Compliance Technology and Self-Enforcing Agreements

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Abstract

These notes analyze a repeated game in which countries are polluting as well as investing in technologies. While folk theorems point out that the first best can be sustained as a subgame-perfect equilibrium when the players are sufficiently patient, we derive the second best equilibrium when they are not. This equilibrium is distorted in that countries over-invest in technologies that are "green" (i.e., strategic substitutes for polluting) but under-invest in adaptation and "brown" technologies (i.e., strategic complements to polluting). It is in particular countries which are small or benefit little from cooperation that will be required to strategically invest in this way. With imperfect monitoring or uncertainty, such strategic investments reduce the need for a long, costly punishment phase and the probability that punishment will be triggered.

1 Introduction

The purpose of this paper is to analyze how the society can succeed with climate treaties of the type signed in Kyoto, 1997, and in Paris, 2015. To be successful, in our view, any treaty must address two major challenges of climate change. First, in the absence of international enforcement bodies, any international treaty must be self-enforcing. In principle, sanctions could be imposed by threatening free-riders with trade barriers, the seizure of infrastructure, or armed conflicts, but such options are never on the table when climate negotiators meet. In the absence of such sanctions, one might hope that countries would follow the treaty in order to motivate other nations to cooperate in the future. This motivation, however, may not always be sufficiently strong. For example, for many years

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it was clear that Canada would not meet its commitments under the Kyoto Protocol. So, in 2011, it simply withdrew.

The second challenge to confront climate change is to develop new and environmentally friendly technology. The importance of new and green technology is recognized in the treaties, but there has been no attempt to negotiate or quantify how much countries should be required to invest in these technologies.[1] Instead, the negotiators focus on quantifying emissions or abatements and leave the investment decision to individual countries. Nevertheless, some countries do invest heavily in green technologies. The European Union aims for 20 percent of its energy to come from renewable sources by 2020, and to increase that number to 27 percent by 2030. China is a still larger investor in renewable energy and has invested heavily in wind energy and solar technology. Other countries have instead invested in so-called "brown" technology: Canada, for example, has developed its capacity to extract unconventional oil such as tar-sands while, at the same time, "Canada risks being left behind as green energy takes off" (The Globe and Mail, September 21st, 2009).

The interaction between the two challenges is poorly understood by economists as well as policymakers. To understand how treaties can address these challenges and how these challenges interact, we need a theory that allows technology investments as well as emission decisions to be made repeatedly. Since the treaty must be self-enforcing, strategies must constitute a subgame-perfect equilibrium (SPE).

There is no such theory in the literature, however, and many important questions have thus not been addressed. First, what is the best (i.e., Pareto optimal) SPE? Second, folk theorems have emphasized that even the first best can be sustained if the players are sufficiently patient, but what distortions occur if they are not? Third, will non-cooperative, selfish investments result in the optimal level of environmentally friendly technologies? Or are there reasons, beyond the traditional argument about technological spillovers, for including technology investments in the agreement? Which kinds of countries ought to invest the most, and in what kinds of technology?

To address these questions, we analyze a repeated extensive form game where, in every period, countries can invest in technology before deciding on emission levels. In the basic model, all decisions are observable and investments are selfish (i.e., there are no technological spillovers). Consequently, equilibrium investments would have been first best if the countries had committed to the emission levels. The first best can also be achieved if the discount factor is sufficiently high, in line with standard folk theorems. For smaller discount factors, however, the best SPE must be distorted. We show that the distortions take the form of over-investments in so-called "green" technologies, i.e., renewable energy or abatement technologies that can substitute for pollution.

[1] Chapter 16 of the Stern Review (2007) identified technology-based schemes as an indispensable strategy for tackling climate change. However, article 114 of the Cancun Agreement 2010 confirmed in Durban in 2011 states that "technology needs must be nationally determined, based on national circumstance and priorities". The 2015 Paris Agreement follows this tradition of letting countries decide on technology themselves.

The reason is that such over-investments reduce a country's temptation to cheat by emitting more rather than less, and they are thus necessary to satisfy the compliance constraint at the emission stage. For so-called "brown" technologies, including drilling technologies and other infrastructure investments that are strategic complements to fossil fuel consumption, investments must instead be less than the first-best amount to satisfy the compliance constraint. Our most controversial result may be that countries should also be required to invest less than the first-best amount in adaptation, i.e., technologies that reduce the environmental harm in a country (and thus also the country's benefit from continuing cooperation and less emissions).

Our analysis is normative since it describes how to define the best possible self-enforcing climate treaty. The comparative statics have important policy implications: Naturally, it is harder to motivate compliance if the discount factor is small, the environmental harm is small, or the investment cost is large. In these circumstances, the best SPE (i.e., the best self-enforcing treaty) requires countries to invest more in green technologies and less in adaptation or brown technologies. If countries are heterogeneous, the countries that are small and face less environmental harm are the most tempted to free-ride. Thus, for compliance to be credible, such countries must invest the most in green technologies or the least in adaptation and brown technologies. This advice contrasts the typical presumption that reluctant countries should be allowed to contribute less in order to satisfy their participation constraint. While the participation constraint requires that a country's net gain of cooperating is positive, the compliance constraint requires that the net gain outweighs the positive benefit of free-riding for one period, before the defection is observed. The compliance constraint is therefore harder to satisfy than the participation constraint in this model, and, to satisfy it, the reluctant countries must invest more in green technologies and less in adaptation and brown technologies.

2 A Model of Compliance Technology

A repeated game consists of a stage game and a set of times when the stage game is played. While we focus on the dynamics and the subgame-perfect equilibria (SPEs) in the next section, we here present the stage game and discuss important benchmarks.

There are n players or countries, indexed by i or $j \in N \equiv \{1, ..., n\}$. In this section, we allow the country size s_i to vary with i, although the average country size is normalized to one. At the emission stage, the countries simultaneously decide between emitting more or less. Let $b_i(\cdot)$ be the *per capita* benefit as an increasing and concave function of country i's per capita emission $g_i \in \{\underline{g}, \overline{g}\}$, while $c_i \sum_{j \in N} s_j g_j$ is the *per capita* environmental cost as a function of aggregate emissions. We assume that the countries' emission decisions constitute a prisoner dilemma. That is, a country i benefits from emitting more for any fixed emission from the other countries, $g_{-i} \equiv \sum_{j \neq i} s_j g_j$, but every country would be better off if everyone emitted less instead of more:

$$b_i\left(\underline{g}, r_i\right) - \left(s_i \underline{g} + g_{-i}\right) c_i < b_i\left(\overline{g}, r_i\right) - \left(s_i \overline{g} + g_{-i}\right) c_i \text{ and} \tag{1}$$

$$b_i\left(\underline{g}, r_i\right) - n\underline{g}c_i > b_i\left(\overline{g}, r_i\right) - n\overline{g}c_i. \tag{2}$$

Variable $r_i \in \Re_+$ is here capturing the fact that a country's benefit depends on more than its emission levels. We will refer to r_i as the country's technology, but r_i can actually be any variable which influences the benefit of emitting. In fact, we also allow r_i to influence a country's environmental cost by letting $c_i \equiv h_i c\left(r_i\right)$, where h_i measures country-specific environmental harm.

To simplify, we use subscripts for derivatives whenever this is not confusing, and we abuse notation by writing $b''_{i,gr} \equiv \partial\left[\left[b_i\left(\overline{g}, r_i\right) - b_i\left(\underline{g}, r_i\right)\right] / \left(\overline{g} - \underline{g}\right)\right] / \partial r_i$. To illustrate the relevance of technologies, we will occasionally refer to the following special types:

Definition 1.
(A) Adaptation technology is characterized by $b''_{i,gr} = 0$ *and* $c'\left(r_i\right) < 0$.
(B) Brown technology is characterized by $b''_{i,gr} > 0$ *and* $c'\left(r_i\right) = 0$.
(C) Clean technology is characterized by $b''_{i,gr} < 0$ *and* $c'\left(r_i\right) = 0$.

Adaptation technologies refer to technologies which help a country to adapt to a warmer or more volatile climate. Such technologies include agricultural reforms or more robust infrastructure, and may even capture the effects of some geo-engineering practices that have strictly local effects. In other words, adaptation technology is useful because it helps the country to adapt to the emissions, since it reduces the environmental cost of emissions, i.e., $c'\left(r_i\right) < 0$. Brown technology can be interpreted as drilling technology, infrastructure that is helpful in extracting or consuming fossil fuel, or other technologies that are complementary to fossil fuel consumption: the complementarity explains $b''_{i,gr} > 0$. In fact, any investment in existing polluting industries are brown, according to our definition. Clean technology, in contrast, is a strategic substitute for fossil fuel and reduces the marginal value of emitting another unit. Thus, $b''_{i,gr} < 0$. This is the case for abatement technology or renewable energy sources, for example. Both brown and clean technology may still be beneficial in that $\partial b_i\left(\cdot\right) / \partial r_i > 0$. We could also allow for an entire vector of technologies, with small modifications of the results.

We now endogenize the technology levels by letting the countries simultaneously, non-cooperatively decide on their r_i's at the investment stage, which is prior to the emission stage. The sequential timing follows whenever there is a positive lag $l > 0$ from the time at which the investment decision is made until the time the technology is ready to be used. The lag implies that if the actual marginal investment cost is, say, $\widehat{k}_i > 0$, then the present-discounted value of this investment cost is $k_i \equiv e^{\rho l}\widehat{k}_i$, when evaluated at the later time of the emission, and where ρ is the discount rate. With this reformulation, we do not need to discount explicitly between the two stages, and a country i's per capita utility can be written as:

$$u_i = b_i\left(g_i, r_i\right) - h_i c\left(r_i\right) \sum_{j \in N} s_j g_j - k_i r_i. \tag{3}$$

Note that it is without loss of generality to assume that the investment cost is linear in r_i, since r_i can enter a country's benefit function in arbitrary ways. If the investment cost were another function $\kappa_i\left(r_i\right)$, we could simply define $\widetilde{b}_i\left(g_i, \kappa_i\left(r_i\right)\right) \equiv b_i\left(g_i, r_i\right)$ and $\widetilde{c}_i\left(\kappa_i\left(r_i\right)\right) \equiv h_i c\left(r_i\right)$, treat $\kappa_i\left(r_i\right)$ as the decision variable, and then proceed as we do below.

Since investments are "selfish", in that they are without spillovers, each country is voluntarily investing the socially optimal amount, conditional on the emission levels. To see this, note that if it happened that $g_i = g \forall i$, the first best would require:

$$r_i^*\left(g\right) \equiv \arg\max_{r_i} b_i\left(g, r_i\right) - ngh_i c\left(r_i\right) - k_i r_i.$$

Clearly, $r_i^*\left(g\right)$ coincides with the noncooperative choice of r_i when country i takes the emission levels as given and everyone emits g. In other words, if the countries could solve their prisoner dilemma by committing to low emission levels in advance, then investments would be socially optimal and the first best would be implemented. These benchmark results provide some preliminary support for the presumption that it is not necessary to contract on investments in addition to emissions.

Proposition 0.
(i) In the first-best, $r_i^* \equiv r_i^*\left(g\right)$ and $g = \underline{g}$.
(ii) In the unique SPE of the stage game, $r_i^* \equiv r_i^*\left(g\right)$ and $g = \overline{g}$.
(iii) If countries had committed to $g_i = \underline{g}$, the outcome, including the equilibrium investments, would be first best.

Remark on stocks and reversibility. It is straightforward to reformulate this model and allow for stocks. Suppose the pollution stock accumulates over time and depreciates only at rate $q^g \in [0, 1]$. As long as the marginal cost of pollution is constant, the stock is payoff-irrelevant in that it does not influence future decisions, and the long-lasting cost of emission can already be accounted for today. To see this in the simplest way, let $c'\left(r_i\right) = 0$ and $\widetilde{h}_i$ be the cost of a marginally larger pollution stock. Then, the present-discounted cost of emitting another unit evaluated at the time of the emission is simply the constant $h_i \equiv \widetilde{h}_i c\left(r_i\right) / \left(1 - \delta q^g\right)$.

Analogously, suppose a fraction $q_i^r \in [0, 1]$ of country i's investments in technology survives to the next period. In this case, one benefit of investing today is that investments can be reduced in the next period. These cost-savings will not be payoff-relevant, however, in the sense that today's choice of r_i will not influence the level of technology in the future; it will only reduce the cost of obtaining that level of technology. Thus, if $\widetilde{k}_i$ were the cost of adding to the technology stock, we can already account for the future cost-savings today and write the net marginal investment cost as $k_i \equiv (1 - \delta q_i^r) \widetilde{k}_i$.[2]

If the q_i^r's are small, then the analysis below is unchanged since countries do need to invest in every period (even off the equilibrium path). The investments are then, in effect, reversible. These assumptions are reasonable in the very long-run context of climate change, in our view.[3] Furthermore, if the q_i^r's were instead large, it would actually be easier to motivate countries to emit less (see Section 3.2 for a further discussion). By ignoring stocks and instead considering the one-period utilities given by (3), it is straightforward to interpret our dynamic game as a simple repeated game.

Remark on assumptions and extensions. In (3), we have assumed that technology investments are selfish in that such investments only affect the investing country's technology. We have also abstracted away from uncertainty and policy instruments, and we permit only two possible emission levels. These assumptions allow us to derive key insights in a simple setting. Starting from Sections 4, we relax all these assumptions and shows that our main results continue to hold. The Appendix discusses time-varying parameters, rather than the stationary ones in our basic model. The key assumptions behind our results are that investments are to some extent observable and decided on before the technology can be used.

3 Self-enforcing Agreements

While the stage game is described above, we here assume that the stage game is played repeatedly in every period $t \in \{1, 2, ..., \infty\}$. We let $\delta \in [0, 1)$ be the common discount factor and $v_i^t = (1 - \delta) \sum_{\tau=t}^{\infty} \delta^{\tau - t} u_i^\tau$ measures country i's continuation value at time t (normalized to per-period utility). The goal of this section is to characterize the "best" (that is, the Pareto optimal) subgame-perfect equilibrium (SPE). Since all parameters are invariant in time, the Pareto optimal SPE is stationary and we skip t-superscripts for simplicity. The Appendix allows for time-varying parameters and contains the proofs which are not in the text.

3.1 The Worst Equilibrium

Note that there is a unique SPE in the one-period stage game described above. Given (1)-(2), more emissions at the emission stage are a dominant strategy for all countries; at the investment stage, emissions are individually optimally set

[2] Of course, the stocks would be payoff-relevant if the environmental harm or the investment costs were nonlinear functions. Allowing for payoff-relevant stocks is reasonable but they open up a host of other issues (some of them are discussed by Harstad 2012 and 2015) that are tangential to the points we are making here.

[3] In long-run problems such as climate change, countries must expect to invest repeatedly partly to maintain the infrastructure and the capacity to produce renewable energy, but also to invest in research and development effort. See, for example, Dockner and Long (1993) or Dutta and Radner (2004).

to $r_i = r_i^* (\overline{g})$. Clearly, these strategies also survive as an SPE in the infinitely repeated game in which the stage game is played in every period. In fact, in every SPE in which $g_i = \overline{g}$, we must have $r_i = r_i^* (\overline{g})$. For any other equilibrium candidate r_i, country i could benefit from deviating to $r_i^* (\overline{g})$ without any risk of reducing v_i. In other words, from country i's point of view, emitting more is the other players' worst strategy (i.e., the minmax strategy), and an SPE cannot be sustained with lower utilities. We refer to this equilibrium as the business-as-usual (BAU) equilibrium and label it with superscript b.

Proposition 1. *The worst SPE is BAU:* $\left(r_i^b, g_i^b\right) = \left(r_i^* (\overline{g}), \overline{g}\right)$*. This equilibrium always exists.*

Of course, the worst equilibrium might be used as a threat to enforce better equilibria. In fact, if a pair (r_i, g_i) can be sustained in *some* SPE, then these actions can (also) be sustained in an SPE where any deviation requires the countries to revert to the worst possible SPE, i.e., BAU forever. Therefore, we can with no loss of generality focus on such simple trigger strategies.

Corollary 1. *If* (r_i, g_i) *can be sustained as an SPE, then it can be sustained as an SPE in which any deviation triggers an immediate reversion to BAU.*

3.2 The Best Equilibrium

Corollary 1 implies that we can, without loss of generality, rely on SPEs that are enforced by simple trigger strategies. We are particularly interested in Pareto optimal SPEs in which every single country emits less. When such an equilibrium is unique, we refer to it as "the best equilibrium".

Definition 2. *An equilibrium is referred to as best if and only if it is the unique Pareto optimal SPE satisfying* $g_i = \underline{g}\ \forall i \in N$.

Since there are two decision-stages in each period, we must consider the temptation to deviate at each of them. At the investment stage, a country must compare the continuation value (v_i) it receives from complying with the SPE by investing r_i, to the maximal continuation value it could possibly obtain by deviating. Since deviating at the investment stage implies that every country will emit more beginning from this period, the compliance constraint at the investment stage is the following:

$$\frac{v_i}{1-\delta} \geq \max_{r_i} b_i\left(\overline{g}, r_i\right) - h_i c\left(r_i\right) n\overline{g} - k_i r_i + \frac{\delta v_i^b}{1-\delta}. \qquad (\mathrm{CC}_i^r)$$

The right-hand side of (CC_i^r) is maximized when $r_i = r_i^* (\overline{g})$, implying that the right-hand side is simply v_i^b. Thus, (CC_i^r) simplifies to $v_i \geq v_i^b$. In other words, as long as every country prefers the SPE to BAU, the compliance constraint for the investment is trivially satisfied.

At the emission stage, the investment cost for this period is sunk and the compliance constraint becomes:

$$b_i\left(\underline{g}, r_i\right) - h_i c\left(r_i\right) n\underline{g} + \frac{\delta v_i}{1-\delta} \geq b_i\left(\overline{g}, r_i\right) - h_i c\left(r_i\right)\left(s_i \overline{g} + \left(n - s_i\right)\underline{g}\right) + \frac{\delta v_i^b}{1-\delta}, \tag{CC_i^g}$$

which implies that:

$$\delta \geq \widehat{\delta}_i\left(r_i\right) \equiv 1 - \frac{v_i - v_i^b}{b_i\left(\overline{g}, r_i\right) - b_i\left(\underline{g}, r_i\right) - s_i h_i c\left(r_i\right)\left(\overline{g} - \underline{g}\right) + v_i - v_i^b}. \tag{4}$$

In the limit as $\delta \to 1$, (CC_i^g) approaches the condition (CC_i^r), i.e., $v_i \geq v_i^b$. For any $\delta < 1$, however, (CC_i^g) is harder to satisfy than (CC_i^r) because of the free-riding incentive. It is not sufficient that the SPE is better than BAU. In addition, the discount factor must be large or the temptation to free-ride must be small.

As indicated in (4), the threshold for the discount factor generally depends on the equilibrium r_i. For first-best investments, $r_i^* \equiv r_i^*\left(\underline{g}\right)$, the threshold is $\overline{\delta}_i \equiv \widehat{\delta}_i\left(r_i^*\right) < 1$. Thus, if $\delta \geq \overline{\delta}_i$ holds for every $i \in N$, every (CC_i^g) holds for first-best investment levels and the best SPE is simply the first best.

If $\delta < \overline{\delta}_i$, however, ($\text{CC}_i^g$) does not hold for $r_i = r_i^*$. To ensure that compliance constraint at the emission stage is satisfied, the temptation to free-ride must be reduced by insisting on an r_i so that $\widehat{\delta}_i\left(r_i\right) \leq \delta$. This requires $r_i > r_i^*$ if $\widehat{\delta}_i'\left(r_i^*\right) < 0$, or $r_i < r_i^*$ if $\widehat{\delta}_i'\left(r_i^*\right) > 0$. It is easy to see that:

$$\widehat{\delta}_i'\left(r_i^*\right) < 0 \text{ if } b_{i,gr}'' < s_i h_i c'\left(r_i^*\right); \tag{G_i}$$

$$\widehat{\delta}_i'\left(r_i^*\right) > 0 \text{ if } b_{i,gr}'' > s_i h_i c'\left(r_i^*\right). \tag{NG_i}$$

Under condition (G_i) for "green" technology, *more* investments relax the compliance constraint by reducing the lower threshold $\widehat{\delta}_i\left(r_i\right)$. Above this threshold, less emission can be sustained as an equilibrium outcome. Under condition (NG_i) for "non-green" technologies, *less* investments relax the compliance constraint.

As the discount factor $\delta < \overline{\delta}_i$ declines further, (CC_i^g) becomes even harder to satisfy and requires investment levels that increasingly differ from the first-best level. Once the discount factor is smaller than a lower threshold referred to as $\underline{\delta}_i < \overline{\delta}_i$, $g_i = \underline{g}$ can no longer be sustained in an SPE. The thresholds are explained in the Appendix, which includes the proofs of the following results.

Proposition 2. *An SPE exists in which* $g_i = \underline{g}\ \forall i \in N$ *if and only if* $\delta \geq \max_i \underline{\delta}_i$. *In this case, the Pareto optimal SPE is unique and it is characterized as follows:*
(i) If $\delta \geq \overline{\delta}_i$, *then* $r_i = r_i^*$ *is first best.*

(ii) If $\delta < \overline{\delta}_i$*, then:*[4]

$$r_i = \min \widehat{\delta}_i^{-1}(\delta) > r_i^* \text{ under } (G_i);$$
$$r_i = \max \widehat{\delta}_i^{-1}(\delta) < r_i^* \text{ under } (NG_i).$$

The result that the first best is achievable when the discount factor is sufficiently large is standard in the literature on repeated games.[5] Thus, the contribution of Proposition 2 is to characterize the distortions that must occur if the discount factor is small. To understand the importance of this characterization, it is useful to once again refer to the special cases in Definition 1. Clearly, condition (G_i) is satisfied for clean technology, while (NG_i) is satisfied for adaptation and brown technology. In other words, if the first best cannot be achieved, countries are only motivated to comply with an agreement and emit less if they have, in advance, invested less in adaptation or brown technologies, or more in clean technologies. Intuitively, the temptation to free-ride is larger after investing in adaptation or brown technology, but smaller after investing in clean technology.

Corollary 2. *Compared to the first best, the Pareto optimal SPE requires the countries to:*
(i) under-invest in adaptation technology;
(ii) under-invest in brown technology;
(iii) over-invest in clean technology.

These strategic investment levels, which are clearly inefficient conditional on the emission levels, must be part of the self-enforcing agreement in the same way as are the small emission levels: any deviation must be triggered by a reversion to BAU.

Distorting the choice of technology in this manner reduces the temptation to deviate from the equilibrium. Note that it is *not* necessary to require so little or so much investment that emitting less becomes a dominant strategy: it is sufficient to ensure that the benefit of emitting more is smaller (although still positive) than the present discounted value of continuing cooperation. Also, note that if technology were long-lasting and not reversible, it would be easier to satisfy the compliance constraint. The reason is simply that the deviation payoff would be less than the BAU payoff if the investments cannot adjust easily.

3.3 Comparative Statics

We are finally ready to discuss important comparative statics. The compliance constraints are not only functions of the technology. They also depend on the

[4] In the following equations, the operators min and max are added since $\widehat{\delta}_i^{-1}(\delta)$ is a correspondence and, of the two values of $\widehat{\delta}_i^{-1}(\delta)$, it is optimal to select the one closest to r_i^*.

[5] The result that folk theorems hold in repeated extensive-form games is due to Rubistein and Wolinsky (1995), who show that the Fudenberg and Maskin (1986) folk theorem can be generalized.

other parameters of the model. Compliance is particularly difficult to motivate if the cost of reverting to BAU is small. The cost of BAU is small if relatively few countries are polluting (i.e., n is small), if the environmental harm is small (i.e., h_i is small), or if the countries heavily discount the value of cooperating in the future (i.e., δ is small). In all these situations, a country i will not find it optimal to comply unless it is requested to invests less in adaptation and brown technologies, or more in clean technologies. The result that investments in clean technologies should decline with the discount factor, for example, is certainly at odds with traditional results in economics.

Furthermore, we show that *all* investments should increase with the investment cost k_i. For adaptation and brown technologies, we have $r_i < r_i^b$. A larger k_i thus reduces the value of BAU (v^b) compared to cooperation, and makes the compliance constraint easier to satisfy. Thus, when k_i increases, r_i can increase towards r_i^* without violating (CC_i^g). For clean technologies on the other hand, we have $r_i > r_i^b$, and a larger k_i again reduces the value of cooperating relative to BAU. The compliance constraint becomes harder to satisfy. As a response, countries must invest even more in clean technologies to satisfy (CC_i^g) when k_i increases.

Proposition 3. *Suppose $\delta \in \left[\max_j \underline{\delta}_j, \overline{\delta}_i\right)$ and consider the Pareto optimal SPE.*
(i) If k_i increases, then r_i increases.
(ii) If δ or s_i decreases, then $|r_i - r_i^*|$ increases.
(iii) If n or h_i decreases, then r_i increases for clean technologies, while r_i decreases for brown technologies, and, assuming $(c')^2/c'' < c$, also for abatement technology.[6]

Note that the comparative statics are country-specific. When environmental harm is heterogeneous, countries subject to the least harm (i.e., those with the smallest h_i) are most tempted to emit more. These "reluctant" countries must be required to invest little in adaptation and brown technologies or more in green technologies. Similarly, a small country is tempted to emit more because it internalizes less of the total harm if it free-rides one period. Small countries must thus be required to invest little in adaptation and brown technology or more in clean technology to counter their incentive to free ride.

Corollary 3. *In the Pareto optimal SPE, the smallest and the most reluctant countries invest the least in adaptation and brown technology, and they invest the most in clean technology.*

The result that countries which are small or have high investment costs ought to invest more in clean technology is in stark contrast to the idea that countries should contribute according to ability and responsibility.

[6] If, instead, $(c')^2/c'' > c$, investing in adaptation technology is so productive that if n, g, or h_i increases, country i's environmental harm $ngh_i c(r_i)$ actually declines when the changes induce the country to invest more in adaptation technology. This is unrealistic, in our view.

The result that countries which are reluctant to cooperate (in that the harm h_i is small) ought to invest more is similarly in contrast to the intuition that such countries must be given a better deal to make them cooperate.

It is true, of course, that countries that are reluctant either because they are small or have high investment costs, or because they are subject to less harm, have participation constraints (i.e., the constraint $v_i \geq v_i^b$) that are more difficult to satisfy than for other countries. However, as we have shown above, the compliance constraint (CC_i^g) is more difficult to satisfy than the participation constraint. Although all countries must obviously benefit from cooperation compared to BAU, they must in addition benefit from cooperation at the stage when they face the possibility of free-riding one period before the others revert to BAU.

Interestingly, the international community appears to recognize these problems. The 2015 Paris Agreement, for instance, emphasizes the importance of transferring technology to the less-developed countries (Articles 66-71). Such technology transfers seem like an ideal way of satisfying the compliance constraints of reluctant countries without violating their participation constraints.

4 Policy Instruments and Continuous Emission Levels

In this section, we study the optimal use of policy instruments, and we permit the emission level to be a continuous variable. It is natural to make these two extensions at the same time, since we cannot pin down a unique emission tax if the emission level continues to be a binary variable. (For example, any sufficiently large emission tax ensures that $\underline{g}$ is preferred to $\overline{g} > \underline{g}$.)

We assume that country i's investment subsidy, ς_i, is set by i just before the investment stage in each period, and it is observable by all countries. The actual investment is made by private investors who receive the subsidy ς_i in addition to the price paid by the consumers. The emission tax, τ_i, is set just before the emission stage, and it represents the cost of polluting paid by the consumers. If the taxes are collected and the subsidies are paid by the national governments, they do not represent actual costs or revenues—from the government's perspective—and their only effect is to influence the decisions g_i and r_i. The agreement between the countries then amounts to setting domestic taxes/subsidies such that the desired SPE is implemented.

Allowing for a continuous g_i complicates the analysis. To proceed, we restrict attention to the case in which g_i and r_i are perfect substitutes in a linear-quadratic utility function:[7]

$$u_i = -\frac{B}{2}\left(\overline{y} - (g_i + r_i)\right)^2 - \frac{K}{2} r_i^2 - c \sum_{j \in N} g_i,$$

[7] This utility function is also considered in Battaglini and Harstad (2014), who do not study SPEs, but instead the Markov-perfect equilibria when countries can commit to the emission levels. The first best and the BAU equilibrium are as in that paper, of course.

where B and K are positive constants. Here, $\overline{y}$ is a country's bliss level for consumption, and consumption is the sum of g_i (energy from fossil fuels) and r_i (energy from renewable energy sources). Since $\partial^2 u_i/\partial g_i \partial r_i < 0$, we explicitly consider only clean technology. We can easily reformulate the utility function such that the investment cost becomes linear,[8] although there is no need to do so here.

Since the emission tax is the only cost of consuming fossil fuel, g_i is chosen by the consumers to satisfy the first-order condition:

$$B\left(\overline{y} - (g_i + r_i)\right) = \tau_i.$$

The left-hand side is also equal to the consumer's willingness to pay for green technology, so private investors invest according to the first-order condition:

$$Kr_i = B\left(\overline{y} - (g_i + r_i)\right) + \varsigma_i = \tau_i + \varsigma_i. \tag{5}$$

Note that the first-best outcome is

$$r^* = \frac{cn}{K} \text{ and } g^* = \overline{y} - \frac{cn}{B} - r^*,$$

which coincides with the equilibrium when the tax and the subsidy are equal to their first-best values:

$$\varsigma^* = 0 \text{ and } \tau^* = cn.$$

In the first best, the emission tax is set at the Pigouvian level and there is no need to additionally regulate investments, since the investors capture the entire surplus associated with their technology investments.

The BAU equilibrium is (the unique SPE in the one-period game):

$$r^b = \frac{c}{K} \text{ and } g^b = \overline{y} - \frac{c}{B} - r^b,$$

which is equivalent to

$$\varsigma^b = 0 \text{ and } \tau^b = c.$$

Thus, the investment subsidy is zero in the first best as well as in BAU.

To follow the same line of reasoning as in the rest of the paper, we here only consider SPEs enforced by the threat of reverting to BAU, despite the fact that BAU is not the harshest penalty when $g < g^b$ is possible. Furthermore, we consider only symmetric SPEs, despite the fact that there can also be asymmetric SPEs that are Pareto optimal.

Naturally, the first best can be achieved when the discount factor is sufficiently large. When δ falls, however, each country finds it tempting to introduce a smaller emission tax than the first-best one. Once δ falls to some threshold, $\overline{\delta}$, the emission-stage compliance constraint starts to bind. For smaller discount

[8] To see this, simply define $\widetilde{r}_i = r_i^2/2$ and rewrite to $u_i = -\frac{B}{2}\left(\overline{y} - \left(g_i + \sqrt{2\widetilde{r}_i}\right)\right)^2 - K\widetilde{r}_i - c\sum_{j\in N} g_i$.

factors, the emission tax must be allowed to fall to satisfy the compliance constraint. The associated increase in emissions can be mitigated by introducing an investment subsidy.

Note that the investment-stage compliance constraints will never bind first. As soon as one country deviates by setting a different investment subsidy, investors in all countries anticipate that cooperation will break down and demand for their technology at the emission stage will be reduced. This lowers investments everywhere, not only in the deviating country. Deviating at the investment stage immediately gives the deviator the BAU payoff, plus the benefit of the other countries' larger investments induced by their subsidies. These subsidies are zero for $\delta \geq \overline{\delta}$ and are small for discount factors close to $\overline{\delta}$. Consequently, some $\underline{\delta} < \overline{\delta}$ exists such that the compliance constraint at the investment stage is not binding when $\delta \in (\underline{\delta}, 1)$. (The proof in the Appendix derives both thresholds.)

Proposition 4. *Consider the symmetric Pareto optimal SPE sustained by the threat of reverting to BAU if a country deviates.*
(i) If $\delta \geq \overline{\delta}$, *the equilibrium is first best:* $\tau = cn$ *and* $\varsigma = 0$.
(ii) If $\delta \in \left[\underline{\delta}, \overline{\delta}\right)$, *the equilibrium is*:

$$\begin{aligned} \tau &= cn - \phi(\delta) \text{ and} \\ \varsigma &= \phi(\delta), \text{ where} \\ \phi(\delta) &\equiv c(n-1)\left(1 - \delta - \sqrt{\delta^2 + \delta B/K}\right) \geq 0. \end{aligned}$$

The function $\phi(\delta)$ decreases toward zero when δ increases to $\overline{\delta}$.[9]

Corollary 4. *The sum of the equilibrium emission tax and the investment subsidy is, for every* $\delta \geq \underline{\delta}$, *equal to nc, the first-best Pigouvian tax level.*

5 Conclusions

To confront global climate change, an environmental treaty must address two primary challenges. The treaty must be self-enforcing, and it must lead to the development of green technology. This paper analyzes these challenges in a joint

[9] The proposition implies that the equilibrium investment level, r_i, given by (5), stays unchanged as the discount factor falls. On the one hand, the fact that a larger g must be tolerated implies that it becomes optimal to invest less in clean technology. On the other hand, the countries can dampen the increase in g by requesting countries to invest more in green technology upfront. These two effects cancel each other out when g and r are perfect substitutes. Relative to the ex post optimal level, however, it is clear that $r - r^*(g)$ is positive and increases as δ falls, just as the equilibrium investment subsidy. The optimal investment level, conditional on the emission level g_i, is decreasing in g_i and given by:

$$r^*(g_i) = \frac{B(\overline{y} - (g_i + r_i))}{K} = \frac{B(\overline{y} - g_i)}{B + K}.$$

framework and uncovers policy-relevant interactions between them. Specifically, we demonstrate that when free-riding is tempting and cooperation difficult to sustain, the best self-enforcing treaty requires countries to over-invest in "green" technology, reducing the temptation to pollute, or under-invest in adaptation or "brown" technology that would have made free-riding more attractive. When countries are heterogeneous, it is particularly countries that are small or reluctant to cooperate (because their environmental harm is relatively small, for example) that are most tempted to pollute. To ensure that compliance by these countries is credible, small or reluctant countries must invest the most in green technology, or the least in adaptation and brown technology.

In a time when the world struggles to develop and reach an agreement on a global climate change treaty, it is natural that the motivation for our analysis has mainly been normative. We believe that the international community has not yet implemented the best possible self-enforcing treaty, and thus do not expect that our predictions are directly observable or consistent with the facts of today. That said, our theory is testable: we do provide a number of predictions that may eventually be compared to the data. Our assumptions are also in line with the facts: Policymakers do have few sanctions available (meaning a treaty must be self-enforcing) and they certainly consider the development of technology to be of great importance. Furthermore, some of the countries that have invested the most in green technology (notably the European Union) are also the ones that have complied with the Kyoto Protocol to the largest extent.[10] Other countries that have instead invested in brown technology (notably Canada and Australia) ended up not complying or increased emissions. Finally, although the 2015 Paris Agreement does not specify how much countries ought to invest in green technology, it does recognize the importance of technology and of technology transfers to less developed countries (Articles 66-71). In our model, such technology transfers appear to be an ideal way of satisfying compliance constraints without violating participation constraints. Future research should empirically clarify the interaction between technology, emissions, and compliance to test the model's predictions.

Theoretical research should also continue. Our simple workhorse model has proven to be sufficiently tractable to be extended in many ways, but our approach is still only a first cut. We have simplified tremendously by abstracting away from payoff-relevant stocks of pollution or technologies. We have focused exclusively on the Pareto optimal subgame-perfect equilibrium, although the actual transition toward such a treaty appears to be characterized by high transaction costs and multiple wars of attritions. By focusing on the best subgame-perfect equilibrium, we have also abstracted from the possibility of opting out of the negotiations at the beginning of the game. When countries are heterogeneous, it may actually be optimal, even for the countries that cooperate, to exclude certain reluctant countries, since these countries may, with some probability, cheat and thus trigger a costly and long-lasting punishment phase. One

[10] "EU over-achieved first Kyoto emissions target" (October 9th, 2013, European Commission: ec.europa.eu/clima/news/articles/news_2013100901_en.htm).

of the goals with this project has been to provide a tractable workhorse model that can be developed along these lines in future research.

6 Appendix

The proofs of the basic results, Propositions 2 − 3, allow for time-varying parameters.

Proofs of Propositions 0-1.

These proofs are in the text.

Proof of Proposition 2.

Since investments are selfish, the Pareto optimal SPE satisfying $g_i = \underline{g}$ is for each $i \in N$ specifying the r_i^t closest to $r_i^{*,t}$ satisfying the compliance constraints. That is, the Pareto optimal SPE solves:

$$\max_{r_i^t} u_i^t = b_i^t\left(r_i^t, \underline{g}\right) - n^t c_i^t\left(r_i^t\right)\underline{g} - k_i^t r_i^t \quad \text{s.t.}$$

$$v_i^t - v_i^{b,t} \geq 0, \tag{CC$_i^{r,t}$}$$

$$\Delta_i^t \equiv -b_i^t\left(\overline{g}, r_i^t\right) + b_i^t\left(\underline{g}, r_i^t\right) + s_i^t h_i^t c\left(r_i^t\right)\left(\overline{g} - \underline{g}\right) + \frac{\delta\left(v_i^{t+1} - v_i^{b,t+1}\right)}{1-\delta} \geq 0. \tag{CC$_i^{g,t}$}$$

(i) Since $v_i^t > v_i^{b,t}$ at $r_i^{*,t}$, both conditions hold if δ is close to 1. A binding (CC$_i^{g,t}$) defines $\widehat{\delta}_i^t\left(r_i^t\right)$ implicitly, and if $\delta \geq \overline{\delta}_i^t \equiv \widehat{\delta}_i^t\left(r_i^{*,t}\right)$, then $r_i^{*,t}$ satisfies both compliance constraints.

(ii) As soon as δ declines below the level $\overline{\delta}_i^t$, (CC$_i^{g,t}$) binds (before (CC$_i^{r,t}$) does). The problem above is then a Kuhn-Tucker maximization problem which can be written as:

$$\max_{r_i^t} u_i^t + \lambda_i^t\left(-b_i^t\left(\overline{g}, r_i^t\right) + b_i^t\left(\underline{g}, r_i^t\right) + s_i^t h_i^t c\left(r_i^t\right)\left(\overline{g} - \underline{g}\right) + \frac{\delta\left(v_i^{t+1} - v_i^{b,t+1}\right)}{1-\delta}\right),$$

where $\lambda_i^t > 0$ is the shadow value of satisfying a strictly binding (CC$_i^{g,t}$). The first-order condition for an interior r_i^t is:

$$0 = \frac{\partial u_i^t}{\partial r_i^t} + \lambda_i^t \frac{\partial}{\partial r_i^t}\left[-b_i^t\left(\overline{g}, r_i^t\right) + b_i^t\left(\underline{g}, r_i^t\right) + s_i^t h_i^t c\left(r_i^t\right)\left(\overline{g} - \underline{g}\right)\right], \tag{6}$$

while the second-order condition is satisfied for u_i^t sufficiently concave. Since $r_i^{*,t}$ is defined by $\partial u_i^t / \partial r_i^t = 0$, r_i^t must increase above $r_i^{*,t}$ to satisfy this first-order condition if the second term in (6) is positive, i.e., if:

$$\frac{\partial}{\partial r_i^t} \frac{b_i^t\left(\overline{g}, r_i^t\right) - b_i^t\left(\underline{g}, r_i^t\right)}{\overline{g} - \underline{g}} < s_i^t h_i^t c'\left(r_i^t\right), \tag{G$_i^t$}$$

while r_i^t must decrease below $r_i^{*,t}$ if

$$\frac{\partial}{\partial r_i^t} \frac{b_i^t\left(\overline{g}, r_i^t\right) - b_i^t\left(\underline{g}, r_i^t\right)}{\overline{g} - \underline{g}} > s_i^t h_i^t c'\left(r_i^t\right), \tag{NG$_i^t$}$$

As δ declines further, $(\mathrm{CC}_i^{g,t})$ can only be satisfied if $\left|r_i^t - r_i^{*,t}\right|$ increases more. Eventually, δ becomes so small than either (i) $\left|r_i^t - r_i^{*,t}\right|$ becomes so large that $(\mathrm{CC}_i^{r,t})$ is violated, (ii) u_i^t and thus $v_i^{t'}$ is reduced so much that $(\mathrm{CC}_i^{g,t'})$ is violated at some earlier date $t' < t$, or (iii) δ reaches zero. We let $\underline{\delta}_i^t$ measure the maximum of these three thresholds. Clearly, $\underline{\delta}_i^t \in \left[0, \overline{\delta}_i^t\right)$. *QED*

Proof of Proposition 3.

If $\delta < \overline{\delta}_i^t$, the Pareto optimal SPE satisfying $g_i^t = \underline{g}$ ensures that r_i^t solves $\widehat{\delta}_i^t(r_i^t) = \delta$, so that $(\mathrm{CC}_i^{g,t})$ binds. As long as $(\mathrm{CC}_i^{g,t})$ binds, we can simply differentiate the left-hand side of $(\mathrm{CC}_i^{g,t})$ to learn how r_i^t must change with the other parameters at time t:

$$\begin{aligned} \frac{\partial r_i^t}{\partial h_i^t} &= -\frac{\Delta_{i,h}^t}{\Delta_{i,r}^t} = -\frac{s_i^t c(r_i^t)\left(\overline{g} - \underline{g}\right)}{\Delta_{i,r}^t}; \\ \frac{\partial r_i^t}{\partial s_i^t} &= -\frac{\Delta_{i,s}^t}{\Delta_{i,r}^t} = -\frac{h_i^t c(r_i^t)\left(\overline{g} - \underline{g}\right)}{\Delta_{i,r}^t}, \text{ where} \\ \Delta_{i,r}^t &= \partial\left[-b_i^t\left(\overline{g}, r_i^t\right) + b_i^t\left(\underline{g}, r_i^t\right)\right]/\partial r_i^t + s_i^t h_i^t\left(\overline{g} - \underline{g}\right)c'\left(r_i^t\right). \end{aligned} \tag{7}$$

Thus, $\partial r_i^t/\partial h_i^t$ and $\partial r_i^t/\partial s_i^t$ are both negative if (G_i^t) holds (then, $\Delta_{i,r}^t > 0$), and otherwise they are positive.

If we could write $b_i^t(\cdot) = \theta_i^t b(\cdot)$, where the importance of consumption, θ_i^t, could be particularly high at times of recessions, then we could show that countries ought to invest more at such times in clean technologies, and less in adaptation or brown technologies, for compliance to be credible:

$$\frac{\partial r_i^t}{\partial \theta_i^t} = -\frac{b\left(\overline{g}, r_i^t\right) - b\left(\underline{g}, r_i^t\right)}{\Delta_{i,r}^t}.$$

If we differentiated Δ_i^t with respect to k_i^t or n^t and r_i^t, we would clearly get $\partial r_i^t/\partial k_i^t = \partial r_i^t/\partial n^t = 0$. The explanation for $\partial r_i^t/\partial k_i^t = 0$, for example, is that r_i^t must be set to satisfy $(\mathrm{CC}_i^{g,t})$, and the investment cost at time is sunk and thus irrelevant at the emission stage in period t. Of course, a larger k_i^t changes $v_i^{t'}$ and $v_i^{b,t'}$ with $t' \leq t$, and therefore the compliance constraints at the earlier periods. Similarly, changes in h_i^t or s_i^t will influence earlier investments as well as r_i^t. To illustrate the total effects without unnecessary notations we henceforth assume that all parameters are time-invariant, as in the model and the text above, and we thus skip the subscripts t.

Stationary parameters: We must now take into account the effects on the continuation value, and we write:

$$\Delta_{i,r} = \frac{\partial}{\partial r_i}\left[-b_i\left(\overline{g}, r_i\right) + b_i\left(\underline{g}, r_i\right)\right] + s_i h_i\left(\overline{g} - \underline{g}\right)c'\left(r_i\right) + \frac{\delta}{1-\delta}\frac{\partial u_i}{\partial r_i}.$$

Under (**??**), $\Delta_{i,r} > 0$ at r_i^* and $\Delta_{i,r}$ remains positive as long as a larger r_i weakens (CC_i^g) (that is, for $\delta > \underline{\delta}_i$). Similarly, under (**??**), $\Delta_{i,r} < 0$ at r_i^* and $\Delta_{i,r}$ remains negative as long as a smaller r_i weakens (CC_i^g) (that is, for $\delta > \underline{\delta}_i$).

(i) Effect of k_i: The compliance constraint depends on k_i because:

$$v_i - v_i^b = -\left[b_i\left(\overline{g}, r_i^b\right) - b_i\left(\underline{g}, r_i\right)\right] + nh_i\left[c\left(r_i^b\right)\overline{g} - c\left(r_i\right)\underline{g}\right] - k_i\left[r_i - r_i^b\right].$$

Suppose, as a start, that r_i does not change in k_i. Then, $\partial u_i/\partial k_i = -r_i$. Furthermore, from the Envelope theorem, $\partial u_i^b/\partial k_i = -r_i^b$. Thus:

$$\frac{dr_i}{dk_i} = -\frac{\Delta_{i,k}}{\Delta_{i,r}} = \frac{\delta}{1-\delta}\frac{r_i - r_i^b}{\Delta_{i,r}}. \tag{8}$$

To see the sign of $r_i - r_i^b$, assume, for a moment, that r_i remains at r_i^*. If so, all investment levels are given by the first-order condition:

$$\frac{\partial b_i\left(g, r_i\right)}{\partial r_i} - h_i c'\left(r_i\right) n g = k_i,$$

which we can differentiate to get

$$\frac{dr_i}{dg} = \frac{-\frac{\partial^2 b_i(g,r_i)}{\partial r_i \partial g} + h_i c'\left(r_i\right) n}{\frac{\partial^2 b_i(g,r_i)}{\left(\partial r_i\right)^2} - h_i c''\left(r_i\right) g n}, \tag{9}$$

where the denominator is simply the second-order condition with respect to r_i, which must be negative. With this, we have:

$$r_i^* - r_i^b = \int_{\underline{g}}^{\overline{g}} \frac{-\frac{\partial^2 b_i(g,r_i)}{\partial r_i \partial g} + h_i c'\left(r_i\right) n}{\frac{\partial^2 b_i(g,r_i)}{\left(\partial r_i\right)^2} - h_i c''\left(r_i\right) g n} dg. \tag{10}$$

Thus, for adaptation and brown technologies, $r_i^* < r_i^b$. For such technologies we also have $r_i \leq r_i^* < r_i^b$ and $\Delta_{i,r} < 0$, so from (8) we have that $dr_i/dk_i > 0$. For clean technologies, Eq. (10) gives $r_i^* > r_i^b$. For clean technologies, we also have $r_i \geq r_i^* > r_i^b$ and $\Delta_r^i > 0$, so from (8) we again have that $dr_i/dk_i > 0$.

(ii) Effect of s_i *and* δ: The effect of s_i on r_i is exactly as in the case with time-dependent variables (7). The effect of δ is trivial. Note that r_i^* does not depend on s_i or δ.

(iii) Effect of n: Consider first the case where $c'\left(r_i\right) = 0$ (brown or clean technologies). In this case, $\partial\left(v_i - v_i^b\right)/\partial n = \left(\overline{g} - \underline{g}\right) h_i c > 0$, so

$$\frac{dr_i}{dn} = -\frac{\Delta_{i,n}}{\Delta_{i,r}} = -\frac{\delta}{1-\delta}\frac{\left(\overline{g} - \underline{g}\right) h_i c}{\Delta_{i,r}},$$

which has the opposite sign of $\Delta_{i,r}$. Note that r_i^* does not depend on n.

Consider next adaptation technologies, where $c'(\cdot) < 0$ but $\partial b_i(\cdot)/\partial r_i = 0$. If r_i were invariant in n, we would have $\partial\left(v_i - v_i^b\right)/\partial n = h_i\left[c\left(r_i^b\right)\overline{g} - c\left(r_i\right)\underline{g}\right]$, but r_i and r_i^b may differ since $r_i^*(g)$ depends on g. If $c(r_i^*(g))\,g$ increased in g, then we would have $c\left(r_i^b\right)\overline{g} > c(r_i^*)\,\underline{g}$. If we use Eq. (9), we can show that $c(r_i^*(g))\,g$ increases in g if:

$$c(r_i^*(g)) - \frac{\left[c'(r_i^*(g))\right]^2}{c''(r_i^*(g))} > 0. \tag{11}$$

So, under this condition, $\partial\left(v_i - v_i^b\right)/\partial n$ and $\Delta_{i,n}$ would be positive for r_i close to r_i^* and, then, $dr_i/dn > 0$.

Effect of h_i: This effect is derived in a similar way. For adaptation technologies, if r_i were invariant in h_i, we would have $\partial\left(v_i - v_i^b\right)/\partial h_i = n\left[c\left(r_i^b\right)\overline{g} - c(r_i)\,\underline{g}\right]$, where the bracket is, as before, positive under condition (11). Then, $\Delta_{i,h} > 0$ and, therefore, $dr_i/dh_i > 0$ for adaptation technologies. For brown technologies we have $\Delta_{i,h} > 0$ and $dr_i/dh_i > 0$ while for clean technologies we have $\Delta_{i,h} < 0$ and $dr_i/dh_i < 0$, even though r_i^* is independent of h_i. *QED*

Proof of Proposition 4.

If we define $d_i \equiv \overline{y} - g_i - r_i$ to be the decrease in consumption relative to the bliss level $\overline{y}$, the first-best emission (and consumption) level is simply given by $d^* = cn/B$, while, in BAU, $d^b = c/B$. We can write the continuation value as:

$$v = d\,(nc - Bd/2) + r\,(nc - Kr/2) - cn\overline{y}.$$

(i) The compliance constraint at the emission stage can be written as:

$$\frac{v}{1-\delta} \geq nc\left(r - r^b\right) - \frac{K}{2}\left(r^2 - \left(r^b\right)^2\right) + c\,(n-1)\left(d - d^b\right) + \frac{v^b}{1-\delta}, \tag{CC$_c^g$}$$

which implies that:

$$\delta \geq \widehat{\delta}(g,r) \equiv 1 - \frac{-cn\overline{y} + d\,(nc - Bd/2) + r\,(nc - Kr/2) - v^b}{nc\,(r - r^b) - \frac{K}{2}\left(r^2 - (r^b)^2\right) + c\,(n-1)\,(d - d^b)}, \text{ so}$$

$$\overline{\delta} \equiv \widehat{\delta}(g^*, r^*) = 1 - \frac{(n-1)^2\,(1/2B + 1/2K)}{(n-1)^2/2K + (n-1)^2/B} = \frac{K}{B + 2K}.$$

(ii) Note that $r_i = r^*$ is both maximizing v and weakening (CC$_c^g$). Given this r^*, the optimal d is the largest d satisfying (CC$_c^g$). Substituting for v and then solving (CC$_c^g$) for the largest d, we get:

$$\begin{aligned} dB &= nc - \phi, \text{ where} \\ \phi(\delta) &\equiv c\,(n-1)\left[1 - \delta - \sqrt{\delta^2 + \delta B/K}\right] \\ &= c\,(n-1)\left[1 - \delta - \sqrt{(1-\delta)^2 - \left(\overline{\delta}^g - \delta\right)/\overline{\delta}^g}\right], \end{aligned}$$

where $\phi(\delta)$ decreases from $c(n-1)$ to 0 as δ increases from 0 to $\overline{\delta}^g$. This d is implemented by the emission tax $nc - \phi$. To ensure $r_i = r^*$, the subsidy must be ϕ.

Note that the investment-stage compliance constraint can be written as

$$\frac{v}{1-\delta} \geq \varsigma (n-1) \frac{c}{K} + \frac{v^b}{1-\delta}, \qquad (\mathrm{CC}_c^r)$$

which always holds when $\varsigma = \phi(\delta) \to 0$. When δ falls, v declines and $\varsigma = \phi(\delta)$ increases. The threshold $\underline{\delta}$ is defined implicitly by requiring (CC_c^r) to hold with equality at $\underline{\delta}$. *QED*