Economic Dynamics and Uncertainty
Final Exam – Fall 2021

Problems 1 and 2 are brief discussions that you can type into INSPERA. If you are consise, half a page should be enough. Please try not to exceed one page. Problem 3 requires calculations and can be answered on paper. It is your responsibility that the submitted document/photo/scan is legible.

1. Income and Substitution Effect (20%).
Write a brief discussion of the intertemporal income and substitution effect. Include in your discussion for each of these effects and a discussion of which of the two dominates and what it implies for present consumption. Then, please briefly discuss the effect of risk aversion and uncertain returns when the intertemporal income and sustitution effect balance each other.

2. Numeric Dynamic Programming (20%).
Write a brief(!) discussion how you would solve a stationary infinite horizon optimization problem numerically on your computer using dynamic programming. Include in your discussion (i) what the main challenge is in solving the Bellman equation rather than a simple two period optimization problem, (ii) how you would represent the value function, (iii) how you would update the value function over the course of your iterations, (iv) when you would consider the Bellman equation solved. We are fine with solving the Bellman equation and you do not have to explain how to generate a time path.

3. Trading-off consumption and leisure (60%).
A representative agent derives utility from consumption C_t and leisure L_t in the form

$$U(C_t, L_t, \bar{C}_t) = \log\left(C_t^{1-\alpha-\gamma}\ L_t^{\gamma}\left(\frac{C_t}{\bar{C}_t}\right)^{\alpha}\right).$$

The consumption level $\bar{C}_t$ reflects the average consumption level in the economy. She discounts the future with discount factor β and has an infinite planning horizon. The economy's production follows a Cobb-Douglas composition of labor $\tilde{L}_t$, capital K_t, and the exogenously evolving technology level A_t yielding output

$$Y_t = A_t\ K_t^{\kappa}\ \tilde{L}_t^{1-\kappa}.$$

A time constraint limits labor $\tilde{L}_t$ and leisure L_t to the available hours N_t, i.e., $\tilde{L}_t + L_t = N_t$. The economy's dynamic equation governs the capital stock

$$K_{t+1} = Y_t - C_t, \tag{1}$$

which assumes full depreciation of capital in the production process. Assume an interior solution to all optimization problems.

i) A representative agent maximizes utility over consumption and leisure subject to the production function taking the average consumption level of the economy $\bar{C}_t$ as given. She uses the production function and evoluation of capital in the economy as her income constraint.[2]

Assume that a linear-affine value function (linear in log-capital and with an additive constant) solves the dynamic programming problem and write down the corresponding Bellman equation.

Note: It is not necessary but can be helpful to express the consumption control in terms of the consumption rate. Then, you should show that the (log) capital equation is $k_{t+1} = a_t + \kappa k_t + (1-\kappa)\tilde{l}_t + \log(1-x_t)$, where we use lower case letters for the log of a variable and x_t for the consumption rate.

ii) Solve the first order condition for leisure.

iii) Solve the first order condition for consumption or for the consumption rate.

iv) Now, assume that a social planner maximizes utility over consumption C_t and leisure L_t subject to the production function, incorporating that $\bar{C}_t = C_t$. You may again pick the consumption rate as the control variable rather than aggregate consumption itself.

Assume again that a linear-affine value function in log-capital solves the dynamic programming problem. Derive the optimal leisure and consumption choices from the perspective of the social planner.

v) Do the expressions for the optimal controls differ or coincide? At this point, can you meaningfullly compare the consumption rates across the two different scenarios? Discuss.

vi) Solve for the shadow value of log-capital in each of the two problems. You can treat them jointly. Do the shadow values differ?

vii) Does the linear-affine value function indeed solve the Bellman equation? Explain.

viii) Using your result for the shadow value of capital, calculate the optimal leisure levels and the optimal consumption rate for the representative agent and for the social planner.

ix) Interpret possible differences you find in leisure and the consumption rate for the representative agent's solution versus the social planner's solution. Discuss your intuition why they differ.

x) Discuss how absolute consumption, GDP, and capital growth differ between the two settings.

If you think that your result and interpretation suggest that you made calculus or other mistakes along the way, you are invited to discuss your suspicion and what you would have expected instead.

[2] This unusual constraint for a representative agent makes your life a lot easier and permits a simpler comparison with the subsequent social planner problem.