Empirical Industrial Organization-Spring 2007

Problem Set 1

Nonlinear econometrics

The due date for this assignment is February 15th.

1. Maximum Likelihood

Consider the linear model with conditional homokedasticity with$\{y_i, x_i\}$ iid

$$y_i = x_i'\beta + \varepsilon_i \qquad\qquad i = 1, 2, ..., n$$

where $\varepsilon|x \sim N(0, \sigma^2)$ and β is a Lx1 vector of parameters.

i. Form the conditional likelihood.

ii. Find the ML estimators for β and σ.

iii. Compute the score and Hessian function

2. Method of Moments

Consider the linear models

$$y_i = x_i'\beta + \varepsilon_i \qquad\qquad i = 1, 2, ..., n$$

i. Assume the following orthogonality condition $E[\varepsilon_t|x_t] = 0$. Describe the appropiate sample moment condition and solve for the MM estimator of β,

$$E[\varepsilon_t|x_t] = 0$$

ii. Now suppose that x_t and ε_t are correlated. Suppose that you have a sets of intruments in the qx1 vector z_t. Suppose that these intruments are valid and therefore $E[z_t\varepsilon_t] = 0$. Describe the appropiate sample moment condition and solve for the MM estimator of β. Does your expression resembles any known estimator?

3. **Computing Maximum Likelihood (ML)**

In this problem you are asked to generate a data generating process (DGP) for a particular distribution and then compute the ML and NLS estimator using a computer software. Consider the following DGP from an exponential function distributed as

$$f(y) = \lambda \exp(-\lambda y)$$

with $\lambda > 0$, y>0 and

$$\lambda = \exp(\beta_1 + \beta_2 x)$$

and $x \sim Normal(1, 1^2)$ and (β_1, β_2)=(2,-1).

i. Compute the mean and variance of the exponential function

ii. Generate 10000 draws from the DGP specified above.

iii. Show the expression for the log likelihood.

iv. Use the command ml in STATA to obtain the estimates for β_1 and β_2.

v. Explain how the STATA estimates the standard errors.

4. **Computing Nonlinear Least Squares (NLS)**

i. Define the sum of squared residuals and explain how to implement NLS procedure.

ii. Using the same dataset constructed above use the nls function in STATA to estimate the parameter.

iii. Explain how the STATA estimates the standard errors.

4. (Bonus question, mandatory for PhD students)

Consider the discrete random variable N having a mass function

$$f_N(n,\theta) = \frac{-(\theta_0)^n}{n\log(1-\theta_0)}$$

for n=1,2,.... and 0<$\theta < 1$.

i. Prove that

$$\sum_{n=1}^{\infty} f_N(n,\theta) = 1$$

(Hint: consider the Maclaurin series expansion of $log(1+x)$ and substitute x=-θ_0)

ii. Find the expected value of N (Hint: Use $\sum_{n=1}^{\infty} = \frac{\rho}{1+\rho}$)

ii. Find the variance of N

iii. Define the method of moment estimator $\widehat{\theta}_{MM}$ of θ_0.

iv. Define the maximum likelihood estimator $\widehat{\theta}_{ML}$ of θ_0.

v. Demostrate that $\widehat{\theta}_{MM}$ and $\widehat{\theta}_{ML}$ are consistent estimators of θ.

vi. Find an expression for the approximate variance of $\widehat{\theta}_{MM}$ and $\widehat{\theta}_{ML}$.

vii. Characterize the asymptotic distribution of $\widehat{\theta}_{MM}$ and $\widehat{\theta}_{ML}$.

Computation:

Consider the following random sample obtained of a measurement of 1000 observations.

N	1	2	3	4	5	6	7	8	9+
Freq	700	205	50	26	10	6	1	1	1

viii. Using the Newton method (using matlab) implement a program to compute the ML estimate of θ_0.

ix. At size 0.05 test the following hypothesis

$$H_0 : \theta_0 = 0.5$$
$$H_1 : \theta_0 \neq 0.5$$

x. At size 0.05 test the following hypothesis

$$
\begin{aligned}
H_0 &: \log \theta_0 = 0.5 \\
H_1 &: \log \theta_0 \neq 0.5
\end{aligned}
$$