Final Exam - 4910 - Environmental Economics
Spring 2018

If you get stuck, and have questions or if the text/problem below is confusing/unclear, please do the following: Make and state clearly your assumptions (/interpretations of the text), and continue your analysis based on that.

Each of problems 1-3 will have equal weight in the grade.

Problem 1. Discounting, CBA, and Climate Change

i) Assume an agent has a CRRA utility function with $u_t = \frac{c_t^{1-\eta}}{1-\eta}$ and the Ramsey equation is $r = \rho + \eta g$. Use this equation as the basis for discounting future consumption steams.

(a) Explain the meaning of each term.

Solution:

- ρ: utility discount rate of pure rate of time preference.
- η: inverse of the intertemporal elasticity of substitution, desire to smooth consumption over time.
- g: consumption growth rate
- r: Either: rate of interest. Or: consumption discount rate (first answer is better, but we would take second as well)

(b) Assume we expect that consumption levels fall in the future. When does this warrant a negative discount rate for a cost-benefit analysis?

Solution: If $-g > \frac{\rho}{\eta}$.

(c) Give two different reasons related to the Ramsey equation why developing countries usually employ higher discount rates than fully developed countries.

Solution:

- opportunity cost argument: They face a higher cost r to borrow at the world market
- they have a higher consumption growth rate g
- they tend to have a higher pure rate of time preference ρ
- or another somewhat reasonable argument related to the Ramsey equation...

(d) You have to evaluate a project with non-market benefits. State and briefly (!) explain two different methods to find the value of such non-market benefits.

Solution: Two out of

- Travel cost method. Estimates the value of a recreational benefit by estimating the (maximal) costs (including travel and time) incurred to visit the site.
- Hedonic pricing. Estimate the value of environmental amenities based on property market values and their location relative to different amenities.
- Contingent valuation. Design a survey that directly asks individuals for their WTP for the non-market goods (or randomizes the amount and asks yes/no, or increases the amount until no,...).
- Choice experiments. Present competing projects and asks which one is preferred (rather than directly asking for the willingness to pay).

(or another reasonable appraoch)

ii) Write down the complete set of equations (including objective function) for a minimalistic integrated assessment model of climate change that includes global atmospheric temperature. You can be general or specific as you spell out the equations. Explain each equation in a couple of sentences. What are the relevant state variables?
Emissions affect production in two ways (directly and indirectly). Explain how. For both channels discuss whether they should have a convex or a concave impact on production.

Solution:

- objective function, e.g., $\sum_t \beta^t u(c_t)$ or similar (including time aggregation)
- production function, e.g., $Y_t = f(K_t, L_t, E_t, T_t)$ or similar, important is that there is a benefit from emissions and a cost from temperature. Alternatively, damage can be specified elsewhere and emission benefits can be modeled as co-product from production plus costly abatement.
- Carbon stock,e.g., $M_t = (1-\delta)M_t + E_t$
- Temperature, e.g., $T_t = g(M_t)$, preferably though $T_{t+1} = g(M_t, T_t)$.
- preferably also an equation of motion for a capital stock: $K_{t+1} = Y_t - C_t$, or more sophisticated as in $K_{t+1} = Y_t - C_t + (1-\delta_k)K_t$

(I omit the required explanation of the equations in this solution key).
The relevant state variables are at least one of atmospheric carbon (or greenhouse

gas) content and temperature (preferably both). Usually an additional stock variable will be (at least one) capital stock.

Emissions directly benefit production, but they also accumulate in the atmosphere and increase temperatures which then harms production.

It's fairly clear that production should be concave in the emission flow unless we hear very good arguments. The channel from emissions over temperature to production can have any curvature. The warming is concave in emissions, but damages (the dependence of production on temperatures) are convex. So we should be fine with any result if the reasoning makes sense.

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Questions 2 and 3 - with solutions in *italics.*

2. Prices vs. quantities

Suppose that $q \geq 0$ measures a firm's/industry's abatement level, and that the cost of abating q is $C(q) = q(\theta + cq)$, where $\theta = 2$ with 50% chance, and $\theta = 4$ with 50% chance. Suppose the society's benefit from abating q is $B(q) = q(8 - q)$. A planner seeks to maximize $B(q) - C(q)$. The planner sets a policy without knowing the realization of θ, but θ is known by the firm/industry when they make their decision.

(i) Suppose the planner specify a quantity requirement (quota), q. What is the optimal q?

Answer:

$$\max_q \left[q(8-q) - \frac{1}{2} q(2+cq) - \frac{1}{2} q(4+cq) \right], \text{ so}$$

$$q = \frac{5}{2(1+c)}$$

(ii) What is the expected deadweight loss, given this optimal q?

We would accept three different ways of answering this:

1) one could derive social welfare for this q, and for the first-best, and take the difference.

2) one could solve it graphically (measuring the triangles)

3) as an in-between alternative, one could work analytically on these trianges to get the deadweight loss:

$$\frac{1}{4(1+c)}$$

(iii) Suppose the planner specifies a subsidy, s, and that the firm/industry maximizes $sq - C(q)$. What is the optimal s, from the planner's point of view?

The firm sets $s - \theta - 2cq = 0$, *so*

$$q(s;\theta) = \frac{s-\theta}{2c}$$

Given this, the optimal s solves

$$\max_s E\left[\left(\frac{s-\theta}{2c}\right)\left(8 - \left(\frac{s-\theta}{2c}\right)\right) - \left(\frac{s-\theta}{2c}\right)\left(\theta + c\left(\frac{s-\theta}{2c}\right)\right)\right]$$

which gives

$$s = 3+\frac{5c}{1+c} = \frac{8c+3}{1+c} \text{ and, by the way,}$$
$$q(s;\theta) = \frac{1}{2c}\left(\frac{8c+3}{1+c}-\theta\right) = \frac{8-\theta+(3-\theta)/c}{2(1+c)}$$

(iv) What is the expected deadweight loss, given this optimal s?

I would accept the same three possible solution methods as in (ii). To follow the third method (which is simplest), remember that the first-best q is $q(\theta) = \frac{8-\theta}{2(1+c)}$, so the absolute value of the difference to $q(s;\theta)$ is $1/2c(1+c)$. The total slope of $B'-C'$ is $2(1+c)$, so each triangle measuring the deadweight loss has the high $1/c$, and area size

$$\frac{1}{2}\frac{1}{2c(1+c)}\frac{1}{c} = \frac{1}{4c^2(1+c)}.$$

(v) When is the deadweight loss of the subsidy smaller than the deadweight loss of the quantity measure? Discuss what determines the answer and the intuition for this result.

Half the point for deriving OR directly guessing the actual answer, which is $c>1$.
Half the point for the intuition/discussion, which should emphasize that what is important is whether B' is large/small relative to C', and the intuition for this.

3. Supply-side policies

Consider n countries and that in each country i, the demand for fossil fuel consumption y_i is $y_i = D(p) = b-p$, where $b>0$ is a constant while p is the fossil fuel price. The supply in country i is $x_i = S(p) = p$. Suppose only country $i=1$ sets a climate policy while all the other countries (or, the consumers and the producers in these other countries) take the price p as given. Fossil fuel is tradable globally.

(i) In words: What do you think happens to the equilibrium choice of y_i and x_i, for $i>1$, when country $i=1$ reduces x_1 while keeping y_1 fixed?

Since a smaller x_1 reduces global supply and increases p, supply goes up and demand goes down in the other countries.

(ii) Can you derive the formula that answers question 3(a) exactly?

Differentiating $D(p)$ we get $dy_i = -dp$ and differentiating $S(p)$ gives $dx_i = dp$, and since global demand equals global supply, we have

$$
\begin{aligned}
0 &= dx_1 - dy_1 + \sum_{i=2}^{n} (dx_i - dy_i) = dx_1 + \sum_{i=2}^{n} (dp + dp) = 2(n-1)\,dp, \text{ so} \\
dp &= -\frac{1}{2(n-1)} dx_1 \text{ and therefore} \\
dy_i &= -dp = \frac{1}{2(n-1)} dx_1 < 0 \\
dx_i &= dp = -\frac{1}{2(n-1)} dx_1 > 0,
\end{aligned}
$$

given that $dx_1 < 0$ when x_1 is reduced.

(iii) Suppose that total pollution is $E = \sum_{i=1}^{n} e_i x_i$, where $e_i > 0$ measures how "dirty" the fossul fuel from country i is. Please derive the condition for when E decreases, if x_1 decreases marginally, while y_1 is hold constant.

Differentiating this equation gives:

$$
\begin{aligned}
dE &= e_1 dx_1 + \sum_{i=2}^{n} e_i dx_i \\
&= e_1 dx_1 + \sum_{i=2}^{n} e_i dp \\
&= \left(e_1 - \sum_{i=2}^{n} e_i \frac{1}{2(n-1)} \right) dx_1
\end{aligned}
$$

So, if x_1 decreases, $dx_1 < 0$ and then $dE < 0$ also if

$$
2e_1 > \sum_{i=2}^{n} e_i \frac{1}{(n-1)},
$$

where the right hand side is the average emission content in the other countries' fuels.

(iv) Suppose we hold fixed every e_i, except for e_2 (this is the emission level from a unit emission in country 2). Suppose e_2 increases. How do you think the larger e_2 will influence country 1's optimal choice of y_1 and x_1?

Intuition is sufficient: Based on the formula above, if e_2 is larger, the average of the other countries has a larger emission content, and then it is less attractive to reduce x_1 since the consequence is that x_2 will increase. Thus, it is likely (and possible to prove) that country 1 finds it optimal to increase x_1 and reduce y_1, relative to before.