1. Tradable permits.

Suppose there are $n = 10$ identical firms and they emit pollution type A (CO_2) as well as B (SO_2). The aggregate harm for the consumers in the society is $c_A e_A + c_B e_B$, where $c_A = 5$, $c_B = 10$ and $e_A = \sum_i e_A^i$ and $\sum_i e_B^i$. But it is costly to reduce emission, so each firm benefits from emitting and has the profit function:

$$\pi_i (e_i) = e_A^i \left(30 - e_A^i\right) + e_B^i \left(30 - e_B^i\right).$$

(a) What is the socially optimal level of CO_2 (e_A^i) ?

Solution key: *clearly, foc with MC=MB gives*

$$\begin{aligned} \left(30 - 2e_A^i\right) &= 5 \\ \left(30 - 2e_B^i\right) &= 10. \end{aligned}$$

(b) Consider a permit market where a certain number of permits is given to the firms and they can trade them. Suppose further that each firm takes as given the prices for buying the right to pollute A, p_A, and B, p_B. Which quantity of e_B^i would firm i like to emit, as a function of the two prices?

Solution key: *clearly, foc gives*

$$\begin{aligned} \left(30 - 2e_A^i\right) &= p_A \\ \left(30 - 2e_B^i\right) &= p_B \end{aligned}$$

(c) To maximize social welfare, how many total (aggregate for all firms) permits for emitting CO_2 (A) should the industry receive as a whole?

Solution key: *with 10 firms and answer (a) above, we get*

$$\sum_i e_A^i = 10 \frac{30 - 5}{2} = 125.$$

(d) If the planner also distribute the optimal number of SO_2 permits (B), what is then the equilibrium market price, p_B?

Solution key: *Similarly for B*

$$\sum_i e_B^i = 10 \frac{30 - 10}{2} = 100.$$

Clearly (by (a) directly or combining the last equation with (b)), we have:

$$p_B = 10.$$

(e) Which allocation(s) of the permits among the firms would you suggest, if you were advising the government? Explain in words and justify your answer.

Solution key: *Equal, for fairness and to minimize the need for trade (given transaction costs / market power), so:*

$$\begin{aligned} e^i_A &= 12,5 \\ e^i_B &= 10. \end{aligned}$$

(f) How would your answer in the previous subquestion (e) change if the firms had heterogeneous abatement costs?

Solution key: *Then there might be a trade-off between fairness (suggesting equal costs or more to those with lowest profit, depending on the fairness criterion), and the need to minimize the transaction costs. Here, please use your judgement and evaluate the student's creativity/judgement when giving points.*

2. Conservation

There are two districts, A and B, and each $i \in \{A, B\}$ has a stock X_i of forest. If an amount $x_i \in [0, X_i]$ is illegally extracted, it is supplied to the market and each unit is sold at price p:

$$p = P - ax$$

where $x = x_A + x_B$. To discourage illegal logging on one unit of the forest, the expected penalty when illegally logging at that unit must be at least as large as the price p. The cost of raising the expected penalty at a unit of forest is c. The marginal value of conserving the forest $(X_i - x_i)$ is measured by the constant v_i for country i.

(a) Based on your intuition, what do you think is the effect of a larger c on x_A, and why?

Solution key: *Larger costs reduces a district's utility, and changes its decision: Less is conserved (so x_A increases), which reduces p.*

(b) District A may take x_B as given when deciding on x_A. Derive a formula showing how x_A depends on A's expectation of x_B. Explain the intuition for your formula.

Solution key: *Raising expected penalty is costly so for part of the forest that is protected (), we raise expected penalty up to the price, p. The cost of this is c. Note that p is endogenous, so therefore the probem is:*

$$\max_{x_A} -c(P - ax)(X_A - x_A) - v_A x_A.$$

with an interior solution we get the foc:

$$ca(X_A - x_A) + c(P - ax) - v_A = 0,$$

which gives

$$x_A = \frac{caX_A + c(P - ax) - v_A}{ca} \text{ or}$$
$$x_A = \frac{caX_A + c(P - ax_B) - v_A}{2ca}.$$

Intuition: If B extracts more, p decreases, and this reduces A's cost of conserving and protecting A's forest. Thus, A conserves more, or equivalently, A cuts less.

(c) Derive a formula showing how the total amount of logging, x, depends on c. Can you explain the similarity / difference to your answer in the first subquestion, above?

Solution key: *With the similar equation (FOC) for B we can sum the two and write:*

$$\begin{aligned} x &= \frac{ca\left(X_A+X_B\right)+c\left(2P-ax\right)-v_A-v_B}{2ca}, \text{ so} \\ x &= \frac{ca\left(X_A+X_B\right)+2cP-v_A-v_B}{3ca} \\ &= \frac{a\left(X_A+X_B\right)+2P-\left(v_A+v_B\right)/c}{3a}. \end{aligned}$$

So a larger c increases x, in line with the intuition in (a).

If countries were symmetric, the change in x is twice the change we were asked to informally discuss in (a).

If they are asymmetric, f.ex. $v_A < v_B$, then, from above:

$$\begin{aligned} x_A &= \frac{caX_A+c\left(P-ax\right)-v_A}{ca} \\ &= \frac{caX_A+cP-v_A}{ca}-\frac{a\left(X_A+X_B\right)+2P-\left(v_A+v_B\right)/c}{3a} \\ &= \frac{2aX_A-aX_B+P-\left(2v_A-v_B\right)/c}{3a}, \end{aligned}$$

so now the effect is asymmetric: If $v_A < v_B/2$, then a larger c decreases x_A. Intuition: A larger c means that the value (v's) are relatively smaller and thus a country increases extraction, everything else equal. But if $v_A < v_B/2$, then B increases extraction by most and the price becomes so small that A overall prefers to protect more (and reduce x_A) given the high c (taking into account the effect on B's expected behavior).

(d) Suppose that B is going to decide on x_B at some specific time, t, while A decide on x_A at a different time, t'. How is x_A, x_B and x depending on whether $t > t'$, $t < t'$, or $t = t'$?

Solution key: *Above we assumed simultaneous moves and our solution for x can be used to derive each district's extraction. We have also derived A's optimal choise given B's, as if B made the choice first. If indeed B moves first, then B takes advantage of this effect and B's problem is:*

$$\begin{aligned} & \max_{x_B} -c\left(P-ax_A-ax_B\right)\left(X_B-x_B\right)-v_Bx_B \\ = \; & \max_{x_B} -c\left(P-a\frac{caX_A+c\left(P-ax_B\right)-v_A}{2ca}-ax_B\right)\left(X_B-x_B\right)-v_Bx_B. \end{aligned}$$

The FOC for x_B is similar to before but with one additional term, since x_B influences x_A. Since B prefers a low p and thus a large x_A, B strategically reduces x_B below the result that would have been optimal if B took A's choice as given, since B would like to increase x_A. In equilibrium, $x_B < x_A$.

(e) Which of these sequences is preferred by district A?

Solution key: *Note: If B moves first, B can set x_B as if they moved at the same time and then x_A would be as in the simultaneous-move game. Since B chooses not to do this, B is better off moving first than with simultaneous move. The effect is that x_B is reduces and the larger p makes A worse off.*

Thus, each district strictly prefers to move first and strictly prefers the least to move last.

(f) Suppose Norway seeks to reduce x. How do you suggest that Norway does this, based on your model?

Solution key: *For this it is useful to consider the equation above:*

$$x = \frac{a(X_A + X_B) + 2P - (v_A + v_B)/c}{3a}.$$

Based on this, Norway can reduce x by a boycott if this can reduce P. Alternatively, as we studied in class, if Norway subsidizes conservation that will imply that the value of conserving forest is increases (in that the v's) are increased. Interestingly, the effect on x is identical if we subsidize by increasing v_A or v_B, even if the two districts are asymmetric in the v's or the sizes (the X'es).

3. Prices vs. Quantities

(a) Weitzman (1974) analyzed the choice between emission taxes and emission quotas when there is uncertainty in the marginal abatement cost Try to explain the trade-offs involved in words, and the intution for the optimal instrument choice.

Solution key*: For example:*

With a tax, firms set marginal abatement cost equal to the tax, so the firm's marginal cost is certain, but this maps into an abatement decision in an uncertain way if the firms' marginal cost shifts. A quota pins down emission level but the variance on the firms' cost is then large. The trade-off is thus whether we should minimize the uncertainty for the firm's cost curve (with a tax) or the emissions (with a quota).

If the marginal social cost of emission is more or less given (by a flat curve), then we can tolerate risk associated with total emission and the tax is better.

If instead the firm's marginal cost is more or less given (by a flat curve), while the marginal social cost of emission is large, then we are rather risk neutral wrt changes in the firms' cost, but not when it comes to the social damage, and then a quota is better as it reduces the uncertainty on the amount of emission.

(b) Can you illustrate the trade-off (from question (a)) in (x,y)-diagrams, where you measure the abatement levels at the horizontal axis?

i. Explain how emissions affect welfare and how this relation will be depicted in a typical integrated assessment model of climate change. You can, but you do not have to, use formulas, but make sure to explain all steps.

Emissions either show up directly in the production function or are released in an explicit energy sector, where energy then enters good production. Ceteris paribus, emissions allow a higher production level. More output implies more consumptions, which increases current welfare and it typically implies as well higher investment which increases future welfare.

But the emissions are released into the atmosphere where they increase the stock of CO2. CO2 causes radiative forcing/increases the atmospheric temperature. An increase in atmospheric temperature affects production/reduces output. The reduced output will generally imply a reduction of consumption, which directly reduces current welfare as well as a reduction of investment which reduces future welfare.

The above is sufficient, an even better answer would include something about the functional forms how these effects are included, like: Emissions enter a box model that increases atmospheric carbon. Temperature is an autoregressive process of own past temperature and ocean temperature as well as a radiative forcing term that depends on the atmospheric CO2 concentration. Production is a function of typical labor, capital, and emissions (or energy which causes emissions).

ii. Explain the difference between the "optimal" scenario in the DICE integrated assessment model and the "Stern" scenario. In what assumptions do they differ? Why? Which is "optimal"? How do they differ (or coincide) in their policy recommendations?

<u>Assumptions:</u>

Main point: The Stern scenario uses a (much) lower pure rate of time preference of 0.1%.

Would be nice: Stern argues that pure time preference is an ethical choice and we cannot reduce the weight on future generations, whereas Nordhaus tries to match observed impatience.

Not needed but great if stated (I have mentioned it but only emphasized pure time preference): The Stern scenario also reduces the desire for intertemporal consumption smoothing by increasing the intertemporal elasticity of substitution to unity.

<u>Policy recommendations:</u>

Because the Stern scenario pays relatively more attention to future damages (and less to consumption smoothing between the poorer present and the richer future, not required), the Stern scenario reduces emissions much more sharply and reaches full decarbonization of the economy much earlier. The implied carbon tax is substantially higher than in the “optimal” scenario.

<u>Which is optimal:</u>

Each scenario is optimal given the respective assumptions.