Economic Dynamics and Uncertainty
Final Exam – Fall 2020

1. Dynamic Programming (12 points, each question 3 points)
We have discussed two different numeric methods to solve (infinite horizon) dynamic programming problems. The two methods differed in using discrete versus continuous function approximation.

i) Why do we have to use approximation methods when solving dynamic programming problems numerically?

ii) Explain the general difference between the two solution approaches.

iii) State and explain all the steps in the solution algorithm where the difference matters (and *only* those steps where it matters!).

iv) Assume you are using continuous function approximation using Chebychev polynomials.
a) How will you pick your interpolation nodes? Are they equidistant?
b) Explain the trade-off between using a low versus high order of the basis. Explain what might help you in your judgment whether the approximation is sufficiently good.

2. Asset Pricing (12 points, each question 3 points)

i) Critically assess the following statement: "The Sharpe ratio can be interpreted as the premium above the market price of non-systemic risk." Please explain your answer carefully.

ii) Discuss the sign of the Sharpe ratio.

iii) Assume an asset's volatility is σ_{R_i}. In the CAPM model, would you expect the asset's risk premium (excess return) to be proportional to σ_{R_i}? If so, what can you say about the proportionality factor? If not, why not?

iv) Discuss the relation betweeen an asset's Sharpe ratio, the volatility of the stochastic discount factor, and the risk-free return factor.

3. Consumption, investment, and Epstein-Zin-Weil preferences
(8 points, each question 4 points)
A special case of Epstein-Zin-Weil preferences is the case where the utility function representating aversion to intertemporal consumption change is logarithmic. The following investment problem is based on an agent exhibiting such preferences with a coefficient of Arrow-Pratt risk aversion RRA $= 1 - \alpha$. The investor's choice problem is

$$\max_{c_1, \boldsymbol{\omega}} \log(c_1) + \frac{\beta}{\alpha} \log\left(\mathbb{E}\, c_2^\alpha\right)$$
$$\text{subject to } c_2 = [W - c_1]\boldsymbol{\omega}^\intercal \tilde{\boldsymbol{R}}$$
$$\text{and } \sum_{i=1}^n \omega_i = 1\ .$$

where W is his/her wealth, $\tilde{\boldsymbol{R}}$ is a vector of stochastic return factors and $\boldsymbol{\omega}$ the vector comprising the portfolio shares.

i) Derive the optimal level of first period consumption.

ii) How does optimal consumption depend on the return factor? Please explain this finding using insights from our class.

4. Intertemporal Optimization (12 points, each question 4 points)
Assume a decision maker optimizes consumption given the following objective and constraints

$$\sum_{t=0}^{\infty} \beta^t u(c_t, B_t)$$

$$\begin{aligned} \text{subject to} \quad K_{t+1} &= F(c_t, K_t, D_t) \\ B_{t+1} &= \alpha K_t \\ D_{t+1} &= D_t + f(B_t). \end{aligned}$$

i) Write down the corresponding Bellman equation. Interpret the value function and give an intuition for Bellman's dynamic programming equation (or Bellman's principle of optimality).

ii) Assume you were given a trial-solution. How would you verify whether the trial solution is correct? Write down the main steps.

iii) How would the Bellman equation change in case the last dynamic equation changes from

$$D_{t+1} = D_t + f(B_t)$$

to the form

$$D_{t+1} = D_t + f(B_t, B_{t-1})?$$