Product differentiation

How far does a market extend?
Which firms compete with each other?
What is an industry?

Products are *not* homogeneous.
Exceptions: petrol, electricity.

But some products are more equal to each other than to other products in the economy. These products constitute an industry.

A market with *product differentiation*.

But: where do we draw the line?

Example:

- beer vs. soda?
- soda vs. milk?

- beer vs. milk?

Two kinds of product differentiation

(i) Horizontal differentiation: Consumers differ in their preferences over the product's characteristics. Examples: colour, taste, location of outlet.

(ii) Vertical differentiation: Products differ in some characteristic in which all consumers agree what is best. Call this characteristic quality. (*quality competition*)

Horizontal differentiation

Two questions:

1. Is the product variation too large in equilibrium?

2. Are there too many variants in equilibrium?

Question 1: A fixed number of firms. Which product variants will they choose?

Question 2: Variation is maximal. How many firms will enter the market?

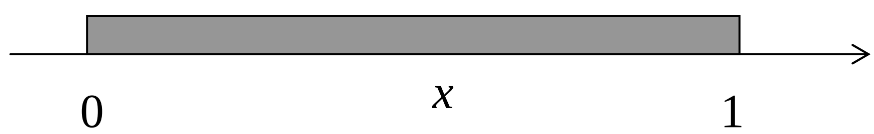
The two questions call for different models.

Variation in equilibrium

Will products supplied in an unregulated market be too similar or too different, relative to social optimum?

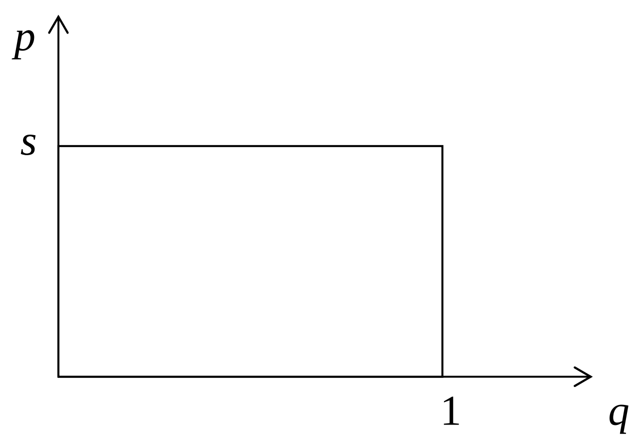
Hotelling (1929)

Product space: the line segment [0, 1].
Two firms: one at 0, one at 1.



Consumers are uniformly distributed along [0, 1].
A consumer at *x* prefers product variant *x*.

Consumers have unit demand:



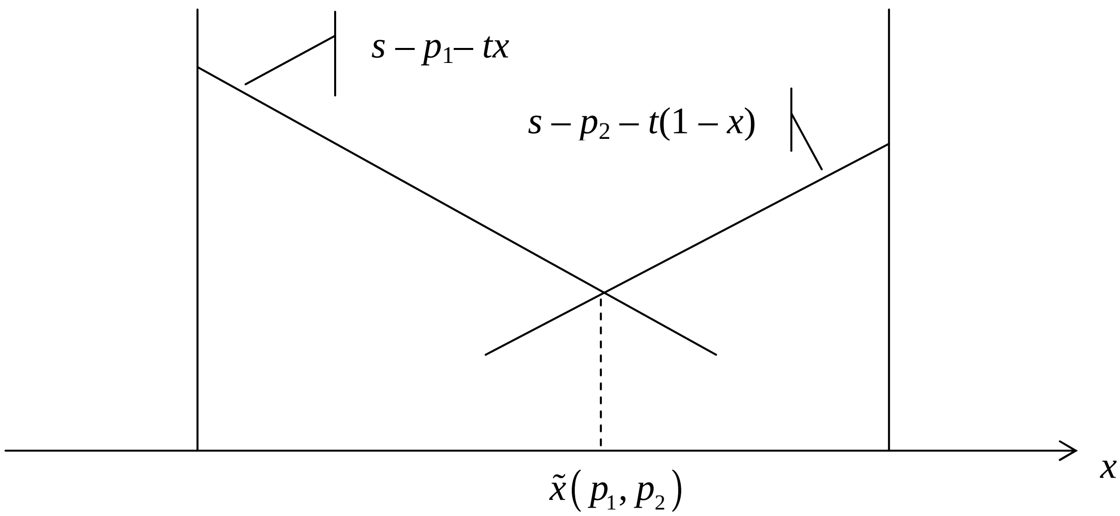
Disutility from consuming product variant y:

$t(|y - x|)$ – ‘‘transportation costs”

Linear transportation costs: $t(d) = td$

Generalised prices (with firm 1 at 0 and firm 2 at 1):

$p_1 + tx$ and $p_2 + t(1 - x)$



The indifferent consumer: $\tilde{x}$

$$s - p_1 - t\tilde{x} = s - p_2 - t(1 - \tilde{x}).$$

$$\Rightarrow \tilde{x}(p_1, p_2) = \frac{1}{2} + \frac{p_2 - p_1}{2t}$$

[But check that: (i) $0 \le \tilde{x} \le 1$; (ii) $\tilde{x}$ wants to buy.]

Normalizing the number of consumers: $N = 1$ (thousand)

$$D_1(p_1, p_2) = \tilde{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t}$$

$$D_2(p_1, p_2) = 1 - \tilde{x} = \frac{1}{2} + \frac{p_1 - p_2}{2t}$$

Constant unit cost of production: c

$$\pi_1(p_1, p_2) = (p_1 - c)\left[\frac{1}{2} + \frac{p_2 - p_1}{2t}\right]$$

Price competition.

Equilibrium conditions: $\frac{\partial \pi_1}{\partial p_1} = 0; \ \frac{\partial \pi_2}{\partial p_2} = 0$

FOC[1]:

$$\underbrace{(p_1 - c)\left(-\frac{1}{2t}\right)}_{\text{increased price reduces sales}} + \underbrace{\frac{1}{2} + \frac{p_2 - p_1}{2t}}_{\text{increased price increases gain per unit sold}} = 0$$

$\Rightarrow$ FOC[1]: $2p_1 - p_2 = c + t$

FOC[2]: $2p_2 - p_1 = c + t$

$\Rightarrow$ $p_1^* = p_2^* = c + t$

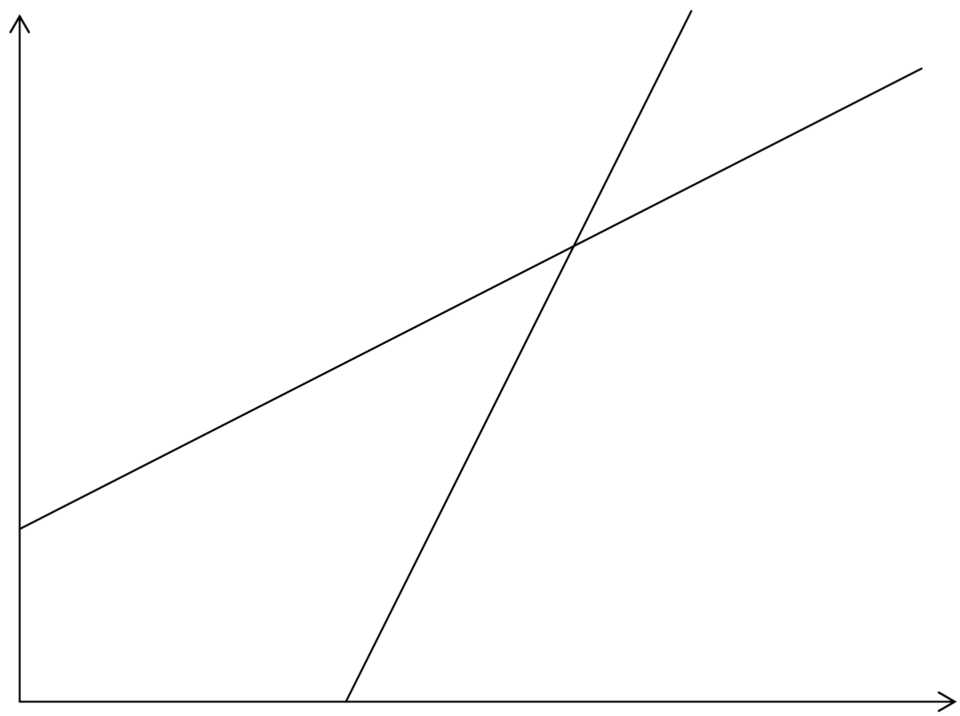
- The indifferent consumer does want to buy if:

$$s \geq c + \tfrac{3}{2}t$$

- Prices are *strategic complements*:

$$\frac{\partial^2 \pi_1}{\partial p_1 \partial p_2} = \frac{1}{2t} > 0$$

Best-response function: $p_1 = \frac{1}{2}(p_2 + c + t)$



The degree of product differentiation: t

Product differentiation makes firms less aggressive in their pricing.

But are 0 and 1 the firms' equilibrium product variants?

Two-stage game of product differentiation:

Stage 1: Firms choose locations on [0, 1].

Stage 2: Firms choose prices.

Linear vs. convex transportation costs.

- Convex transportation costs analytically tractable – but economically less meaningful?

Assume quadratic transportation costs.

Stage 2:
Firms 1 and 2 located at a and $1 - b$, $a \geq 0$, $b \geq 0$, $a + b \leq 1$.

The indifferent consumer:

$$p_1 + t(\tilde{x} - a)^2 = p_2 + t(1 - b - \tilde{x})^2$$

$$\tilde{x} = a + \frac{1}{2}(1 - a - b) + \frac{p_2 - p_1}{2t(1 - a - b)}$$

$$D_1(p_1, p_2) = \tilde{x}, \quad D_2(p_1, p_2) = 1 - \tilde{x}$$

$$\pi_1(p_1, p_2) = (p_1 - c)\left[a + \frac{1}{2}(1 - a - b) + \frac{p_2 - p_1}{2t(1 - a - b)}\right]$$

Equilibrium conditions: $\frac{\partial \pi_1}{\partial p_1} = 0$; $\frac{\partial \pi_2}{\partial p_2} = 0$

FOC[1]: $2p_1 - p_2 = c + t(1 - a - b)(1 + a - b)$

FOC[2]: $2p_2 - p_1 = c + t(1 - a - b)(1 - a + b)$

Equilibrium:

$$p_1 = c + t(1 - a - b)\left(1 + \frac{a - b}{3}\right)$$
$$p_2 = c + t(1 - a - b)\left(1 + \frac{b - a}{3}\right)$$

- Symmetric location: $a = b \Rightarrow p_1 = p_2 = c + t(1 - 2a)$
- A firm's price decreases when the other firm gets closer: $\frac{dp_1}{db} < 0$.
- Stage-2 outcome depends on locations:
  $p_1 = p_1(a, b)$, $p_2 = p_2(a, b)$

Stage 1:

$\pi_1(a, b) = [p_1(a, b) - c]D_1(a, b, p_1(a, b), p_2(a, b))$

$$\frac{d\pi_1}{da} = D_1 \frac{\partial p_1}{\partial a} + (p_1 - c)\left[\frac{\partial D_1}{\partial a} + \frac{\partial D_1}{\partial p_1}\frac{\partial p_1}{\partial a} + \frac{\partial D_1}{\partial p_2}\frac{\partial p_2}{\partial a}\right]$$

$$= \underbrace{\left[D_1 + (p_1 - c)\frac{\partial D_1}{\partial p_1}\right]}_{=0}\frac{\partial p_1}{\partial a} + (p_1 - c)\left[\frac{\partial D_1}{\partial a} + \frac{\partial D_1}{\partial p_2}\frac{\partial p_2}{\partial a}\right]$$

$$\frac{d\pi_1}{da} = (p_1 - c)(\underbrace{\frac{\partial D_1}{\partial a}}_{\substack{\text{direct}\\ \text{effect;}\\ >0}} + \underbrace{\overbrace{\frac{\partial D_1}{\partial p_2}}^{>0}\overbrace{\frac{\partial p_2}{\partial a}}^{<0}}_{\substack{\text{strategic}\\ \text{effect;}\\ <0}})$$

Moving toward the middle:
A positive direct effect vs. a negative strategic effect.

$$\frac{\partial D_1}{\partial a} = \frac{1}{2} + \frac{p_2 - p_1}{2t(1-a-b)^2} = \frac{1}{2} + \frac{b-a}{3(1-a-b)}$$

$$= \frac{3-5a-b}{6(1-a-b)} > 0, \text{if } a \le \frac{1}{2}$$

$$\frac{\partial p_2}{\partial a} = \frac{2}{3}t(a-2) < 0$$

$$\frac{\partial D_1}{\partial p_2} = \frac{1}{2t(1-a-b)} > 0$$

$$\frac{\partial D_1}{\partial a} + \frac{\partial D_1}{\partial p_2}\frac{\partial p_2}{\partial a} = \frac{3-5a-b}{6(1-a-b)} + \frac{a-2}{3(1-a-b)} = -\frac{3a+b+1}{6(1-a-b)} < 0$$

Equilibrium: $a^* = b^* = 0$.

Strategic effect stronger than direct effect.
Maximum differentiation in equilibrium.

Social optimum:

No quantity effect. Social planner wants to minimize total transportation costs. (Kaldor-Hicks vs. Pareto)

In social optimum, the two firms split the market and locate in the middle of each segment: ¼ and ¾.

In equilibrium, product variants are too different.

- Crucial assumption: convex transportation costs.

- Also other equilibria, but they are in mixed strategies.
  [Bester *et al*., ‘‘A Noncooperative Analysis of Hotelling’s Location Game”, *Games and Economic Behavior* 1996]

- Multiple dimensions of variants: Hotelling was almost right
  [Irmen and Thisse, ”Competition in multi-characteristics spaces: Hotelling was almost right”, *Journal of Economic Theory* 1998]

- Head-to-head competition in shopping malls: Consumers’ shopping costs.
  [Klemperer, “Equilibrium Product Lines”, *AER* 1992]

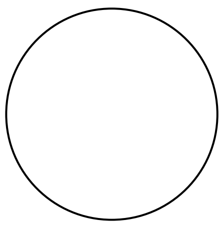
Have we really solved the problem whether or not the equilibrium provision of product variants has too much or too little differentiation?

Too many variants in equilibrium?

A model without location choice.

Focus on firms' entry into the market.

The circular city



Circumference: 1
Consumers uniformly distributed around the circle.
Number of consumers: 1
Linear transportation costs: $t(d) = td$
Unit demand, gross utility = s

Entry cost: f

Unit cost of production: c

Profit of firm i: $$\pi_i = \begin{cases} (p_i - c)D_i - f, & \text{if it enters,} \\ 0, & \text{otherwise} \end{cases}$$

Two-stage game.

Stage 1: Firms decide whether or not to enter. Assume entering firms spread evenly around the circle.

Stage 2: Firms set prices.

If *n* firms enter at stage 1, then they locate a distance 1/*n* apart.

Stage 2: Focus on symmetric equilibrium.

If all other firms set price *p*, what then should firm *i* do?

Each firm competes directly only with two other firms: its neighbours on the circle.

At a distance $\tilde{x}$ in each direction is an indifferent consumer:

$$p_i + t\tilde{x} = p + t\left(\frac{1}{n} - \tilde{x}\right)$$

$$\tilde{x} = \frac{1}{2t}\left(p + \frac{t}{n} - p_i\right)$$

Demand facing firm *i*:

$$D_i(p_i, p) = 2\tilde{x} = \frac{1}{n} + \frac{p - p_i}{t}$$

Firm *i*'s problem:

$$\max_{p_i} \pi_i = (p_i - c)\left(\frac{1}{n} + \frac{p - p_i}{t}\right) - f$$

$$\frac{\partial \pi_i}{\partial p_i} = \left(\frac{1}{n} + \frac{p - p_i}{t}\right) - (p_i - c)\frac{1}{t} = 0$$

$$2p_i - p = c + \frac{t}{n}$$

In a symmetric equilibrium, all prices are equal. $\Rightarrow p_i = p$.

$$p = c + \frac{t}{n}$$

Stage 1:
How many firms will enter?

$$D_i = \frac{1}{n}$$

$$\pi_i = (p - c)\frac{1}{n} - f = \frac{t}{n^2} - f$$

$$\pi = 0 \Rightarrow n = \sqrt{\frac{t}{f}}$$

$$\Rightarrow p = c + \frac{t}{\sqrt{t/f}} = c + \sqrt{tf}$$

Condition: Indifferent consumer wants to buy:

$$s \geq p + \frac{t}{2n} = c + \frac{3}{2}\sqrt{tf} \iff f \leq \frac{4}{9t}(s-c)^2$$

Exercise 7.3: What if transportation costs are quadratic?

[Exercise 7.4: What if fixed costs are large?]

<u>Social optimum</u>: Balancing transportation and entry costs.

Average transportation cost: $t\,(\frac{1}{2}\tilde{x}) = \frac{t}{2}\frac{1}{2n} = \frac{t}{4n}$

The social planner's problem:

$$\min_n \left(nf + \frac{t}{4n} \right)$$

FOC: $f - \frac{t}{4n^2} = 0 \quad \Rightarrow n^* = \frac{1}{2}\sqrt{\frac{t}{f}} < n^e$

Too many firms in equilibrium.

Private motivation for entry: business stealing
Social motivation for entry: saving transportation costs

[Exercise: What happens with n^e/n^* as N (number of consumers) grows?]

Advertising

- informative
- persuasive

Persuasive: shifting consumers' preferences?

Focus on informative advertising.

Hotelling model, two firms fixed at 0 and 1, consumers uniformly distributed across [0,1], linear transportation costs td, gross utility s.

A consumer is able to buy from a firm if and only if he has received advertising from it.

φ_i – fraction of consumers receiving advertising from firm i

Advertising costs: $A_i = A_i(\varphi_i) = \frac{a}{2}\varphi_i^2$

Potential market for firm 1: φ_1.
Out of these consumers, a fraction $(1 - \varphi_2)$ have not received any advertising from firm 2.
The rest, a fraction φ_2 out of φ_1, know about both firms.

Firm 1's demand:

$$D_1 = \varphi_1\left[(1-\varphi_2)+\varphi_2\left(\frac{1}{2}+\frac{p_2-p_1}{2t}\right)\right]$$

A simultaneous-move game.

Each firm chooses advertising and price.

Firm 1's problem:

$$\max_{p_1,\varphi_1} \pi_1 = (p_1 - c)\varphi_1\left[(1-\varphi_2) + \varphi_2\left(\frac{1}{2} + \frac{p_2 - p_1}{2t}\right)\right] - \frac{a}{2}\varphi_1^2$$

Two FOCs for each firm.

FOC[p_1]: $\varphi_1\left[(1-\varphi_2) + \varphi_2\left(\frac{1}{2} + \frac{p_2 - p_1}{2t}\right)\right] - (p_1 - c)\frac{\varphi_1\varphi_2}{2t} = 0$

FOC[φ_1]: $(p_1 - c)\left[(1-\varphi_2) + \varphi_2\left(\frac{1}{2} + \frac{p_2 - p_1}{2t}\right)\right] - a\varphi_1 = 0$

$$\Rightarrow \quad p_1 = \frac{1}{2}(p_2 + c - t) + \frac{t}{\varphi_2}$$

$$\varphi_1 = \frac{1}{a}(p_1 - c)\left[(1-\varphi_2) + \varphi_2\left(\frac{1}{2} + \frac{p_2 - p_1}{2t}\right)\right]$$

Firms are identical ⇒ Symmetric equilibrium

$$p = \frac{1}{2}(p + c - t) + \frac{t}{\varphi}$$

$$\Rightarrow\ p = c + t\left(\frac{2}{\varphi} - 1\right)$$

$$\varphi = \frac{1}{a}(p - c)\left[(1 - \varphi) + \varphi\frac{1}{2}\right]$$

$$\varphi = \frac{1}{a}t\left(\frac{2}{\varphi} - 1\right)\left(1 - \frac{\varphi}{2}\right)$$

$$\Rightarrow\ \varphi = \frac{2}{1 + \sqrt{\frac{2a}{t}}}$$

Condition: $\frac{a}{t} \geq \frac{1}{2}$

$$\Rightarrow p = c + \sqrt{2at}$$

Condition: $s \geq c + t + \sqrt{2at}$ $(\geq c + 2t)$

- $\frac{\partial \varphi}{\partial a} < 0, \quad \frac{\partial p}{\partial a} > 0$

Firms' profit:

$$\pi = \frac{2a}{\left(1+\sqrt{\frac{2a}{t}}\right)^2}$$

- $\frac{\partial \pi}{\partial t} > 0; \quad \frac{\partial \pi}{\partial a} > 0!$

An increase in advertising costs increases firms' profits.

Two effects of an increase in *a* on profits:

A direct, negative effect.
An indirect, positive effect: $a\uparrow \;\rightarrow\; \varphi\downarrow \;\rightarrow\; p\uparrow$

Firms profit collectively from more expensive advertising.

Crucial assumption: convex advertising costs.

What about the market for advertising?
[Kind, Nilssen & Sørgard, *Marketing Science* 2009]

Social optimum

Average transportation costs

among fully informed consumers: $t/4$.

among partially informed consumers: $t/2$.

The social planner's problem:

$$\max_{\varphi} \varphi^2\left(s-c-\frac{t}{4}\right)+2\varphi(1-\varphi)\left(s-c-\frac{t}{2}\right)-2\frac{a}{2}\varphi^2$$

$$\varphi^* = \frac{2(s-c)-t}{2(s-c)+2a-\frac{3}{2}t}$$

[Condition: $t \leq 2(s-c)$]

Special cases:

(i) $\frac{a}{t} \to \frac{1}{2}$: $\varphi^e \to 1$

$$\varphi^* \to 1-\frac{t}{4(s-c)-t} < 1$$

Too much advertising in equilibrium

(ii) $\frac{a}{t} \to \infty$: $\varphi^e \to 0$

$$\varphi^* \to \frac{1}{1+\frac{a}{s-c}} > 0$$

Too little advertising in equilibrium

Vertical product differentiation

Quality competition

Consumers agree on what is the best product variant.
But they differ in their willingness to pay for quality.

s – quality
θ – measure of a consumer's taste for quality.

If a consumer of type θ buys a product of quality s at price p, her net utility is:

$$U = \theta s - p$$

$F(\theta)$ – cumulative distribution function of consumer type

$F(\theta')$ – fraction of consumers with type $\theta \leq \theta'$.

Unit demand: If $\theta s - p \geq 0$, then a consumer of type θ buys one unit of the good.

One firm:

At price p, its demand is $D(p) = 1 - F\left(\frac{p}{s}\right)$.

Two firms:

Suppose $s_1 < s_2$, $p_1 < p_2$. The indifferent consumer:

$$\tilde{\theta} s_1 - p_1 = \tilde{\theta} s_2 - p_2$$

$$\tilde{\theta} = \frac{p_2 - p_1}{s_2 - s_1}$$

Product 2 *quality dominates* product 1 if:

$$\tilde{\theta} < \frac{p_1}{s_1} \Leftrightarrow \frac{p_2}{s_2} < \frac{p_1}{s_1}$$

Otherwise $\left(\frac{p_2}{s_2} \geq \frac{p_1}{s_1}\right)$, demand is:

$$D_1(p_1, p_2) = F\left(\frac{p_2 - p_1}{s_2 - s_1}\right) - F\left(\frac{p_1}{s_1}\right)$$

$$D_2(p_1, p_2) = 1 - F\left(\frac{p_2 - p_1}{s_2 - s_1}\right)$$

Assume:

Consumers uniformly distributed across $[\underline{\theta}, \bar{\theta}]$

Consumers sufficiently different:

$$\bar{\theta} > 2\underline{\theta}$$

(avoiding quality dominance in equilibrium)

Firm 2 is the high-quality producer: $s_2 > s_1$.

Production costs independent of quality: c

Equilibrium in prices

$$\tilde{\theta} = \frac{p_2 - p_1}{s_2 - s_1}$$

Firm 1's profit: $\pi_1 = (p_1 - c)\left(\frac{p_2 - p_1}{s_2 - s_1} - \max\left[\underline{\theta}, \frac{p_1}{s_1}\right]\right)$

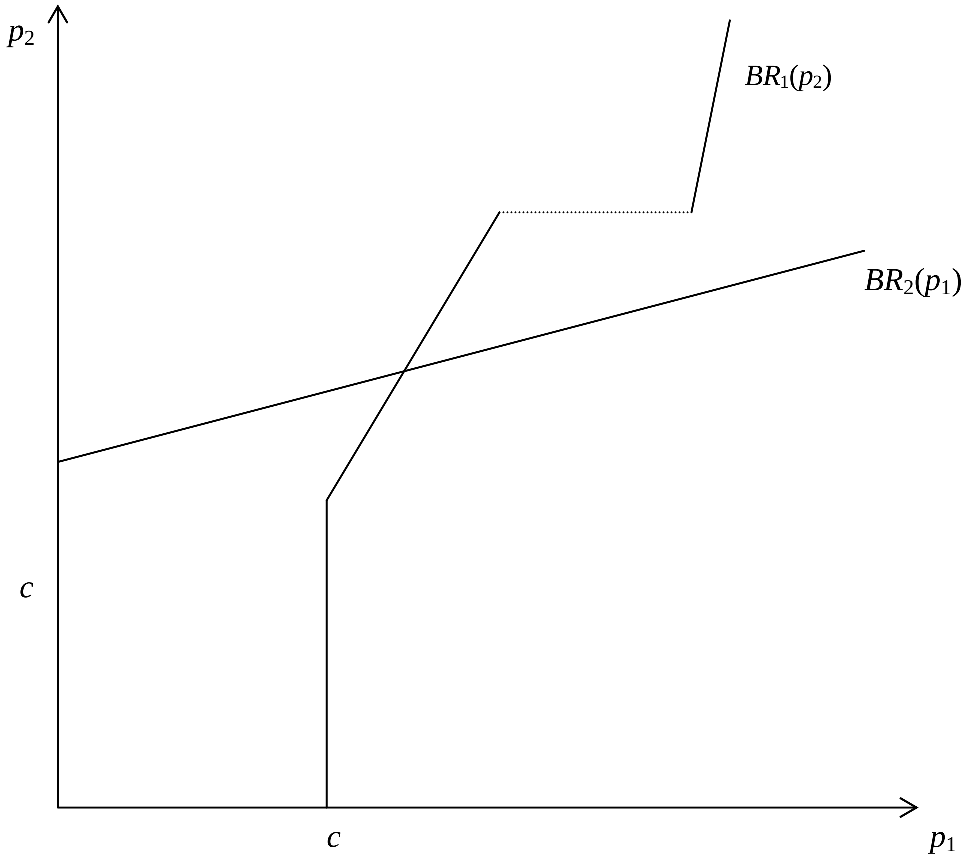
Best response of firm 1:

$$p_1 = \begin{cases} \frac{1}{2}\left[c + \frac{s_1}{s_2} p_2\right], \text{ if } p_2 > c + \underline{\theta}(s_1 + s_2) \\ \frac{1}{2}\left[c + p_2 - \underline{\theta}(s_2 - s_1)\right], \text{if } c + \underline{\theta}(s_1 + s_2) \geq p_2 \geq c + \underline{\theta}(s_2 - s_1) \\ c, \text{ if } p_2 < c + \underline{\theta}(s_2 - s_1) \end{cases}$$

Firm 2's profit: $\pi_2 = (p_2 - c)\left(\overline{\theta} - \frac{p_2 - p_1}{s_2 - s_1}\right)$

Best response of firm 2:

$$p_2 = \frac{1}{2}\left[c + p_1 + \overline{\theta}(s_2 - s_1)\right]$$



Equilibrium prices:

$$p_1 = c + \frac{1}{3}\left(\overline{\theta} - 2\underline{\theta}\right)\left(s_2 - s_1\right)$$
$$p_2 = c + \frac{1}{3}\left(2\overline{\theta} - \underline{\theta}\right)\left(s_2 - s_1\right)$$

Condition for the market being *covered*, $\underline{\theta} \geq \frac{p_1}{s_1}$:

$$c \leq \tfrac{1}{3}[\underline{\theta}(2s_1 + s_2) - (\overline{\theta} - \underline{\theta})(s_2 - s_1)]$$

- The high-quality firm sets the higher price:

$$p_2 - p_1 = \tfrac{1}{3}(\bar{\theta} + \underline{\theta})(s_2 - s_1) > 0$$

- The high-quality firm has the higher demand:

$$\tilde{\theta} = \frac{p_2 - p_1}{s_2 - s_1} = \tfrac{1}{3}(\bar{\theta} + \underline{\theta}) < \tfrac{1}{2}(\bar{\theta} + \underline{\theta})$$

$$D_1 = \tilde{\theta} - \underline{\theta} = \tfrac{1}{3}(\bar{\theta} - 2\underline{\theta})$$

$$D_2 = \bar{\theta} - \tilde{\theta} = \tfrac{1}{3}(2\bar{\theta} - \underline{\theta})$$

- The high-quality firm has the higher profit:

$$\pi_1(s_1, s_2) = (p_1 - c)D_1 = \tfrac{1}{9}(\bar{\theta} - 2\underline{\theta})^2(s_2 - s_1)$$

$$\pi_2(s_1, s_2) = (p_2 - c)D_2 = \tfrac{1}{9}(2\bar{\theta} - \underline{\theta})^2(s_2 - s_1)$$

- Firms' profits are increasing in the quality difference

<u>Two-stage game</u>

Stage 1: Firms choose qualities
Stage 2: Firms choose prices

Stage 1 – feasible quality range: $[\underline{s}, \bar{s}]$
Assume: $c \leq \tfrac{1}{3}[\underline{\theta}(2\underline{s} + \bar{s}) - (\bar{\theta} - \underline{\theta})(\bar{s} - \underline{s})]$

In equilibrium: $s_1 = \underline{s}$, $s_2 = \bar{s}$ (or the opposite).

- Asymmetric equilibrium
- Maximum differentiation

What if …

- $c > \frac{1}{3}[\underline{\theta}(2\underline{s} + \bar{s}) - (\bar{\theta} - \underline{\theta})(\bar{s} - \underline{s})]$
  - the low-quality firm will choose a quality above $\underline{s}$.

- $\bar{\theta} < 2\underline{\theta}$
  - only one firm active in the market:
    $p_1 = c$, $D_1 = 0$, $\pi_1 = 0$
    $p_2 = c + \frac{1}{2}\bar{\theta}(\bar{s} - \underline{s})$, $D_2 = 1$, $\pi_2 = \frac{1}{2}\bar{\theta}(\bar{s} - \underline{s})$
  - natural monopoly: low consumer heterogeneity makes price competition too intense for the low-quality firm

Natural duopoly for a range of consumer heterogeneity "above" $\bar{\theta} > 2\underline{\theta}$.

<u>Vertical differentiation:</u> the number of firms determined by *consumer heterogeneity*.

<u>Horizontal differentiation:</u> the number of firms determined by *market size*.