UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Exam: ECON4910 – Environmental Economics

Date of exam: Tuesday, June 6, 2006 **Grades will be given: Thursday, June 15**

Time for exam: 09:00 a.m. – 12:00 noon

The problem set covers 2 pages

Resources allowed:
- No resources allowed

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

Question 1

There are N firms that emit a certain type of pollutant. The government has decided to cap the total allowed amount of emissions in the economy at an aggregate level X. There are two types of firms that produce the pollutant. There are m firms with an individual benefit from pollution, x, given by $c(x) = Ax - Bx^2$ and n firms with individual benefit functions $k(x) = \alpha x - \beta x^2$. Obviously, $N = m + n$.

a) Given the cap on aggregate levels, find the most cost-effective distribution of emissions. You do not need to discuss the possibility of solutions where one or more of the firms have zero emissions in optimum.
b) The firms are issued emission permits. The total amount of permitted emissions is equal to X. Assume that the firms are allowed to trade emission permits. Derive the firms demand for permits and find the competitive equilibrium in the emission permit market. How does the market outcome depend on the initial distribution of emission permits?
c) Find an expression for the marginal aggregate benefit of emissions. Use this expression to examine properties of the aggregate benefit function.

Question 2

Consider the following model of a stock pollutant. Let the instantaneous benefit from pollution, denoted u, be given by $\ln(u)$ where $\ln(\cdot)$ is the natural logarithm. The variable u can only take positive values. The consumption leads to pollution which accumulates according to the differential equation $dx/dt = u - \delta x$. The instantaneous damage from stock pollution is given by $\frac{1}{2}x^2$.

Overleaf..

a) Formulate the regulation of this pollutant as an optimal control problem.
b) Derive the optimality conditions.
c) Give a qualitative characterization of the optimal path by calculating steady state(s) and drawing a phase diagram.
d) How is the solution affected by a higher value of δ?

Question 3

Two firms manufacture the same product. Let Firm 1 manufacture x_1 and Firm 2 manufacture x_2. The product is exported and sold at a fixed price p. The firms have the same quadratic cost structure and produce at a cost given by $C_1(x_1) = cx_1^2$ and $C_2(x_2) = cx_2^2$. Note that the parameter c is the same for both firms. A consequence of production is pollution. Pollution is deposited in a vulnerable ecosystem according to the following formula: pollution = $z = \beta x + y$. Ecologists decide that the ecosystem can be sustained as long as $z \leq Z$. In your calculations, assume that $p = c = 1$, $\beta > 0$ and $Z > 0$.

a) Find the production levels x and y such that the social surplus generated by production is maximised without harming the ecosystem.
b) Calculate the shadow price on the constraint $z \leq Z$. Explain how changes in β and Z affect the optimal solution.
c) Assume $\beta = 1$. Ecologists and economists work together and decide that if z exceeds ½ the damage is given by $D(z) = \gamma(z - ½)$. γ is a positive constant. If $z \leq ½$, the damage is zero. Use the results from a) and b) and find the optimal level of z for different values of γ.