Economic Dynamics and Uncertainty
Final Exam – Fall 2021

Problems 2 and 3 are brief discussions that you can type into INSPERA. If you are concise a paragraph for each question will be enough. Problem 1 requires calculations and can be answered on paper. It is your responsibility that the submitted document/photo/scan is legible. If you get stuck or think that a question is ambiguous explain your concern and pick your interpretation or make an explicit assumption if needed, and move on.

1. Optimal consumption with fixed income (50%).
Consider a consumer with an initial wealth W_0 who receives a fixed income F at the beginning of each period. The interest rate is r and there is a perfect credit market so the consumer can borrow on future income as long as he is able to pay back in the final period. We will assume that $r = 0$. The consumer chooses consumption C_t each period and maximizes

$$\sum_{t=0}^{T} \beta^t \ln(C_t)$$

subject to the wealth accumulation equation

$$W_{t+1} = (W_t - C_t) + F$$

(Remember that $r = 0$.) Let $V_T(W)$ denote the value function when there are T periods left (in addition to the current, i.e., here $W = W_0$).

i) Solve the case $T = 1$, i.e., find the optimal C_0 and the value function $V_1(W_0)$.

ii) Solve the problem for $T = 2$, i.e., find optimal C_0 and the value function $V_2(W)$

iii) Since the utility function is $\ln C$, we may guess that the value function in the infinite horizon case ($T = \infty$) is of the form

$$V(W) = A \ln W + B$$

Check whether this guess is correct.

iv) Based the results about the cake eating economy in the textbook and on a) and b) above, explain why we may expect that V_T is of the form

$$V_T(W_0) = \left(\sum_{t=0}^{T-1} \beta^t \right) \ln(W_0 + TF) + B$$

v) Given the value function above, what happens to initial consumption C_0 in the T period problem, as $T \to \infty$.

2. Numerical solution (30%).
If we did not know the analytical solution to Problem 1 above, we could try solve the problem numerically using Chebychev polynomials. We then only consider the case $T < \infty$.

i) Explain briefly how you would solve the problem numerically.

ii) Is there any way you could evaluate the accuracy of the numerical solution?

3. Asset Pricing (20%).
Please comment on the statement: "A high covariance of an asset's return with consumption explains a high risk premium". In particular, explain the intuition why this statement might (or might not) hold.