ECON4830 - **Economic Dynamics and Uncertainty**
Final Exam – Fall 2023

Each roman numbered subquestion counts equally to the exam. The final starred subquestion 2.vi*) is a bonus question.

1. Asset Pricing (40%).

You are advising an educated economist who is ignorant about asset pricing on how to value a given asset. She is convinced that the correlations with the return on $\tilde{x}$ should be the crucial factor in determining the asset's return and heard that there is a model of the form

$$\mathbb{E}\,\tilde{R}_i = \alpha + \beta(\tilde{R}_i, \tilde{R}_x)\eta$$

that she can use to price the asset, but is not quite sure about the meaning of this equation.

i) Please explain the different terms in the equation.

ii) Explain how she can get from the equation above to the alternative form

$$\mathbb{E}\,\tilde{R}_i - R_f = \beta(\tilde{R}_i, \tilde{R}_x)\left(\mathbb{E}\,\tilde{R}_x - R_f\right).$$

Explain steps and possible assumptions and relate the terms back to the earlier equation.

iii) The economist tells you that she had not seen the previous equation, but this one instead:

$$\bar{R}_i - R_f = \rho_{i,M}\ \sigma_i\ S_e$$

Explain the different terms in this equation, and state which factor pricing model this equation most likely corresponds to. Explain the insights deriving from this equation.

iv) The economists tells you that she's not convinced that the pricing factor of the previous model is really the only determinant for asset returns. She asks you whether there might be models that use more than one factor. Give a briefly(!) verbal answer (no formulas needed). If your answer is affirmative, motivate why there might be more than one factor in such a model. If it is negative, explain why not.

2. Dynamic Programming (60%). A social planner with infinite planning horizon and utility discount factor β maximizes a representative agent's utility $U(c_t, x_t)$, where x_t is the level of biodiversity and consumption c_t increases utility but also destroys some of the biodiversity. Utility is increasing and concave in both arguments. The law of motion for biodiversity is

$$x_{t+1} = f(x_t) - g(c_t)$$

with some given functions f and g.

Note: Biodiversity is a measure for species diversity. We reduce biodiversity by destroying species, e.g., through habitat destruction or climatic change.

i) Spell out the social planner's Bellman equation including all necessary substitutions and/or constraints required to solve the problem.

ii) Derive the Euler equation from the dynamic programming equation.

iii) Interpret the Euler equation.
If you did not succeed in deriving the Euler equation, assume the Euler equation has a term of the form $\beta\frac{\partial U(c_{t+1},x_{t+1})}{\partial x_{t+1}}g'(c_t)$ on the right hand side and explain why this novel term appears in the Euler equation and interpret it.

iv) Let's think about a generalization where the agent has two types of consumption. One type of consumption labeled c_2 relies on activities that destroys biodiversity, the other labeled c_1 does not. The utility derived from consumption is now $U(ac_{1,t}^s + (1-a)c_{2,t}^s, x_t)$. Let's assume that consumption level c_1 evolves exogenously and the equation of motion for biodiversity is now

$$x_{t+1} = f(x_t) - g(c_{2,t}).$$

How does this change in the model affect the Euler equation (intertemporal trade-off) previously derived?

Hint: Think, rather than re-solving the problem. You can can answer this question even if you have not solved (or solved correctly/fully) for the Euler equation above.

v) What would be missing in our model to meaningfully let the representative agent optimize over the consumption level c_2.

Hint: The correct answer is related to questioning whether the original trade-off in the model with only c_t really contained the comprehensive trade-off we might want in a more realistic model.

Bonus: vi*) Let's return to the original optimization problem posed in the first part of this problem. Do you think the equation captures well a model of biodiversity loss? Think about how easy or difficult it is to regrow biodiversity as compared to eliminating biodiversity. Discuss how you might want to modify the model to better capture biodiversity loss. No need to actually solve or resolve the model.