1: the same

and so on

T infinite (or indefinite)

At period τ, firm i maximizes

$$\sum_{t=\tau}^{\infty} \delta^{t-\tau} \pi^i (p_{1t}, p_{2t}), \qquad \delta = \frac{1}{1+r}$$

The best response to (c, …) is (c, …).

But do we have other equilibria?

Can $p > c$ be sustained in equilibrium?

Trigger strategies: If a firm deviates in period t, then both firms set $p = c$ from period $t + 1$ until infinity.

[Optimal punishment schemes? Renegotiation-proofness?]

Monopoly price: $p^m = \arg\max\,(p - c)D(p)$
Monopoly profit: $\pi^m = (p^m - c)D(p^m)$

A trigger strategy for firm 1:

- Set $p_{10} = p^m$ in period 0
- In the periods thereafter,
  - $p_{1t}(H_t) = p^m$, if $H_t = (p^m, p^m, \ldots, p^m, p^m)$
  - $p_{1t}(H_t) = c$, otherwise

If a firm *collaborates*, it sets $p = p^m$ and earns $\pi^m/2$ in every period.

The *optimum deviation*: $p^m - \varepsilon$, yielding $\approx \pi^m$ for one period.

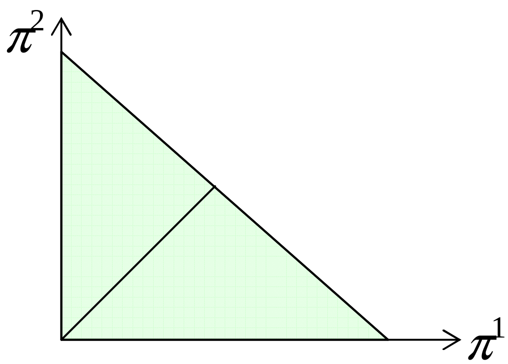
An equilibrium in trigger strategies exists if:

$$\frac{\pi^m}{2}(1 + \delta + \delta^2 + \ldots) \geq \pi^m + 0 + 0 + \ldots$$

$$\Leftrightarrow \frac{1}{2}\frac{1}{1-\delta} \geq 1 \Leftrightarrow \delta \geq \frac{1}{2}$$

The same argument applies to collusion on any price $p \in (c, p^m]$. $\Rightarrow$ Infinitely many equilibria.

The Folk Theorem.



<u>Collusion when demand varies</u>

Demand stochastic.

Periodic demand is

low: $D_1(p)$ with probability ½

high: $D_2(p)$ with probability ½

$D_1(p) < D_2(p)$, $\forall\, p$.

The demand shocks are *i.i.d.*

Each firm sets its price after having observed demand.

What are the best collusive strategies for the two firms?
Trigger strategies: A deviation is followed by $p = c$ forever.

What are the best collusive prices? One price in low-demand periods and one in high-demand periods: p_1 and p_2.

$\pi_s(p)$ – total industry profit in state s when both firms set p.

With prices p_1 and p_2 in the two states, each firm's expected net present value is:

$$V = \sum_{t=0}^{\infty} \delta^t \left[\frac{1}{2} \frac{D_1(p_1)}{2}(p_1 - c) + \frac{1}{2} \frac{D_2(p_2)}{2}(p_2 - c) \right]$$

$$= \frac{1}{4(1-\delta)} [D_1(p_1)(p_1 - c) + D_2(p_2)(p_2 - c)]$$

$$= \frac{\pi_1(p_1) + \pi_2(p_2)}{4(1-\delta)}$$

The best possible collusive price in state s is:

$$p_s^m = \arg\max\, (p - c)D_s(p),\ s = 1, 2.$$

$$\pi_s^m = (p_s^m - c)D_s(p_s^m),\ s = 1, 2.$$

If the firms can collude on these prices, then:

$$V = \frac{\pi_1^m + \pi_2^m}{4(1-\delta)}$$

A deviation in state s receives a gain equal to: π_s^m

For (p_1^m, p_2^m) to be equilibrium prices, we must have:

$$\pi_s^m \leq \tfrac{1}{2}\pi_s^m + \delta V \iff \pi_s^m \leq 2\delta V$$

The difficulty is state 2 (high-demand), since $\pi_1^m < \pi_2^m$.

The equilibrium condition becomes:

$$\pi_2^m \leq 2\delta \frac{\pi_1^m + \pi_2^m}{4(1-\delta)}$$

$$\iff \delta \geq \frac{2}{3 + \dfrac{\pi_1^m}{\pi_2^m}} \equiv \delta_0$$

$$0 < \frac{\pi_1^m}{\pi_2^m} < 1 \Rightarrow \tfrac{1}{2} < \delta_0 < \tfrac{2}{3}$$

But what if $\delta \in [\tfrac{1}{2}, \delta_0)$? Can we still find prices at which the firms can collude?

The problem is again state 2. We need to set p_2 so that

$$\pi_2(p_2) \le 2\delta \frac{\pi_1^m + \pi_2(p_2)}{4(1-\delta)}$$

$$\Rightarrow \pi_2(p_2) = \frac{\delta}{2-3\delta}\pi_1^m$$

$$\tfrac{1}{2} \le \delta < \tfrac{2}{3} \Rightarrow \tfrac{\delta}{2-3\delta} \ge 1 \quad \Rightarrow \pi_2 \ge \pi_1$$

So: prices below monopoly price in high-demand state – during boom. Could even be that $p_2 < p_1$.

But is this a price war?

More realistic demand conditions:
Autocorrelation – business cycle.
Collusion most difficult to sustain just as the downturn starts.

Haltiwanger & Harrington, *RAND J Econ* 1991
Kandori, *Rev Econ Stud* 1991
Bagwell & Staiger, *RAND J Econ* 1997

[Exercise 6.4]

Empirical studies of collusion

- the railroad cartel
  - Porter *Bell J Econ* 1983
  - Ellison *RAND J Econ* 1994

- collusion among petrol stations
  - Slade *Rev Econ Stud* 1992

- collusion in the soft-drink market: prices and advertising
  - Gasmi, *et al.*, *J Econ & Manag Strat* 1992

- collusion in procurement auctions
  - Porter & Zona *J Pol Econ* 1993 (road construction)
  - Pesendorfer *Rev Econ Stud* 2000 (school milk)

Infrequent interaction

Suppose the period length doubles.

$$\delta \to \delta^2$$

Collusion feasible if:

$$\delta^2 \geq \frac{1}{2} \iff \delta \geq \frac{1}{\sqrt{2}} \approx 0.71$$

Multimarket contact

Market A: Frequent interaction, period length 1.
Collusion if $\delta \geq ½$.

Market B: Infrequent interaction, period length 2.
Collusion if $\delta^2 \geq ½$.

(How could frequency vary across markets?)

What if both firms operate in both markets?
Can the firms obtain collusion in both markets even in cases where $\delta^2 < ½ < \delta$?

A deviation is most profitable when both markets are open.

Deviation yields: $2\pi^m$
Collusion yields:
$[\pi^m/2]$ every period, plus
$[\pi^m/2]$ every second period (starting today)

Collusion can be sustained if:

$$\frac{\pi^m}{2}[1 + \delta + \delta^2 + \ldots] + \frac{\pi^m}{2}[1 + \delta^2 + \delta^4 + \ldots] \geq 2\pi^m$$

$$\Leftrightarrow \ \frac{1}{2}\frac{1}{1-\delta} + \frac{1}{2}\frac{1}{1-\delta^2} \geq 2$$

$$\Leftrightarrow \ 4\delta^2 + \delta - 2 \geq 0 \ \Leftrightarrow \ \delta \geq \frac{\sqrt{33}-1}{8} \approx 0.59$$

Secret price cuts, or:

Price coordination when supervising the partners is difficult

Own demand observable
Market demand *not* observable
Other firms' prices *not* observable

When own demand is low, is it because market demand is low, or because partners default?

Punishment ($p = c$) is necessary.
But punishment forever?

Can firms coordinate prices without being able to observe each other's prices?

Punishment starts when one observes low demand.
Punishment phase lasts for a finite number of periods.
Even colluding firms have periods of ''price wars".

Model: Two firms; homogeneous products; $MC = c$.

In each period: firms set prices; consumers choose the firm with the lower price.

Market demand is either:
$D = 0$, with probability α;
$D = D(p)$, with probability $(1 - \alpha)$.

Both firms know it if at least one firm has zero profit in a period. Either:

- market demand is zero and both firms have zero profit, or
- one firm has cut its price and knows that the other firm has zero profit

Strategy:

- Start with $p = p^m$.
- Set $p = p^m$ until (at least) one firm has zero profit.
- If this happens, then set $p = c$ for T periods.
- After T periods, return to $p = p^m$ until (at least) one firm has zero profit.

  And so on.

Is there an equilibrium in which each firm plays this strategy?

T must be determined.

Two phases:

- Colluding phase
- Punishment phase

V^+ = net present value of a firm in the colluding phase

V^- = net present value of a firm *at the start of* the punishment phase

$$V^+ = (1-\alpha)\left(\frac{\pi^m}{2} + \delta V^+\right) + \alpha\delta V^-$$

$$V^- = \delta^T V^+$$

Equilibrium condition:

$$V^+ \geq (1-\alpha)(\pi^m + \delta V^-) + \alpha\delta V^- = (1-\alpha)\pi^m + \delta V^-$$

$$\Leftrightarrow (1-\alpha)\left(\frac{\pi^m}{2} + \delta V^+\right) + \alpha\delta V^- \geq (1-\alpha)\pi^m + \delta V^-$$

$$\Leftrightarrow \delta\left(V^+ - V^-\right) \geq \frac{\pi^m}{2}$$

$$\Leftrightarrow V^+\delta\left(1-\delta^T\right) \geq \frac{\pi^m}{2}$$

$$V^{+} = (1-\alpha)\left(\frac{\pi^{m}}{2} + \delta V^{+}\right) + \alpha\delta^{T+1}V^{+}$$

$$V^{+} = \frac{(1-\alpha)\frac{\pi^{m}}{2}}{1-(1-\alpha)\delta - \alpha\delta^{T+1}}$$

$$\frac{(1-\alpha)\frac{\pi^{m}}{2}}{1-(1-\alpha)\delta - \alpha\delta^{T+1}}\delta\left(1-\delta^{T}\right) \geq \frac{\pi^{m}}{2}$$

$$2\delta(1-\alpha) + (2\alpha - 1)\delta^{T+1} \geq 1$$

The best equilibrium has the highest possible V^{+}.

The firms' problem:

$\max_{T}$ V^{+}, such that: $2\delta(1-\alpha) + (2\alpha - 1)\delta^{T+1} \geq 1$

But: $dV^{+}/dT < 0$. So we restate the problem.

min T, such that: $2\delta(1-\alpha) + (2\alpha - 1)\delta^{T+1} \geq 1$

T = 0 is too low – there has to be some punishment, even under collusion:

$$2\delta(1-\alpha) + (2\alpha - 1)\delta = \delta < 1$$

And the lefthand side must be increasing in T:

$$\frac{d}{dT}\left[2\delta(1-\alpha)+(2\alpha-1)\delta^{T+1}\right]$$

$$=(2\alpha-1)\delta^{T+1}\underbrace{\ln\delta}_{<0}>0 \Leftrightarrow \alpha<\frac{1}{2}$$

If $\alpha \geq$ ½, then collusion is impossible: The probability of zero market demand is too large.

If $\alpha <$ ½, then $2\alpha - 1 < 0$. But $(2\alpha - 1)\delta^{T+1} \to 0$ as $T \to \infty$.

Equilibrium condition satisfied for some T if also

$$2\delta(1-\alpha) \geq 1$$

All in all: Collusion can occur in equilibrium if:

- $\alpha <$ ½
- $\delta \geq \frac{1}{2}\frac{1}{1-\alpha}$

T is chosen as the lowest integer that satisfies:

$$2\delta(1-\alpha) + (2\alpha - 1)\delta^{T+1} \geq 1$$

Example: δ = ¾, α = ¼. Condition: $(¾)^{T+1} \leq$ ¼ $\Rightarrow T^* = 4$.
But often T^* is smaller: $\delta = 0.9$, $\alpha = 0.2 \Rightarrow T^* = 2$.

Price rigidities

- Menu costs
- Price reactions not punishments, but attempts to regain market share

Suppose

- a price is fixed for two periods
- firms alternate at setting price

Duopoly with alternating price setting

- A discrete price grid

- *Markov strategies*: strategies based only on directly payoff-relevant information

Example: A trigger strategy is *not* Markov; no price from the past has a *direct* effect on a firm's profit today, only an *indirect* effect, because other firms use trigger strategies.

A restriction to Markov strategies would be too strong when moves are simultaneous. Here, moves are alternating.

Model: duopoly; each firm's price fixed for two periods; firm 1 sets price in odd-numbered periods (1 – 3 – 5 – …), firm 2 in even-numbered periods (2 – 4 – 6 – …).

Markov reaction functions:

Let p_{it} be the price set by firm i in period t.

Firm 1's reaction function:

$$p_{1,\,2k+1} = R_1(p_{2,\,2k}), \quad k = 0, 1, 2, \ldots$$

Firm 2's reaction function:

$$p_{2,\,2k+2} = R_2(p_{1,\,2k+1}), \quad k = 0, 1, 2, \ldots$$

Markov perfect equilibrium: An equilibrium in Markov reaction functions. At the start of each subgame, the firm that makes the move chooses an optimum strategy, given the restriction only to pay attention to payoff-relevant information, and given the other firm's equilibrium strategy.

The two firms at any point in time:
‘‘the active" and ‘‘the other"

Consider the active firm's decision today.

Suppose the other firm set the price p_h last period; this is also its price today. – We are in state h.

V_h – the active firm's net present value in state h.
W_h – the other firm's net present value in state h.

Tomorrow, the roles are changed.

Profit per period: π(own price, the other's price)

$$\Rightarrow \; V_h = \max_k \left[\pi\left(p_k, p_h\right) + \delta W_k \right]$$

A symmetric equilibrium: $R_1(\cdot) = R_2(\cdot) = R(\cdot)$

Mixed strategy: A firm may be indifferent between one or more prices, and in equilibrium, the other firm has beliefs about which of these prices will be chosen. These beliefs will then constitute the firm's mixed strategy.

α_{hk} – the probability (according to the other firm's beliefs) that a firm in state h chooses price p_k.

Note: $\sum_k \alpha_{hk} = 1$

A symmetric equilibrium can be described by a *transition matrix*: Suppose there are H possible prices.

$$\text{from state } h \left\{ \underbrace{\begin{bmatrix} \alpha_{11} & \cdots & \cdots & \alpha_{1H} \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \alpha_{H1} & \cdots & \cdots & \alpha_{HH} \end{bmatrix}}_{\text{to state } k} \right. = A$$

<u>Equilibrium conditions</u>

$$V_h = \sum_k \alpha_{hk} [\pi(p_k, p_h) + \delta W_k]$$
$$W_k = \sum_l \alpha_{kl} [\pi(p_k, p_l) + \delta V_l]$$

These are the values of V_h and W_k that follow from the transition matrix A.

$[V_h - \pi(p_k, p_h) - \delta W_k]\alpha_{hk} = 0, \ \forall\ h, k.$

$V_h \geq \pi(p_k, p_h) + \delta W_k, \ \forall\ h, k.$

Complementary slackness: If $\alpha_{hk} > 0$, it must be because $V_h = \pi(p_k, p_l) + \delta W_k$, that is, because p_k maximizes the firm's net present value in state h.

$$\sum_k \alpha_{hk} = 1, \ \forall\ h$$

$\alpha_{hk} \geq 0, \forall\ h, k.$

Example:

$D(p) = 1 - p$; $c = 0$

The price grid: $p_h = \frac{h}{6}$, $h = 0, \ldots, 6$.
Competitive price: $p_0 = 0$. Monopoly price: $p_m = p_3 = ½$.

Two (symmetric Markov perfect) equilibria (at least):

1. ‘‘Kinked demand curve”: The other firm does *not* follow you if you increase the price but undercuts you if you decrease the price.

$R(1) = R(\frac{5}{6}) = R(\frac{2}{3}) = R(\frac{1}{2}) = R(0) = \frac{1}{2}$;
$R(\frac{1}{3}) = \frac{1}{6}$; $R(\frac{1}{6}) \in \{\frac{1}{6}, \frac{1}{2}\}$.

- Either the game starts in state 3 and stays there, or it ends there sooner or later (absorbing state).

- A mixed strategy in state 1 – a waiting game (‘‘war of attrition”): Each firm is indifferent between meeting p_1 with p_1, and making a short-term sacrifice in order to get the monopoly price from next period on.

- The equilibrium is sustainable only if each firm is able to supply the whole market demand at $p_1 = \frac{1}{6}$: $D(\frac{1}{6}) = \frac{5}{6}$. In the absorbing state 3, each firm sells $\frac{1}{2}D(p_3) = \frac{1}{4}$ but needs to keep an excess capacity of $\frac{5}{6} - \frac{1}{4} = \frac{7}{12}$.

2. Price war: The firms undercut each other.

$R(1) = R(\frac{5}{6}) = \frac{2}{3}$; $R(\frac{2}{3}) = \frac{1}{2}$; $R(\frac{1}{2}) = \frac{1}{3}$;
$R(\frac{1}{3}) = \frac{1}{6}$; $R(\frac{1}{6}) = 0$; $R(0) \in \{0, \frac{5}{6}\}$.

- Unstable prices: no absorbing state.
- Edgeworth cycle.
- Again a waiting game. But now the price jumps beyond the monopoly price.

- Multiple equilibria, even when we restrict attention to Markov strategies.
- Fewer equilibria than in an ordinary repeated game.
- $p = c$ is no longer an equilibrium; there is always *some* price collusion in equilibrium.

Other cases of dynamic price competition

- Brand loyalty / consumer switching costs
- Durable goods