# Auctions

Auction:

- One seller and a small number of potential buyers

The mirror image –
Contract auction / Procurement auction:

- One buyer and a small number of potential sellers.
- The buyer decides on the purchasing procedure, potential sellers bid their prices.

## When are auctions used?

- A unique object
  - well defined? indivisible?

- Uncertainty about who should get the object / the contract

- Uncertainty about the object's value / the project costs

- Commitment to selling / buying procedure

## Alternatives to auctions

Market

- decides who gets the object / project
- but how to determine the price?

Bargaining

- determines the price
- but how to determine who is the counterpart?

Handing out for free

- beauty contest
- lobbying costs

## Two concerns with an auction

- For society - *efficiency*: Is the object bought by the bidder with the highest willingness to pay?
- For the seller: Is the price the highest possible?

## Several auction procedures

How are these questions affected by the procedure chosen?

## Various kinds of auctions

- Sealed bids vs. open bids

- Open bids

    - Ascending bids – English auction
        - bidders submit higher and higher bids until only one bidder remains
        - art, collectibles

    - Descending bids – Dutch auction
        - seller starts with a high price and cries out lower and lower prices until a bidder accepts
        - flowers (Netherlands), fish (Israel), tobacco (Canada)

- Sealed bids

    - First price:
        - The bidder with the highest bid wins and pays his bid.
        - real estate, government procurement

    - Second price:
        - The bidder with the highest bid wins and pays an amount equal to the second highest bid.
        - Vickrey auction [Vickrey, *J Finance* 1961]
            - William Vickrey, Nobel laureate 1996
        - stamps etc. [Lucking-Reiley, *J Econ Perspectives* 2000]

## Basic model

- Bidders are risk neutral
- Bidders' valuations are different but independent
- Each bidder knows only his own valuation
- Seller doesn't know any bidder's valuation
- No observable differences among the bidders
- Reservation price?

## Bidder behaviour

(i) English auction

- continuing bidding is profitable as long as
  own valuation > current high bid

- this strategy is independent of what other bidders do
  (dominant strategy)

- the winner is the one with highest valuation
  $\rightarrow$ efficiency

- price is (just above) second highest evaluation

## (ii) Sealed-bid second-price auction

bidder B's valuation = $v$
bidder B's bid = $b$
largest bid from others = $a$

- With a valuation of $v$, what should be bidder B's bid, $b$?
  Distinguish between two cases:
  - $a > v$: B's decision does not matter
  - $a < v$: B wins if $b > a$, and earns $(v – a)$
- Bidding $b < v$ reduces B's chances to win but does not affect what he has to pay if he wins.
- Optimum bid: $b = v$
  (dominant strategy)
- The winner is the one with highest valuation
  → Efficiency
- The price equals second-highest valuation
- English auction and sealed-bid second-price auction are equivalent with respect to winner and price.
- Contract auction:
  - winner is the one with lowest cost
  - price equals second-lowest cost
- Calculating the bid is easy

## (iii) Sealed-bid first-price auction

- Bidder trades off two concerns:
  Bidding $b < v$
    - reduces his chances to win; not good.
    - reduces the price he has to pay if he wins; good.

- This trade-off makes the optimum bid lower than $v$.

- The bidder knows that other bidders think the same way: All bidders bid below their valuation. This makes the optimum bid even lower.

- This also holds for (iv) Dutch auction

- The winner is the one with the highest valuation

- The price equals highest bid, which is lower than highest valuation

- Expected price = Expected second-highest valuation

- Calculating bid is difficult

Equilibrium bid – sealed-bid first-price auction

$n$ bidders, $v_i \in [v_l, v_h]$, $i \in \{1, \ldots, n\}$
cumulative distribution function: $F(v_i)$, $i \in \{1, \ldots, n\}$

Let's focus on a symmetric equilibrium. Bidders are not identical, since valuations differ. But there are no observable differences, so their valuations are all drawn from the same cdf.

In a symmetric equilibrium, there exists some function $B(v)$, which is the same for all players, so that if one's valuation is $v$, the equilibrium bid is $B(v)$.

Consider bidder $i$. He does not know the other bidders' $v$s but believes that their bids depend on their valuations according to the function $B(v)$. Assume: $B' > 0$.

$\Rightarrow$ A bid of $b$ implies a valuation equal to $B^{-1}(b)$.

The probability that $i$'s bid $b_i$ is the winning bid =

$$[F(B^{-1}(b_i))]^{n-1}$$

Bidder $i$'s expected profit:

$$\pi_i = [v_i - b_i][F(B^{-1}(b_i))]^{n-1}$$

Optimum bid satisfies: $\frac{\partial \pi_i}{\partial b_i} = 0$

$$\Rightarrow \frac{d\pi_i}{dv_i} = \frac{\partial \pi_i}{\partial v_i} + \frac{\partial \pi_i}{\partial b_i}\frac{db_i}{dv_i} = \frac{\partial \pi_i}{\partial v_i} = [F(B^{-1}(b_i))]^{n-1}$$

In a symmetric equilibrium: $b_i = B(v_i), \forall\, i. \Rightarrow v_i = B^{-1}(b_i)$

In equilibrium, bidders' beliefs about each other's valuations are correct.

$$\Rightarrow \frac{d\pi_i}{dv_i} = [F(v_i)]^{n-1}$$

Assume (reasonably): $\pi_i = 0$ if $v_i = v_l. \Rightarrow B(v_l) = v_l.$

Integration:

$$\pi(v_i) = \int_{v_l}^{v_i} [F(x)]^{n-1} dx$$

Two expressions for bidder *i*'s profit – must be equal.

$$\pi_i = [v_i - b_i][F(B^{-1}(b_i))]^{n-1} = \int_{v_l}^{v_i} [F(x)]^{n-1} dx$$

$$\Rightarrow B(v_i) = b_i = v_i - \frac{\int_{v_l}^{v_i} [F(x)]^{n-1} dx}{[F(v_i)]^{n-1}}$$

Common for all four kinds of auctions (in the base model):

- Efficiency: Object to the bidder with highest valuation (or lowest cost)

- Revenue equivalence: All four kinds give the seller the same expected income.

- An increase in the number of bidders increases the expected price.
    - the more bidders, the higher is the expected second-highest valuation.

Difference among the auctions:

- Bid more difficult to calculate in sealed-bid first-price and Dutch auctions than in sealed-bid second-price and English auction.

## Seller's reservation price

Revenue equivalence in the basic model: Seller indifferent between auction procedures. But what about a reservation price?

A parallel situation: The monopolist's problem

A monopolist trades off two concerns:

- wants to sell large quantities → low price
- wants to earn a profit per unit sold → high price

Optimum trade-off: Price above marginal cost

Auction: Seller trades off the same two concerns:

- wants to sell the object → low reservation price
- wants to earn a profit if the object is sold
  → high reservation price

The two highest valuations: $v_1$, $v_2$
Reservation price: $r$

Three cases:
(i) $v_1 > v_2 > r$: increasing $r$ has no effect
(ii) $v_1 > r > v_2$: increasing $r$ increases the price
(iii) $r > v_1 > v_2$: increasing $r$ reduces the chances to sell

Optimum reservation price with 1 bidder

Bid = $r$ or nothing
Seller's own valuation: $v_0$
Seller's expected profit:

$$\pi(r) = r[1 - F(r)] + v_0 F(r)$$

FOC: $[1 - F(r)] - rf(r) + v_0 f(r) = 0$

$$\Rightarrow v_0 = r - \frac{1 - F(r)}{f(r)} \equiv J(r)$$

i.e., marginal cost = marginal revenue

$$\Rightarrow r = J^{-1}(v_0)$$

Generally:

If highest bidder has valuation $v$, his expected gain is

$$\frac{1 - F(v)}{f(v)}$$

so that the expected price in this case is

$$v - \frac{1 - F(v)}{f(v)} = J(v)$$

The seller sells only if $J(v) \geq v_0$ for the highest bid

$$\Rightarrow r = J^{-1}(v_0)$$

Effiency with a reservation price:

- With a reservation price, the object may not be sold, even if a bidder exists with $v > v_0$.
- *Ex-ante* efficiency vs. *ex-post* efficiency.

## Some extensions

### (i) Observable differences among the bidders

Example:
Public procurement – domestic vs. foreign firms.
Suppose foreign firms are more cost effective than domestic ones.

- English auction and sealed-bid second-price auction are still efficient.

- Sealed-bid first-price auction no longer efficient: it is possible to win the auction without having the lowest cost.

- It is optimum for the procurer to discriminate between bidder groups, and one is no longer certain that the project is won by the lowest-cost bidder.

- In the example: It is optimum to discriminate in favour of the domestic firms. This favouring
  - increases the chance of getting an inefficient supplier, but also
  - lowers the bid from the efficient firms

(ii) Risk-averse bidders

- In a sealed-bid first-price auction, risk-averse bidders bid *higher* than risk-neutral ones. An increase in the bid
  (1) increases the chance of winning, and therefore getting something
  (2) reduces what one earns in case of winning.

With risk aversion, (1) gets more important than (2)

- Contract auction: Risk averse bidders bid more aggressively than risk neutral bidders.

- The seller gains more in a sealed-bid first-price auction than in a sealed-bid second-price auction.

(iii) Correlated valuations

- Extreme case: identical valuations. Bidders do not know the object's true value but have access to different pieces of information about this value. No bidder knows what other bidders know.

- More common in auctions than in contract auctions?

  Auctions:
  - buying for resale
  - exclusive rights

  Contract auction
  - pioneering projects with great cost uncertainty for all potential suppliers

- “Winner’s curse”
  - Bidders base bids in a sealed-bid auction on estimates. The bidder with the most optimistic estimate wins.
  - If you win, then you will wish to revise your estimate: The winner is the most optimistic one.
  - But this is taken into consideration in the bids: Bids are even lower because of the “winner’s curse”.

- In an English auction, bidders learn from each other during the bidding process. This reduces the winner’s-curse problem.
  - With correlated values, an English auction is preferred by the seller to the other kinds.

- Asymmetric information
  - one bidder knows the object’s true value
  - US offshore oil and gas lease auctions
    - Porter, *Econometrica* 1995

## Other issues

- Collusion
  - second-price auction better for sustaining collusion among bidders than first-price auction
  - open bids better than closed bids
  - contract auctions: *Norsk Standard*

- divisible objects
  - securities, quotas

- combined bids
  - petroleum: price on exploration right + production fee
  - vague projects: price + content

- entry costs, number of bidders, participation fee

- auctioning incentive contracts

- competition *for* a market vs. competition *in* a market

## Efficiency of auctions

- Which auction procedure to use?
  - revenue equivalence
  - easily calculated bids

    → sealed-bid second-price auction

  But: risk aversion? correlated values?

- Which objects are sold most effectively in an auction?
  - unique object
  - uncertainty about willingness to pay:
    how large? who?

- Does price affect efficiency?
  - one unit – no quantity effects from price change
  - divisible objects (quotas, securities): quantity effects

## Repeated auctions

- Less aggressive bidding today in order not to reveal one's high valuation before future auctions (the "ratchet" effect)
  - better to have large projects? negotiating renewal with current supplier?
- Capacity constraints: The winner of a contract today may not have capacity to participate in the next round.