Exam ECON 4910 Spring 2018

Questions 1, 2 and 3 - with solutions in *italics.*

If you need to make additional assumptions, please state them clearly in the answer.

1. Discounting.

Assume a CRRA utility function with $u_t = 4\sqrt{c_t}$ and suppose consumers maximize $\sum_{t=1}^{\infty} \delta^t u_t$ with $\delta = 0.95$.

(i) What is the difference between discounting utility and discounting consumption in this example? Please derive both and discuss the difference.

Solution key: Utility is discounted by the factor 0.8, but the discount factor on consumption measures how much one unit of consumption is worth relative to one unit of consumption one period earlier.

The corresponding discount rate is given by:

$$\begin{aligned} e^{-\rho} &= 0.95 \text{ or} \\ \rho &= -\ln(95/100), \end{aligned}$$

which is roughly 5 percent.

From Ramsey's formula (which you can derive, as we did in class):

$$r = \rho + \eta\mu = 0.05 + \frac{1}{2}\mu,$$

where μ is the consumption growth rate.

(ii) How much should one discount future consumption, if the growth rate of consumption is 3% a year?

Solution key: Apply Ramsey's formulae from the slides, with $\eta = 1/2$ and $\mu = 0.03$ to get:

$$r = \rho + \eta\mu = 0.05 + \frac{1}{2}0.03 = 0.065,$$

or 6.5 percent.

(iii) Please discuss arguments for and against using this discount rate when evaluating climate change policies.

Solution key: *Following the discussion from the class, for example, we should take into account:*

- the very long term means more uncertainty and that translates into a smaller discount rate because of risk aversion.

- disagreements in what constitute the right discount factor also implies that when opinions are aggregated, a benevolent social planner will find it optimal to apply a lower discount rate for very long term decisions.

- time inconsistent preferences are also such that one applies lower discount rates for long term decisions.

- Finally, in this simple model there is a simple and single consumption good. For climate change, we need a different good. The price /value of this good is likely to change over time, and increase if the population becomes larger, and these effects should also be taken into account.

2. Prices vs. quantities

Suppose that $q \geq 0$ measures a firm's/industry's abatement level, and that the cost of abating q is $C(q) = q(\theta + cq)$, where $\theta = 3$ with 40% chance, and $\theta = 6$ with 60% chance. Suppose the society's benefit from abating q is $B(q) = q(10 - q)$. A planner seeks to maximize $B(q) - C(q)$. The planner sets a policy without knowing the realization of θ, but θ is known by the firm/industry when they make their decision.

(i) Suppose the planner specify a quantity requirement (quota), q. What is the optimal q?

Answer:

$$\begin{aligned} & \max_q \left[q(10-q) - (0.4)\, q(3+cq) - (0.6)\, q(6+cq)\right], \text{ so} \\ q \quad = \quad & \frac{10 - 1.2 - 3.6}{2(1+c)} = \frac{2.6}{1+c}. \end{aligned}$$

(ii) Suppose the planner instead introduces an emission tax. What is the optimal level of the tax?

Answer: With a tax t, the polluter's FOC gives $\theta + 2cq = t$ so $q = \left(\frac{\theta - t}{2c}\right)$. So the expected B-C becomes:

$$\mathrm{E}_\theta \left[\left(\frac{\theta - t}{2c}\right) \left(10 - \left(\frac{\theta - t}{2c}\right)\right) - \left(\frac{\theta - t}{2c}\right) \left(\theta + c\left(\frac{\theta - t}{2c}\right)\right) \right].$$

The derivative wrt t is:

$$\mathrm{E}_\theta \left[10\left(\frac{\theta - t}{2c}\right) + 2\left(\frac{\theta - t}{4c^2}\right) + \frac{\theta}{2c} + 2c\left(\frac{\theta - t}{4c^2}\right) \right],$$

which is different from zero unless the following FOC holds (2OC holds trivially):

$$\begin{aligned} t \quad &= \quad \mathrm{E}\left[\theta + \frac{\theta}{2c} \Big/ \left(\frac{10}{2c} + \frac{1}{2c^2} + \frac{1}{2c}\right)\right] \\ &= \quad 4.8\left[1 + \frac{c}{11c+1}\right] \end{aligned}$$

(iii) What is the best of these two instruments?

Answer: There are many ways of answering these questions. Following the arguments from the lectures we will learn that taxes are best if and only if $C''(q) > B''(q)$, which here boils down to $c > 1$.

(iv) Can you propose regulatory instruments that are better than both alternatives considered above?

Answer: Of course. In class we discussed how price ceilings and price floors could improve on the outcome (and lead to the first best when there are only two possible values for the shock).

3. Supply-side policies

Consider n countries and that in each country i, the demand for fossil fuel consumption y_i is $y_i = D(p) = D - 2p$, where $D > 0$ is a constant while p is the fossil fuel price. The supply in country i is $x_i = S(p) = 2p$. Suppose only country $i = 1$ sets a climate policy while all the other countries (or, the consumers and the producers in these other countries) take the price p as given. Fossil fuel is tradable globally. Suppose country 1 internalizes the harm $H(.)$, as a function of all emissions. The other countries do not care about the harm, for simplicity.

(i) What is the main concern (or, what are the main concerns) if country 1 tries to sets its policies in order to reduce global emission?

Answer: That other countries will take opposite actions, so that the overall effect is diminished (i.e., carbon leakage).

(ii) Can you derive a formula for how another country responds, when country 1 reduces its supply?

Answer: Supply equal demand means:

$$\begin{aligned} y_1 + (n-1)[D - 2p] &= x_1 + (n-1)2p, \text{ so} \\ p &= \frac{y_1 - x_1 + (n-1)D}{4(n-1)}. \end{aligned}$$

Therefore,

$$\begin{aligned} \frac{\partial y_i}{\partial (y_1 - x_1)} &= \frac{-2}{4(n-1)} \text{ and} \\ \frac{\partial x_i}{\partial (y_1 - x_1)} &= \frac{2}{4(n-1)}. \end{aligned}$$

(iii) What is the carbon leakage rate in this case? What do you think can make the actual real-world leakage rate different from this number?

Answer: The leakage rate is 50 percent: If country 1 consumes one unit less, the rest of the world consumes half a unit more and extracts half a unit less, for example. In reality, the leakage rate will depend on the slopes of the curves, the time horizon, whether they indeed take the price as given, etc.