Legally Binding Environmental Agreements*

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Lecture Notes - 2016

1. Introduction

These notes study dynamic games where both pollution stocks and technology stocks cumulate over time. Furthermore, and more importantly, the notes allow for so-called "legally binding agreements," where we assume that countries may be able to (in part) commit to certain emission levels.

If countries can commit to any future action (emission as well as technology), then the first best can easily be achieved. In reality, however, commitment is only imperfect: Countries may not be able to commit to the very long term, and some actions may not be possible to measure or observe, or commit to, at all.

For example, the importance of developing new and green technology has been recognized in the treaty texts, but the "technology needs must be nationally determined, based on national circumstances and priorities."[1] Thus, both past and future agreements are likely to prescribe emission levels rather than investments in technology. To some extent, commitments on emissions will motivate countries to invest in new technology. However, there are large externalities or technological spillovers associated with such investments. When surveying the literature, IPCC (2014, Ch. 13:50) concludes that "the evidence indicates a systematic [positive] impact of IP protection on technology transfer." It may thus be comforting that

[1] The quote is from §114 in the Cancun Agreement (UNFCCC, 2011), confirmed by the Durban Platform (UNFCCC, 2012).

the Agreement on Trade Related Aspects of Intellectual Property Rights (TRIPS) of 1994 generally commits all countries to create and enforce standard intellectual property rights. TRIPS, however, allows for exceptions to the exclusive rights of patents for public policy reasons. It provides for the possibility of compulsory licensing and royalty-free compulsory licensing has indeed been advocated for as a way to encourage technology transfers (IPCC, 2014, chap. 13:50; 15:47). Also the 2015 Paris agreement focused on emissions, while investments in new technology was not something anyone tried to specify. Furthermore, the commitments under the Paris agreement is only to 2025 (for some countries) or 2030 (for others).

This raises several important questions. How valuable is such an agreement with a relatively short duration? What is the optimal term of this agreement? How should the term, and the design more generally, be influenced by actual intellectual property rights and the requirements for licensing?

These lecture notes present a dynamic framework in which countries both pollute and invest in technology during every period. The pollution as well as the technology stocks depreciate but accumulate over time. The technology, which could involve either renewable energy sources or abatement technology, reduces the need to pollute. While the model has a large number of subgame perfect equilibria, I focus on the symmetric Markov perfect equilibrium (MPE) since it is simple, robust, and unique. If countries cannot commit to any future action, the game is a dynamic version of the common-pool problem. If countries can contract on every future action, they easily implement the first-best outcome. The more realistic and interesting situation arises when countries contract on emission but not investment levels.

While these notes abstract from more general functional forms, heterogeneity among the countries, and renegotiation, they intend to make a number of important policy lessons:

First, climate agreements can *reduce* welfare relative to business as usual. When future negotiations are anticipated, countries may fear being held up by the others when negotiating emission quotas.[2] This hold-up problem reduces the incentive to invest, and every country may be worse off with short-term agreements than in the business-as-usual scenario in which no negotiations or agreements ever occur. Specifically, climate agreements are likely to be harmful if intellectual property rights are weak, the commitment period is short, and the number of

[2] The *New York Times* reports that "Leaders of countries that want concessions say that nations like Denmark have a built-in advantage because they already depend more heavily on renewable energy" (October 17, 2008: A4).

countries large.

Following the pessimistic result above, the optimal climate treaty is characterized. If quotas are negotiated before a country invests, it cannot be held up by the other countries–at least not before the agreement expires. Thus, countries invest more when the agreement is long-term. Nevertheless, countries underinvest compared to the optimum if technological spillovers are positive or intellectual property rights are weak. To compensate for this and to encourage further investments, the best (and equilibrium) treaty is tougher, in that it stipulates lower emissions, relative to what is optimal ex post, once the investments are sunk. The weaker the intellectual property rights, the tougher the optimal (and equilibrium) treaty.

Finally, the optimal length of the agreement is characterized. On the one hand, a longer time horizon is required to minimize the hold-up problem and to maximize the incentive to invest in technology. On the other hand, the future marginal cost of pollution is uncertain and stochastic in the model, and it is hard to predict the ideal quotas in the far future. The optimal length trades off these concerns. If intellectual property rights are strengthened, for example, the optimal length decreases.

The optimal climate treaty is a function of trade policies, but the reverse is also true: if the climate treaty is relatively short-term, it is more important to strengthen intellectual property rights, reduce tariffs, or subsidize technological trade. Negotiating such trade policies is thus a strategic substitute for a tough climate agreement.

By analyzing environmental agreements as incomplete contracts in a dynamic game, I contribute to three strands of literature.

The model is presented in the next section. Section 3 presents the (complete contracting) first-best outcome as well as the (noncooperative) business-as-usual scenario. The fact that short-term agreements can be harmful is shown in Section 4, while Section 5 characterizes the optimal agreement. Section 6 discusses trade policies and shows that the main results hold whether or not side transfers are available or the emission permits are tradable. The final section concludes and the proofs follow in the appendix.

2. A Dynamic Framework

2.1. A Model

Pollution is a public bad. Let G represent the stock of greenhouse gases, and assume that the environmental cost for every country $i \in N \equiv \{1, ..., n\}$ is given by the quadratic cost function:

$$C(G) = \frac{c}{2} G^2.$$

Parameter $c > 0$ measures the importance of climate change.

The stock of greenhouse gases G is measured relative to its "natural" level: G would, were it not for emissions, tend to approach zero over time, and $1 - q_G \in [0, 1]$ measures the fraction of G that naturally depreciates every period. The stock may nevertheless increase if country i's emission level g_i is positive:

$$G = q_G G_- + \sum_{i \in N} g_i + \theta. \tag{2.1}$$

By letting G_- represent the stock of greenhouse gases in the previous period, subscripts for periods can be skipped whenever this is not confusing.

The time-varying shock θ_t is arbitrarily distributed i.i.d. over time with mean 0 and variance σ^2. It is quite realistic to let the depreciation and accumulation of greenhouse gases be uncertain. The main impact of θ_t is that it affects the marginal cost of pollution. In fact, the model is essentially unchanged if the level of greenhouse gases is simply $\widehat{G}_t \equiv q_G G_{t-1} + \sum_N g_{i,t}$, while the marginal cost of pollution is stochastic and given by $\partial C_t / \partial \widehat{G}_t = c\Theta_t + c\widehat{G}_t$, where $\Theta_t \equiv q_G \Theta_{t-1} + \theta_t$. For either formulation, a larger θ_t increases the marginal cost of emissions. Note that, although the shock θ_t is i.i.d. across periods, it has a long-lasting impact through its effect on G (or on Θ).

The benefit of polluting g_i units is that country i can consume g_i units of energy. Country i may also be able to consume alternative or renewable energy, depending on its stock of nuclear power, solar technology, or windmills. Let R_i measure this stock and the amount of energy it can produce. The total amount of energy consumed is thus:

$$y_i = g_i + R_i, \tag{2.2}$$

and the associated benefit for i is:

$$B_i(y_i) = -\frac{b}{2} (\overline{y}_i - y_i)^2. \tag{2.3}$$

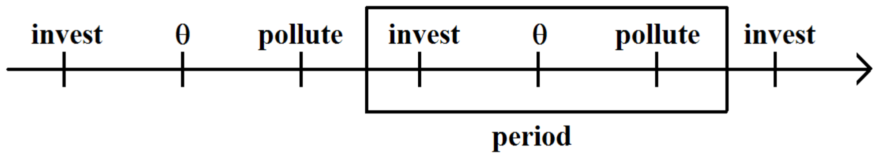

Figure 2.1: *The investment and emission stages alternate over time.*

The benefit function is thus concave and increasing in y_i up to i's bliss point $\overline{y}_i$, which can vary across countries. The bliss point represents the ideal energy level if there were no concern for pollution: a country would never produce more than $\overline{y}_i$ due to the implicit costs of generating, transporting, and consuming energy. The average $\overline{y}_i$ is denoted $\overline{y}$. Parameter $b > 0$ measures the importance of energy.

Note that green technology can be alternatively interpreted as abatement technology. Suppose R_i measures the amount by which country i can reduce (or clean) its potential emissions at no cost. If energy production, measured by y_i, is generally polluting, the actual emission level of country i is given by $g_i = y_i - R_i$, implying (2.2), as before.

The technology stock R_i evolves in a natural way. On the one hand, the technology might depreciate at the expected rate of $1 - q_R \in [0, 1]$. On the other hand, when r_i measures country i's investment in the current period, then:

$$R_i = q_R R_{i,-} + r_i.$$

As described by Figure 2.1, the investment stages and the pollution stages alternate over time. Without loss of generality, a *period* is defined as starting with the investment stage and ending with the pollution stage; in between, θ is realized. Information is symmetric at all stages.

It is important for tractability that investments and emissions are decided on at different points in time.[3] On the other hand, it is *not* important that the uncertainty θ be realized between the investment stage and the emission stage, rather than vice versa. I do not require the $r_{i,t}$'s or the $g_{i,t}$'s to be strictly positive.[4]

[3] This assumption can be endogenized. Suppose the countries can invest at any time they want but $\xi \in (0, 1)$ measures time required for the technology to mature or be built. Then, in equilibrium all countries will invest at time $t - \xi \in (t - 1, t)$.

[4] In principle, one could permit $g_{i,t} < 0$ by employing direct air capture or carbon dioxide

A country's objective is to maximize the present-discounted value of its utilities, i.e., its continuation value of the game:

$$U_{i,t} = \sum_{\tau=t}^{\infty} \delta^{\tau-t} u_{i,\tau},$$

where δ is the common discount factor and

$$u_i = B_i(y_i) - C(G) - kr_i + e \sum_{j \in N \setminus i} r_j. \tag{2.4}$$

Thus, parameter k captures country i's private cost of investing and e captures possible externalities associated with other countries' investments. When K represents the net *social* cost of technology investments, then

$$k = K + (n-1)e.$$

The simple externality e may represent traditional technological spillovers, diffusion, imitation, licensing, or trade. In particular, if traditional measures of intellectual property rights (IPR) are strengthened, then e and k tend to decrease while K stays constant. While this may be intuitive to some readers, Section 6.2 provides a careful micro-foundation for the externality e, and shows how it decreases in IPR-policies but increases in tariffs on trade.

2.2. Definition of an Equilibrium

There is typically a large number of subgame perfect equilibria in dynamic games, and refinements are necessary. This note focuses on Markov perfect equilibria (MPEs) in which strategies are conditioned only on the payoff-relevant stocks (G and $\mathbf{R} \equiv \{R_1, ..., R_n\}$).

There are several general reasons for selecting the Markov perfect equilibria. First, Markov perfect strategies are simple, since they do not depend on the history in arbitrary ways, which simplifies the analysis as well. Second, experimental evidence suggests that players tend toward Markov perfect strategies in complex environments. Third, focusing on the MPEs is quite standard when studying games with stocks.

removal (CDR) methods, while $r_{i,t} < 0$ is possible if green infrastructure, such as expensive silicon in solar panels, can be employed for other purposes.

In addition, Markov perfection is particularly attractive in the present model. In contrast to much of the literature, there is a unique symmetric MPE in the present game. This sharpens the predictions and makes institutional comparisons possible. The restriction to *symmetric* MPEs means that if every country faces identical continuation values at the investment stage, then we select the equilibrium where they invest the same amount.[5]

This equilibrium coincides with the unique symmetric subgame perfect equilibrium if time were finite but approached infinity.[6] This is particularly important in the present context, since the equilibrium is then robust to the introduction of real-world aspects that would make the effective time horizon finite. For example, since fossil fuel is an exhaustible resource, the emission game may indeed have a finite time horizon in the real world. Similarly, politicians' term-limits or short time horizon may force them to view time as expiring. Finally, since the unique MPE makes it impossible to enforce agreements by using trigger strategies, it becomes meaningful to focus instead on settings where countries can negotiate and contract on emission levels–at least for the near future.

This note does not attempt to explain *how* countries can commit, but domestic ratification is seldom meaningless. In the United States, for example, the Supremacy Clause (Article VI, paragraph 2 of the US Constitution) states that "all Treaties made... shall be the supreme Law of the Land..." Thus, US states are bound to uphold a signed treaty, even in the presence of conflicting state laws. However, since nations' ability to commit may in general be imperfect, I analyze alternative scenarios where countries cannot commit at all (Section 3.3), where they can sign complete contracts (Section 3.2), or where they contract on some but not all issues of interest (Sections 4-6). The last scenario turns out to be most interesting analytically. This is also the scenario best describing current climate negotiations. Note that I do not allow countries to commit to rules for how they should negotiate in the future.[7]

At the negotiation stage, we will assume the bargaining outcome is efficient and symmetric *if* it should happen that the game and the payoffs are symmetric. This

[5] Since the investment cost is linear, there are asymmetric MPEs in which the countries invest different amounts. In fact, if parameter b, c or k varied across countries, the MPE would have to be asymmetric. In the present model, however, the asymmetric equilibria would cease to exist if there were a slightly convex cost function for the investments $r_{i,t}$.

[6] This fact can easily be seen by the recursive nature of the proofs.

[7] For example, a commitment to uniform quotas could raise efficiency for short-term agreements, and would implement the first best if $e = 0$ and the uniform quota was determined by a majority requirement short of unanimity.

condition is satisfied whether we rely on (i) the Nash Bargaining Solution, with or without side transfers, (ii) the Shapley value, or instead (iii) noncooperative bargaining in which one country is randomly selected to make a take-it-or-leave-it offer specifying quotas and side payments.

3. Benchmarks

3.1. Preliminaries

While the $n+1$ stocks in the model are a threat to its tractability, the analysis is simplified by two of the model's deliberately chosen features. First, one can utilize the additive form in (2.2). By inserting (2.2) into (2.1), we get:

$$\begin{aligned} G &= q_G G_- + \theta + \sum_{i \in N} \widetilde{y}_i - R, \text{ where} && (3.1) \\ R &\equiv \sum_{i \in N} R_i = q_R R_- + \sum_{i \in N} r_i \text{ and} && (3.2) \\ \widetilde{y}_i &\equiv y_i + \overline{y} - \overline{y}_i . \end{aligned}$$

Together with $u_i = -b(\overline{y} - \widetilde{y}_i)^2/2 - cG^2/2 - kr_i + \sum_{j \in N \setminus i} er_j$, the R_is as well as the $\overline{y}_i$s are eliminated from the model. All countries are thus *symmetric* in the model when it comes to deciding on $\widetilde{y}_i$ and r_i, regardless of any heterogeneity in $\overline{y}_i$ or $R_{i,-}$. Furthermore, the R_is are *payoff-irrelevant* as long as R is given: if the other players do not condition their strategies on the R_is, there is no reason for i to do so, either. The Markov-perfect strategies are not contingent on technology differences, a country's continuation value U_i is a function of only G_- and R_-, and we can write it as $U(G_-, R_-)$, without the subscript i.

Second, the linear investment cost is utilized to prove that the continuation value must be linear in R and in G. This simplifies the analysis and leads to a unique symmetric MPE for each scenario analyzed below.

LEMMA 1. (i) *There is a unique symmetric Markov perfect equilibrium whether contracts are absent, complete, or incomplete.*

(ii) *In each case, the continuation value* $U(G_-, R_-)$ *has the properties:*

$$\begin{aligned} U_R &= \frac{q_R K}{n}, \\ U_G &= -\frac{q_G K}{n}\left(1-\delta q_R\right). \end{aligned}$$

The lemma holds for *every* scenario analyzed below; it is proven in the appendix when each case is solved.

3.2. Complete "Contracts": The First-Best

If investments as well as emissions were contractible, the countries would agree to the first-best outcome. This follows from the observation (made in the previous subsection) that the bargaining game is symmetric, even if the R_is or the $\overline{y}_i$s differ. The outcome is thus efficient and coincides with the case in which a benevolent planner makes all decisions in order to maximize the sum of utilities.

PROPOSITION 1. (i) *The first-best emission levels are functions of the technology stocks,* $\mathbf{R} \equiv \{R_1, ..., R_n\}$:

$$g_i^*(\mathbf{R}) = \overline{y}_i - R_i - \frac{cn\left(n\overline{y} + q_G G_- + \theta - R\right) + \delta q_G\left(1-\delta q_R\right)K}{b+cn^2}. \tag{3.3}$$

(ii) *The symmetric first-best investments are:*

$$r_i^* = \overline{y} - \frac{q_R}{n}R_- + \frac{q_G}{n}G_- - \left(1-\delta q_R\right)\left(\frac{1-\delta q_G}{cn^2} + \frac{1}{b}\right)K.$$

(iii) *Combined, the first-best pollution stock is:*

$$G^* = \sum_{i \in N} g_i^*(\mathbf{R}^*) + q_G G_- = \frac{\left(1-\delta q_G\right)\left(1-\delta q_R\right)}{cn}K + \frac{b}{b+cn^2}\theta. \tag{3.4}$$

The dynamics are noteworthy. If the random pollution shock θ_t happens to be large, the emissions are reduced somewhat but the total stock, G_t, is nevertheless increasing in θ_t. In the following period, the countries' investments are so much larger, and the optimal emission levels are thus so much smaller, that the stock

G_{t+1} returns to the original level (independent of θ_t). Since the large technology stock survives to period $t+2$, investments are then reduced. The steady state is reached after two periods–thanks to the linear investment cost:[8]

COROLLARY 1. *In the first-best outcome, a large θ_t reduces g_t and g_{t+1} but raises r_{t+1}:*

$$\begin{aligned}\frac{\partial g_t}{\partial \theta_t} &= -\rho \text{ and } \frac{\partial g_{t+1}}{\partial \theta_t} = -q_G(1-\rho) = -\frac{\partial r_{t+1}}{\partial \theta_t} = \frac{1}{q_R}\frac{\partial r_{t+2}}{\partial \theta_t}, \text{ where}\\ \rho &\equiv \frac{cn}{b+cn^2} \in (0,1).\end{aligned}$$

3.3. No Contracts: Business as Usual

In the other extreme scenario, neither emissions nor investments are negotiated. This noncooperative situation is referred to as "business as usual."

PROPOSITION 2. *With business as usual, countries pollute too much and invest too little:*

$$\begin{aligned}r_i^{bau} &= \overline{y} - \frac{q_R}{n}R_- + \frac{q_G}{n}G_- \\ &\quad - \left[\frac{(b+cn)^2}{cb(b+c)n}\left(e(n-1) + \left(1-\frac{\delta q_R}{n}\right)K\right) - (1-\delta q_R)\frac{\delta q_G}{cn^2}K\right] \\ &< r_i^*, \\ g_i^{bau}\left(\mathbf{R}^{bau}\right) &= \overline{y}_i - R_i - \frac{c(n\overline{y} + q_G G_- + \theta - R) + \delta q_G(1-\delta q_R)K/n}{b+cn} \\ &> g_i^*\left(\mathbf{R}^{bau}\right) > g_i^*\left(\mathbf{R}^*\right). \qquad (3.5)\end{aligned}$$

The first inequality in (3.5) states that each country pollutes too much compared to the first-best levels, conditional on the investments.

[8]With linear contribution costs, it is a typical result that an MPE can reach the steady state in a single step. Also my model would feature a one-step transition with shocks in the technology stock. However, as described by my corollaries, with a pollution stock (and nonlinear emission benefits) as well as technology stocks, the transition lasts two periods when the shock regards the pollution. If investment costs were convex, the transition would be more gradual (and reasonable).

Furthermore, note that country i pollutes less if the existing level of pollution is large and if i possesses good technology, but pollutes more if the other countries' technology level is large, since these countries are then expected to pollute less. In fact, the equilibrium level of consumption reduction $\overline{y}_i - y_i = \overline{y}_i - R_i - g_i^{bau}$ is the same across countries no matter what the differences in technology are. While perhaps surprising at first, the intuition is straightforward. Every country has the same preference (and marginal utility) when it comes to reducing its consumption level *relative* to its bliss point. The *marginal* impact on G is also the same for every country: one *more* energy unit generates one unit of emissions. The technology is already utilized to the fullest possible extent, so consuming more energy results in more pollution.

Therefore, a larger R, which reduces G, must increase every y_i. This implies that if R_i increases but R_j, $j \neq i$, is constant, then $g_j = y_j - R_j$ must increase. If a country has better technology, it pollutes less, but as a result, all other countries pollute more. Clearly, this effect reduces a country's willingness to pay for technology, and constitutes another reason why investments are suboptimally low, reinforcing the impact of weak intellectual property rights. The suboptimal investments make it necessary to pollute more, implying the second inequality in (3.5) and a second reason why pollution is higher than its first-best level.

Intuitively, a country may want to invest less in order to induce other countries to pollute less and to invest more in the following period. In addition, a country may want to pollute more today to induce others to pollute less (and invest more) in the future. These dynamic considerations make this dynamic common-pool problem more severe than its static counterpart. The transition to the steady state is also slower than in the first best:

Corollary 2. *With business as usual, Corollary 1 continues to hold if ρ is replaced by $\rho^{bau} = c/(b + cn) < \rho$.*

4. Short-Term Agreements

If countries can commit to the immediate but not the distant future, they may negotiate a short-term agreement. If the agreement is truly short-term, it will be difficult to develop new technology during the time span of the agreement and the relevant technology is given by earlier installations. This interpretation of short-term agreements can be captured by the timing shown in Figure 4.1.

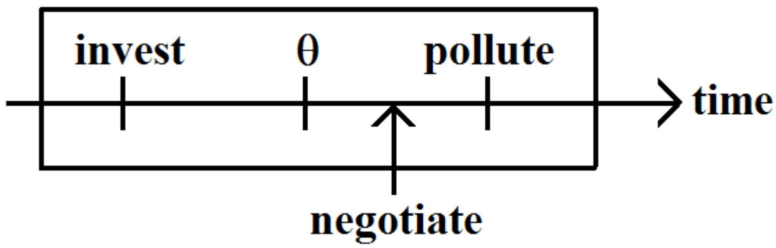

Figure 4.1: *The timing for short-term agreements.*

Technically, negotiating the emission level g_i is equivalent to negotiating $\widetilde{y}_i$ as long as the technology stock R_i is sunk and observable (even if it is not verifiable). Just as in Section 3.1, equations (3.1)-(3.2) imply that the R_is are payoff-irrelevant, given R. Even if countries have different R_is, they face the exact same marginal benefits and costs of reducing y_i relative to $\overline{y}_i$, whether negotiations succeed or not. Symmetry thus implies that $\widetilde{y}_i$ is the same for every country in the bargaining outcome and efficiency requires them to be optimal. Consequently, the emission levels are equal to the first-best, conditional on past investments.

Intuitively, if country i has better technology, its marginal benefit from polluting is smaller, and i thus pollutes less with business as usual. This gives i a poor bargaining position and the other countries can offer i a smaller emission quota. At the same time, the other countries negotiate larger quotas for themselves, since the smaller g_i (and the smaller G) reduce the marginal cost of polluting. Countries anticipate this hold-up problem and are therefore discouraged from investing.

Consequently, although emission levels are ex post optimal, actual emissions are larger compared to the first-best levels since the hold-up problem discourages investments and makes it ex post optimal to pollute more.

Proposition 3. *With short-term agreements, countries pollute the optimal amount, given the stocks, but investments are suboptimally low:*

$$\begin{aligned} r_i^{st} &= r_i^* - \left(\frac{b+cn^2}{bcn}\right)\left(e+\frac{K}{n}\right)(n-1) < r_i^*, \\ g_i^{st}\left(\mathbf{R}^{st}\right) &= g_i^*\left(\mathbf{R}^{st}\right) > g_i^*\left(\mathbf{R}^*\right). \end{aligned}$$

Deriving and describing this outcome are relatively simple because Lemma 1 continues to hold for this case, as is proven in the appendix. In particular, U_G and U_R are exactly the same as with business as usual. This does *not* imply, however, that the continuation value U itself is identical in the two cases: the levels can be different. This equivalence does imply, however, that, when deriving actions and utilities for one period, it is irrelevant whether there will be a short-term agreement in the next (or any future) period. This makes it convenient to compare short-term agreements to business as usual. For example, such a comparison will be independent of the stocks, since U_G and U_R are identical in the two cases.

By comparison, the pollution level is indeed less under short-term agreements than under business as usual. For welfare, however, it is also important to know how investments differ in the two cases.

PROPOSITION 4. *Compared to business as usual, short-term agreements lead to:* (i) *lower pollution,*

$$\mathrm{E}g^{st}\left(r^{st}\right) = \mathrm{E}g^{bau}\left(r^{bau}\right) - \frac{n-1}{n\left(b+c\right)}\left(e\left(n-1\right) + \left(1 - \frac{\delta q_R}{n}\right)K\right);$$

(ii) *lower investments,*

$$r_i^{st} = r_i^{bau} - \frac{\left(n-1\right)^2}{n\left(b+c\right)}\left(e\left(n-1\right) + \left(1 - \frac{\delta q_R}{n}\right)K\right);$$

(iii) *lower utilities if intellectual property rights are weak while the period is short, i.e., if*

$$\left(e + \frac{K}{n}\right)^2\left(n-1\right)^2 > \left(1 - \delta q_R\right)^2 + \frac{\left(b+c\right)\left(bc\sigma\right)^2}{\left(b+cn^2\right)\left(b+cn\right)^2}. \tag{4.1}$$

Part (ii) is a negative result. Short-term agreements discourage, rather than encourage, investments. The reason is the following. First, the hold-up problem is exactly as strong as the crowding-out problem in the noncooperative equilibrium; in either case, each country enjoys only $1/n$ of the total benefit generated by its investments. In addition, when an agreement is expected, everyone anticipates that the pollution stock will be smaller. A further decline in emissions, made possible by new technology, is then less valuable.[9]

[9] A counterargument is that, if an agreement is expected, it could become *more* important to invest to ensure a decent energy consumption level. This effect turns out to be smaller in the model above.

As part (iii) shows, even utility levels can be smaller with short-term agreements. Intuitively, this is the case if investments are already well below the optimal level, so that a further fall is particularly harmful. Thus, short-term agreements are bad if intellectual property rights are weak (e is large), the number of countries is large, and the period for which the agreement lasts is very short. If the period is short, δ and q_R are large, while the uncertainty from one period to the next, determined by σ, is likely to be small. All changes make (4.1) reasonable, and it always holds when the period is very short (i.e., when $\delta q_R \rightarrow 1$ and $\sigma \rightarrow 0$).

A large variance σ implies that business as usual is worse since the transition following a shock θ is then too slow (Corollary 2). Short-term agreements, on the other hand, coincide with the first-best except for a reduction in investment levels that are independent of θ. Thus, the transition following a shock is just as fast as in the first-best outcome:

COROLLARY 3. *With short-term agreements, Corollary 1 continues to hold.*

5. The Optimal Agreement

The hold-up problem under short-term agreements arises because the g_is are negotiated after investments are made. If the time horizon of an agreement is longer, however, it is possible for countries to develop technologies within the time frame of the agreement. The other countries are then unable to hold up the investing country, since the quotas have already been agreed to, at least for the near future.

To analyze such long-term agreements, let the countries negotiate and commit to emission quotas for T periods. The next subsection studies equilibrium investment as a function of such an agreement. Taking this function into account, the second subsection derives the optimal (and equilibrium) emission quotas, given T. Finally, the optimal T is characterized.

If the agreement is negotiated just before the emission stage in period 0, then the quotas and investments for that period are given by Proposition 3. For the subsequent periods, it is irrelevant whether the quotas are negotiated before the first emission stage or instead at the start of the next period, since no information is revealed and no strategic decisions are made in-between. To avoid repeating earlier results, I will focus on the subsequent periods, and thus implicitly assume that the T-period agreement is negotiated at the start of period 1, as described by Figure 5.1.

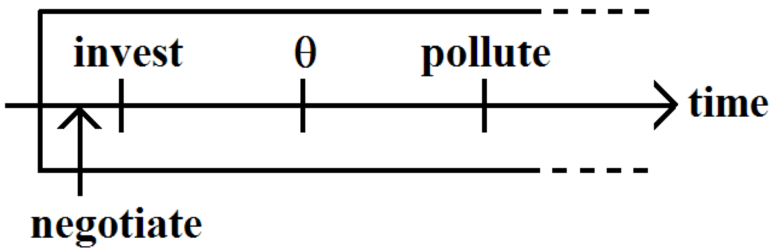

Figure 5.1: *The timing for long-term agreements.*

5.1. Equilibrium Investments Depend on the Agreement

When investing in period $t \in \{1, 2, ..., T\}$, countries take the quotas as given. A country is willing to pay more for innovations and investments if its quota, $g_{i,t}$, is small, since it is going to be very costly to comply if the consumption level $y_{i,t} = g_{i,t} + R_{i,t}$ is also small. In anticipation of this, innovations and investments decrease in $g_{i,t}$.

Nevertheless, compared to the investments that are first-best conditional on the quotas, $r^*_{i,t}(g_{i,t})$, equilibrium investments are too low for two reasons. First, the positive externality e is not taken into account by the investing countries. Second, a country anticipates the hold-up problem in period $T+1$, when a new agreement is to be negotiated. A large technology stock in period $T+1$ means that it will be relatively inexpensive for i to reduce emissions, and the other countries will demand that i cut more. Anticipating this, countries invest less in the last period, particularly if that period is short (δ is large), the technology long-lasting (q_R large), and the number of countries large (n large).

Proposition 5. *Equilibrium investments are:*
(i) *decreasing in the quota $g_{i,t}$ and the externality e;*
(ii) *less than the efficient level, $r^*_{i,t}(g_{i,t})$, if $e > 0$, for any given quota and period;*

(iii) *less in the last period than in earlier periods if* $\delta q_R > 0$. *Formally:*

$$\begin{aligned} r_{i,t}\left(g_{i,t}\right) &= r^*_{i,t}\left(g_{i,t}\right) - \left(e + \frac{\delta q_R K}{n}\right)\left(\frac{n-1}{b}\right) \text{ for } t = T \\ &\leq \text{ (strict if } \delta q_R > 0) \\ r_{i,t}\left(g_{i,t}\right) &= r^*_{i,t}\left(g_{i,t}\right) - e\left(1 - \delta q_R\right)\left(\frac{n-1}{b}\right) \text{ for } t < T \\ &\leq \text{ (strict if } e > 0) \\ r^*_{i,t}\left(g_{i,t}\right) &= \overline{y}_i - q_R R_{i,-} - g_{i,t} - \left(1 - \delta q_R\right) K/b. \end{aligned}$$

5.2. The Optimal Quotas

At the emission stage, the ex post optimal pollution level is, as before, given by $g_i^*\left(\mathbf{R}^{lt}\right)$, where $\mathbf{R}^{lt}$ is the equilibrium technology vector under long-term agreements. However, the countries anticipate that the negotiated $g_{i,t}$s will influence investments in technology: the smaller the quotas, the larger the investments. Thus, since the investments are suboptimally low, the countries have an incentive to commit to quotas that are actually smaller than the expected $g_i^*\left(\mathbf{R}^{lt}\right)$ to further encourage investments.

PROPOSITION 6. (i) *The negotiated quotas are strictly smaller than the ex post optimal levels if* $e > 0$*:*

$$g_{i,t} = \mathrm{E}g_i^*\left(\mathbf{R}^{lt}\right) - e\left(1 - \delta q_R\right)\left(\frac{n-1}{b + cn^2}\right) \text{ for } t < T. \tag{5.1}$$

(ii) *For the last period, the negotiated quotas are strictly smaller than the ex post optimal quotas if either* $e > 0$ *or* $\delta q_R > 0$*:*

$$g_{i,t} = \mathrm{E}g_i^*\left(\mathbf{R}^{lt}\right) - \left(e + \frac{\delta q_R K}{n}\right)\left(\frac{n-1}{b + cn^2}\right) \text{ for } t = T. \tag{5.2}$$

If the technological spillover e is larger, the last terms of (5.1)-(5.2) are larger, and every negotiated $g_{i,t}$ is smaller relative to $g_i^*\left(\mathbf{R}^{lt}\right)$. The small quotas mean that the agreement is demanding or *tough* to satisfy.[10] Encouraging investments in this way is especially important in the last period, since, according to Proposition

[10] Interestingly, the equilibrium quotas, as described by Proposition 6, are in fact equal to the

5, investments are particularly low then. Thus, the optimal agreement is tougher to satisfy over time.[11]

5.3. The Optimal Length

If the countries are able to make commitments for any future period, they can negotiate the agreement length, T. Since, as noted before, the countries are symmetric at the negotiation stage (regardless of any differences in R_is or $\overline{y}_i$s), they will agree on the optimal T. This trades off two concerns. On the one hand, investments are particularly low at the end of the agreement, before a new agreement is to be negotiated. This hold-up problem arises less frequently, and is delayed, if T is large. On the other hand, the stochastic shocks cumulate over time and affect the future marginal costs of pollution. Thus, the emission allowances should ideally depend on the shocks (as in the first best).

In general, the optimal length of an agreement depends on the regime that is expected to replace it. This is in contrast to the other parts of the contract studied above, which have been independent of the future regime. When the time horizon is chosen, it is better to commit to a longer-term agreement if everyone expects that, once it expires, the new regime will be bad (e.g., business as usual).

However, if future as well as present negotiators are able to contract on emissions, then we can anticipate that the next agreement is also going to be optimal. Under this assumption, the optimal term is derived and characterized in the appendix.

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first-best emission levels *if* investments had been first-best:

$$g_{i,t} = \mathrm{E}g_i^* \left(\mathbf{R}^*\right).$$

When the optimal quotas are selected, there are, as noted, good reasons for selecting small quotas in order to induce investments. As a counterargument, the suboptimally low investments make it ex post optimal to permit larger emission levels. These two effects turn out to cancel each other out, as the appendix shows. The technical reason is that, in this equilibrium as well as in the first-best outcome, y_i is independent of g_i, so a smaller g_i only reduces G and increases R_i. Since the marginal cost of increasing R_i is constant, the optimal G is the same in this equilibrium and in the first-best outcome.

[11] This conclusion would be strengthened if the quotas were negotiated just before the emission stage in the first period. Then, the first-period quotas would be ex post optimal since these quotas would, in any case, have no impact on investments. It is easy to show that these quotas are expected to be larger than the quotas described by Proposition 6 - whether or not this is conditioned on investment levels.

Proposition 7. (i) *The optimal length is finite, $T^* < \infty$, if and only if:*

$$\left(e + \frac{K}{n}\right)\left(e\left[2 - \delta q_R\right] + \frac{\delta q_R K}{n}\right) < \frac{bcq_G^2\sigma^2}{q_R\left(1 - q_G^2\right)\left(1 - \delta q_G^2\right)\left(n - 1\right)^2}.$$

(ii) *Under this condition, T^* increases in e, n, q_R, and K, but decreases in b, c, and σ.*

If θ were known or contractible, the optimal agreement would last forever. Otherwise, the length of the agreement should be shorter if future marginal costs are uncertain (σ large) and important (c large). On the other hand, a larger T is preferable if the underinvestment problem is severe. This is the case if the intellectual property rights are weak (e large), the technology is long-lasting (q_R large), and the number of countries large. If b is large while K is small, then consuming the right amount of energy is more important than the concern for future bargaining power. The hold-up problem is then relatively small, and the optimal T declines.

6. Trade Policy

So far, the note has focused on the treaty's depth and duration. This section shows that the results are unaffected by the presence of side payments or tradable permits, and it shows how to endogenize the externalities, the intellectual property rights, and tariffs/subsidies on technological trade.

6.1. Tradable Pollution Permits and Side Payments

Note that in every bargaining situation above, the countries are identical when considering $\widetilde{y}_i$ and r_i, regardless of any differences in the $\overline{y}_i$s or in the R_is. Thus, side transfers would be used neither on nor off the equilibrium path.

The discussion above has ignored trade in pollution permits. However, if the permits were tradable, no trade would take place in equilibrium, and the possibility for such trade (off the equilibrium path) would change neither the equilibrium investments nor the emission quotas.

Proposition 8. (i) *Propositions 1-7 survive whether or not side payments are available.*

(ii) *Propositions 1-7 survive whether or not emission permits are tradable.*
(iii) *With tradable permits, the equilibrium permit prices under short-term agreements (p_{st}), the last-period of long-term agreements (p_T), the earlier periods of long-term agreements (p_t), and in the first best (p_*), are given by:*

$$\begin{aligned} \mathrm{E}p_{st} &= \left[e\,(n-1) + K\left(1 - \frac{\delta q_R}{n}\right)\right] n > \\ p_T &= e\,(n-1) + K\left(1 - \frac{\delta q_R}{n}\right) > \\ p_t &= [e\,(n-1) + K]\,(1 - \delta q_R) \geq \text{ (strict if } e > 0) \\ p_* &= K\,(1 - \delta q_R)\,. \end{aligned}$$

Interestingly, the permit price would increase toward the end of the agreement. Then, investments in green technology decline and the demand for fossil fuel goes up. However, even at $t < T$, the permit price is higher than it would have been in the first-best outcome (i.e., if investments were contractible). The reason is that the agreement is tougher than what is ex post optimal in order to motivate investments when technological spillovers are positive. The expected price is highest for short-term agreements, since the technology stock is then small, as is the corresponding consumption level. For each scenario, the equilibrium permit price is larger if intellectual property rights are weak.

6.2. Technology Spillovers, Diffusion, IPR, and Trade

This subsection provides a microfoundation for the externality e. The model above can allow for imitation, licensing, or tariffs, or all three in combination.

Imitation: When country j has invested $r_{j,t-1}$ units in technology in period $t-1$, country $i \neq j$ may learn and benefit from those investments. Parts of j's investment may have required innovations or generated new ideas, or there may simply be traditional learning by doing or by using when j upgrades its technology stock. To capture such technological spillovers in a simple way, let parameter $\varphi \in [0,1]$ be the fraction of $r_{j,t-1}$ which another country i can copy and adopt in the following period t.[12] A one-period lag between the time of one

[12] Note that the spillover is here related to the total upgrade of investments, $r_{j,t}$. One would obtain similar results if the spillover came instead from the cumulated stock, $R_{j,t}$. If the spillover only came from j's *new* technology the results would be similar but the analysis more complicated.

country's investment and the time of diffusion to another country is reasonable.[13]

Let $\underline{k}$ be the unit cost of investing in *new* technology, while the unit cost of adopting spillovers is only $(1-\gamma)\underline{k}$, where $\gamma \geq 0$ measures the cost savings. For example, $1-\gamma \approx 0.65$ appears as an estimate for the imitation cost, relative to new investments.

Spillovers: With these possibilities for adopting spillovers from $r_{j,t-1}$, country i's cost when choosing the total upgrade $r_{i,t}$ becomes $\underline{k}r_{i,t} - \varphi\gamma\underline{k}\sum_{j\neq i} r_{j,t-1}^N$.[14] Since the second term is fixed in period t, we can account for i's imitation benefit already in period $t-1$, exactly as represented by (2.4), if we just discount that future benefit by writing $e = \delta\varphi\gamma\underline{k}$. The private investment cost is then $k = \underline{k}$ and the net social cost when investing becomes

$$K = [1 - (n-1)\,\delta\varphi\gamma]\,\underline{k}. \tag{6.1}$$

Intellectual property rights (IPR): There are several ways of adding intellectual property rights to the model.

(i) Suppose the IPR is enforced (or judged valid) with probability α.

(ii) Or, suppose i can imitate $(1-\alpha)\varphi r_{j,t-1}$ for free, while the remaining fraction α requires the consent of j.

(iii) Alternatively, consider IPR as a way to raise the cost of imitation. For example, write the imitation cost as $[1-\gamma(1-\alpha)]\underline{k}$, increasing from $(1-\gamma)\underline{k}$ to $\underline{k}$ when the IPR-policy α increases from 0 to 1.

For any of these three alternatives, the externality would be $e = \delta\varphi\gamma(1-\alpha)\underline{k}$ if there were no licensing or technological trade.

Licensing or technological trade: The IPR policy, measured by α, inefficiently limits diffusion and thus there are gains from trade. Consider the moment before i can imitate j, and suppose i and j negotiate a fee permitting i to use the technology $\varphi r_{j,t-1}$. Assume the licensing fee is determined by the generalized Nash bargaining solution where $\beta \in [0,1]$ is the seller's bargaining-power index. Finally, suppose private purchasers of technology in country i pay an ad valorem

[13] Any imitation lag is enough to generate a one-period lag in the model: as footnote 11 explains, if $\xi \in (0,1)$ measures the time required for the technology to mature or be built before it can be used at emission stage t, then equilibrium investments will take place at time $t-\xi$. If imitators in other countries face any additional lag, these countries will certainly not be able to use the technology before the emission stage $t+1$. The lag implies that i's investment does not reduce j's immediate abatement cost (if it did, then technological spillovers could motivate i to invest more).

[14] To see this, note that when i adopts $r_{i,t}^A = \varphi\sum_{j\neq i} r_{j,t-1}^N$, the level of new technology must be $r_{i,t}^N = r_{i,t} - r_{i,t}^A$. The total cost of $r_{i,t}$ is thus $\underline{k}r_{j,t-1}^N + \underline{k}(1-\gamma)r_{i,t}^A = \underline{k}r_{i,t} - \varphi\gamma\underline{k}\sum_{j\neq i} r_{j,t-1}^N$.

tariff τ (or subsidy if $\tau < 0$) when buying technology. The tariff revenues are collected by the importing country, but it will improve the importer's terms of trade.

PROPOSITION 9. *With IPR modelled as (i), (ii) or (iii), i's payoff can be written as (2.4) where the externality is:*

$$e = \left(1 - \frac{\alpha\beta}{1+\tau}\right) \delta\gamma\varphi\underline{k}. \tag{6.2}$$

Thus, if the strength of the IPR, given by $\alpha\beta/(1+\tau)$, decreases, then the externality e increases and so does the private investment cost $k = K + (n-1)e$, while the social investment cost K is unchanged and as given by (6.1) (as shown in the proof in the appendix).

Note that, since the spillovers, fees, and revenues following $r_{j,t-1}$ are accounted for in period $t-1$, the payoff-relevant states at the beginning of period t are simply captured by the stocks G_{t-1} and $\mathbf{R}_{t-1} \equiv \{R_{1,t-1}, ..., R_{n,t-1}\}$.

6.3. Endogenous Trade Policies

Propositions 4-9 have shown that the optimal climate treaty depends on the intellectual property rights, the tariffs, or the technological subsidies, but we may also ask the reverse question: What is the best technological policy as a function of the climate policy?

If tariffs, for example, are determined at the country level with no commitment in advance, then each country sets the highest possible tariff after its neighbors have invested, since a high tariff improves the importer's terms of trade and raises the technological spillover. This, in turn, implies that short-term agreements are likely to be worse than business as usual while the optimal treaty becomes both tougher and more long-lasting. The same conclusion can be drawn if countries unilaterally and with no commitment decide on the intellectual property rights they provide to foreign exporters of technology.

If countries can negotiate and contract on IPR policies, however, the situation is rather different. Whether climate agreements are short-term, long-term, or absent, one can search for the socially optimal policy (α, β, τ), or simply the e that would follow according to (6.2). For any value of e which we would like to implement, we can, for example, derive the necessary subsidy $-\tau$ on licensing or technological trade by rewriting (6.2) as:

$$-\tau = 1 - \frac{\alpha\beta\delta\gamma\varphi\underline{k}}{\delta\gamma\varphi\underline{k} - e}. \tag{6.3}$$

PROPOSITION 10. *The optimal subsidy $-\tau$ and intellectual property rights (α or β) are larger if the agreement is short-term or absent. The optimal policy follows from:*
(i) *Equation (6.4) for short-term agreements as well as for business as usual;*
(ii) *Equation (6.5) for a long-term agreement's last period;*
(iii) *Equation (6.6) for a long-term agreement, except for its last period.*

$$e^*_{st} = e^*_{bau} = -\frac{K}{n} < \tag{6.4}$$

$$e^*_{lt,T} = -\frac{\delta q_R K}{n} < \tag{6.5}$$

$$e^*_{lt,t} = 0, \ t < T. \tag{6.6}$$

Given the optimal e, we can use (6.3) to derive the optimal subsidy implementing that e. For long-term agreements, for example, (6.6) requires $e^*_{lt,t} = 0$ for $t < T$; this is implemented by the subsidy $-\tau = 1 - \alpha\beta$.

If the climate treaty is short-term, the hold-up problem is larger and it is more important to encourage investments by protecting intellectual property rights, reducing tariffs, or subsidizing technological trade. Such trade agreements are thus strategic substitutes for climate treaties: weakening cooperation in one area makes further cooperation in the other more important. As before, the optimal agreement will also be the equilibrium when the countries negotiate, since they are symmetric at the negotiation stage (with respect to $\overline{y}_i - y_{i,t}$), no matter what their technological differences are.

If the subsidy can be freely chosen and set in line with Proposition 10, then short-term agreements are actually first-best: while the optimal subsidy induces first-best investments, the negotiated emission levels are also first-best, conditional on the investments. Long-term agreements are never first-best, however, due to the stochastic and noncontractible θ.

7. Conclusions

While mitigating climate change will require emission reduction as well as the development of new technology, recent agreements have focused on short-term emissions. What is the value of such an agreement? What is the optimal term of such an agreement, and how does it depend on existing intellectual property rights? To address these questions, this note analyzes a framework in which countries over time both pollute and invest in environmentally friendly technologies. The analysis generates a number of important lessons.

First, short-term agreements can be worse than business as usual. This may be surprising given that the noncooperative game is a particularly harmful dynamic common-pool problem. With business as usual, countries pollute too much, not only because they fail to internalize the externality, but also because polluting now motivates the other countries to pollute less and invest more in the future. However, countries invest less when they anticipate being held up in future negotiations. If investments are valuable (for example, due to large technological spillovers), then short-term agreements are worse than no agreement.

Second, the optimal agreement is described. A tough agreement, if long-term, encourages investments. The optimal and equilibrium agreement is therefore tougher and longer-term if, for example, technologies are long-lasting and intellectual property rights weak.

Trade policies and climate treaties interact. If technologies can be traded, high tariffs or low subsidies discourage investments; to counteract this, the climate treaty should be tougher and longer-term. If the climate treaty is absent or relatively short-lasting for exogenous reasons, then intellectual property rights should be strengthened, tariffs should be lowered, or licensing of green technology should be subsidized. Negotiating intellectual property rights or trade policies is thus a strategic substitute to a tough climate treaty: if one fails, the other becomes more important.

8. Appendix

While U_i is the continuation value for a subgame starting with the investment stage, let W_i represent the (interim) continuation value at (or just before) the emission stage. To shorten equations, define $m \equiv -\delta\partial U_i/\partial G_-$, $z \equiv \delta\partial U_i/\partial R_-$, $\widetilde{R} \equiv q_R R_-$, $\widetilde{G} \equiv q_G G_- + \theta$, and $\widetilde{y}_i \equiv y_i + \overline{y} - \overline{y}_i$, where $\overline{y} \equiv \sum_N \overline{y}_i/n$. As above, $k \equiv K + (n-1)\,e$.

Proof of Lemma 1 for the business-as-usual scenario. Just before the emission stage, θ is known and the payoff-relevant states are R and $\widetilde{G}$.[15] A country's (interim) continuation value is $W(\widetilde{G}, R)$. Since country i takes r_j, $j \neq i$, as given, deciding on r_i is equivalent to deciding on $R = q_R R_- + \sum_{j\in N\setminus i} r_j + r_i$. Thus, i's investment decision must ensure that the following problem is solved:

$$\max_R \mathrm{E}W(\widetilde{G}, R) - k\left(R - q_R R_- - \sum_{j\in N\setminus i} r_j\right), \tag{8.1}$$

where expectations are taken w.r.t. θ. In this problem, the level of R_- is clearly payoff-irrelevant and the equilibrium R must be independent of R_-. Thus, when all countries invest the same amount, a marginally larger R_- implies that R will be unchanged and that every r_i will decline by q_R/n units. It follows that:

$$\frac{\partial U}{\partial R_-} = \frac{q_R\,(k-(n-1)\,e)}{n} \equiv \frac{q_R K}{n}. \tag{8.2}$$

At the emission stage, a country's first-order condition for y_i is:

$$0 = b\,(\overline{y} - \widetilde{y}_i) - c\left(\widetilde{G} + \sum_N \widetilde{y}_j - R\right) + \delta U_G(\widetilde{G} - R + \sum_N \widetilde{y}_j, R), \tag{8.3}$$

which implies that all $\widetilde{y}_i$s are identical. From (8.2), we know that $U_{RG} = U_{GR} = 0$, and that U_G cannot be a function of R. Therefore, (8.3) implies that $\widetilde{y}_i$, G, and thus $B\,(\widetilde{y}_i - \overline{y}) - C\,(G) \equiv \gamma\,(.)$ are functions of $\widetilde{G} - R$ only. Hence, write

[15] As explained in Section 3, there is no reason for one country, or one firm, to condition its strategy on R_i, given R, if the other players are not doing so. Ruling out such dependence is consistent with the definition of MPE.

$G = \chi\left(\widetilde{G} - R\right)$. When we substitute into (8.1), the corresponding first-order condition becomes:

$$\frac{\partial \mathrm{E}\left[\gamma\left(q_G G_- + \theta - R\right) + \delta U\left(\chi\left(q_G G_- + \theta - R\right), R\right)\right]}{\partial R} = k. \tag{8.4}$$

This requires $q_G G_- - R$ to be a constant, say, ξ, which is independent of the stocks. Thus, $\partial r_i / \partial G_- = q_G/n$ and U becomes:

$$\begin{aligned} U\left(G_-, R_-\right) &= \mathrm{E}\gamma\left(\xi + \theta\right) - Kr + \mathrm{E}\delta U\left(\chi\left(\xi + \theta\right), R\right) \\ &= \mathrm{E}\gamma\left(\xi + \theta\right) - K\left(\frac{q_G G_- - \xi - q_R R_-}{n}\right) + \mathrm{E}\delta U\left(\chi\left(\xi + \theta\right), q_G G_- - \xi\right) \Rightarrow \\ \frac{\partial U}{\partial G_-} &= -K\left(\frac{q_G}{n}\right) + \delta U_R q_G = -\frac{K q_G}{n}\left(1 - \delta q_R\right). \end{aligned}$$

With U_G and U_R pinned down, (8.3) and (8.4) give a unique solution. □

Proof of Proposition 1. Since the proof is analogous to the next proof, it is omitted.

Proof of Proposition 2. From (8.3), we have:

$$\widetilde{y}_i = \overline{y} - \frac{m + cG}{b} \Rightarrow y_i = \overline{y}_i - \frac{m + cG}{b} \Rightarrow \tag{8.5}$$

$$G = \widetilde{G} + \sum_N \left(y_i - R_i\right) = \widetilde{G} - R + n\left(\overline{y} - \frac{m + cG}{b}\right) = \frac{b\overline{y}n - mn + b\left(\widetilde{G} - R\right)}{b + cn} \tag{8.6}$$

$$y_i = \overline{y}_i - \frac{m}{b} - \frac{c}{b}\left(\frac{b\overline{y}n - mn + b\left(\widetilde{G} - R\right)}{b + cn}\right) = \overline{y}_i - \frac{c\overline{y}n + c\left(\widetilde{G} - R\right) + m}{b + cn} \Rightarrow$$

$$g_i = y_i - R_i = \overline{y}_i - \frac{c\overline{y}n + c\left(\widetilde{G} - R\right) + m}{b + cn} - R_i.$$

Simple algebra and a comparison to the first-best gives (3.5). Interim utility (after investments are sunk) can be written as:

$$W_i^{bau} \equiv -\frac{c}{2}G^2 - \frac{b}{2}\left(\overline{y}_i - y_i\right)^2 + \delta U(G, R) = -\frac{c}{2}\left(1 + \frac{c}{b}\right)G^2 - \frac{Gmc}{b} - \frac{m^2}{2b} + \delta U(G, R).$$

Since $\partial G/\partial R = -b/(b+cn)$ from (8.6), equilibrium investments are given by:

$$k = \mathrm{E}\partial W_i^{bau}/\partial R = c\left(1+\frac{c}{b}\right)\left(\frac{b}{b+cn}\right)\mathrm{E}G + \frac{bm(1+c/b)}{b+cn} + z. \tag{8.7}$$

The second-order condition holds since EW is concave in R. Taking expectations of G in (8.6), substituting in (8.7), and solving for R give:

$$\begin{aligned} R &= \overline{y}n + \mathrm{E}\widetilde{G} - k\frac{(b+cn)^2}{bc(b+c)} + \frac{m}{c} + z\frac{(b+cn)^2}{bc(b+c)} \Rightarrow \quad (8.8)\\ r_i = \frac{R-q_R R_-}{n} &= \overline{y} + \frac{q_G G_-}{n} - k\frac{(b+cn)^2}{bc(b+c)n} + \frac{m}{cn} + z\frac{(b+cn)^2}{bc(b+c)n} - \frac{q_R R_-}{n}. \end{aligned}$$

Simple algebra and a comparison to the first-best concludes the proof. □

Proof of Proposition 3. At the emission stage, the countries negotiate the g_is. Variable g_i determines $\widetilde{y}_i$. Since countries have symmetric preferences over $\widetilde{y}_i$ (in the negotiations as well as in the default outcome), the $\widetilde{y}_i$s must be identical in the bargaining outcome. Consequently, efficiency requires:

$$0 = b(\overline{y} - \widetilde{y}_i)/n - c\left(\widetilde{G} - R + \sum \widetilde{y}_i\right) + \delta U_G(\widetilde{G} - R + \sum \widetilde{y}_i, R). \tag{8.9}$$

The rest of the proof of Lemma 1 continues to hold: R will be a function of only G_-, so $U_{R_-} = q_R K/n$. This makes $\mathrm{E}\widetilde{G} - R$ a constant and $U_{G_-} = -q_G(1-\delta q_R)K/n$, just as before. The comparative static becomes the same, but the *levels* of g_i, y_i, r_i, u_i, and U_i are obviously different from the previous case.

The first-order condition (8.9) becomes:

$$\begin{aligned} 0 &= -ncG + b\overline{y} - b\widetilde{y}_i - nm \Rightarrow y_i = \overline{y}_i - \frac{nm+ncG}{b}.\\ G &= \widetilde{G} + \sum_j (y_j - R_j) = \widetilde{G} + n\left(\overline{y} - \frac{nm+ncG}{b}\right) - R \Rightarrow\\ G &= \frac{b\overline{y}n - mn^2 + b\left(\widetilde{G} - R\right)}{b+cn^2}. \quad (8.10) \end{aligned}$$

The second-order condition holds trivially. Note that the interim utility can be written as:

$$W_i^{st} = -\frac{c}{2}G^2 - \frac{b}{2}\left(\frac{nm+ncG}{b}\right)^2 + \delta U(G,R).$$

Since (8.10) implies $\partial G/\partial R = -b/(b+cn^2)$, equilibrium investments are given by:

$$\begin{aligned} k &= \mathrm{E}\frac{\partial W_i^{st}}{\partial R} = \mathrm{E}G\left(c+\frac{c^2n^2}{b}\right)\left(\frac{b}{b+cn^2}\right)+\frac{cmn^2}{b}\left(\frac{b}{b+cn^2}\right)+m\left(\frac{b}{b+cn^2}\right)+z \\ &= c\mathrm{E}G+m+z. \end{aligned} \tag{8.11}$$

The second-order condition holds since EW is concave. Next, take the expectation of (8.10) and combine it with (8.11) to solve for R to get:

$$R^{st} = q_G G_- + n\overline{y} + \frac{m}{c} - \left(\frac{b+cn^2}{b}\right)\left(\frac{k}{c}-\frac{z}{c}\right).$$

The proof is completed by comparing r_i^* to $r_i^{st} = (R^{st} - q_R R_-)/n$:

$$\begin{aligned} r_i^{st} &= \overline{y} - \frac{q_R R_-}{n} + \frac{q_G G_-}{n} - \left(\frac{b+cn^2}{bcn}\right)(k-\delta U_R) - \frac{\delta U_G}{cn} \\ &= r_i^* - K\left(\frac{b+cn^2}{bcn^2}\right)\left(\frac{nk}{K}-1\right) < r_i^*. \ \square \end{aligned}$$

Proof of Proposition 4. Part (i) and (ii) follow from simple algebra when emissions and investment levels for business as usual are compared to short-term agreements. When these levels are substituted into u_i, which in turn should be substituted in $U = u_i + \delta U_+(.)$, we can compare U^{bau} and U^{st} . $\square$

Proof of Proposition 5. In the last period, investments are given by:

$$\begin{aligned} k &= b(\overline{y}_i - g_{i,T} - R_{i,T}) + z \Rightarrow \\ \widetilde{y}_i - \overline{y} &= -\frac{k-z}{b},\ R_{i,T} = \overline{y}_i - g_{i,T} - \frac{k-z}{b} \Rightarrow && (8.12) \\ r_{i,T} &= \overline{y}_i - g_{i,T} - \frac{k-z}{b} - q_R R_{i,T-1}. && (8.13) \end{aligned}$$

Anticipating the equilibrium $R_{i,T}$, country i can invest q_R fewer units in period T for each invested unit in period $T-1$. Thus, in period $T-1$, equilibrium investments are given by:

$$\begin{aligned} k &= b(\overline{y}_i - g_{i,T} - R_{i,T}) + \delta q_R k \Rightarrow && (8.14) \\ R_{i,T-1} &= \overline{y}_i - g_{i,T-1} - \frac{k(1-\delta q_R)}{b} \Rightarrow \\ r_{i,T-1} &= \overline{y}_i - g_{i,T-1} - \frac{k(1-\delta q_R)}{b} - q_R R_{i,T-2}. \end{aligned}$$

The same argument holds for $T-t$, $t \in \{1,...T-1\}$. Proposition 5 follows since the socially optimal R_i and r_i, given g_i, are:

$$R_i^* = r_i^* + q_R R_{i,-} = \overline{y}_i - g_i - \frac{K(1-\delta q_R)}{b}. \ \square$$

Proof of Proposition 6. In the bargaining game, the default is the business-as-usual outcome, where everyone faces the same utility. Note that negotiating the g_is is equivalent to negotiating the r_is, given (8.13). Given identical preferences regarding the r_is, symmetry requires that r_i, and thus $\varsigma_t \equiv \overline{y}_i - g_{i,t} - q_R R_{i,t-1}$, are the same for every country in equilibrium.

For the last period, (8.13) becomes

$$r_{i,T} = \varsigma_T - \frac{k - \delta q_R K/n}{b}.$$

When we take equilibrium investments into account, the utility for the last period can be written as:

$$U_i = -\frac{1}{2b}(k-z)^2 - \mathrm{E}C(G) - K r_{i,T} + \delta U(G,R).$$

Efficiency requires U_i to be maximized w.r.t. ς, taking into account that $g_i = \overline{y}_i - q_R R_{i,-} - \varsigma$ and $\partial r_i / \partial \varsigma = 1 \forall i$. The f.o.c. is:

$$n\mathrm{E}cG - K - n\delta U_G + n\delta U_R = 0 \Rightarrow \mathrm{E}cG + m + z = K/n. \qquad (8.15)$$

The second-order condition holds trivially.

For $t < T$, $r_{i,t} = r_{j,t} = r_t$, given by:

$$r_t = \varsigma_t - \frac{k(1-\delta q_R)}{b}.$$

Note that for every $t \in \{2,...T\}$, $R_{i,t-1}$ is given by the quota in the *previous* period:

$$\begin{aligned} r_t &= \left(\overline{y}_i - g_{i,t} - q_R\left(\overline{y}_i - g_{i,t-1} - \frac{k(1-\delta q_R)}{b}\right)\right) - \frac{k(1-\delta q_R)}{b} \\ &= \overline{y}_i(1-q_R) - g_{i,t} + q_R g_{i,t-1} - (1-q_R)\frac{k(1-\delta q_R)}{b}. \qquad (8.16) \end{aligned}$$

All countries have the same preferences over the ς_ts. Dynamic efficiency requires that the countries not be better off after a change in the ς_ts (and thus the $g_{i,t}$s), given by $(\Delta\varsigma_t, \Delta\varsigma_{t+1})$, such that G is unchanged after two periods, i.e., $\Delta\varsigma_{t+1} = -\Delta\varsigma_t q_G$, $t \in [1, T-1]$. From (8.16), this implies:

$$\begin{aligned}
-n\mathrm{E}C'(G_t)\Delta\varsigma_t + \Delta g_t K + \delta(\Delta\varsigma_{t+1} - \Delta g_t q_R)K - \delta^2\Delta g_{t+1}q_R K &\leq 0 \forall \Delta\varsigma_t \Rightarrow \\
(1-\delta q_R)(1-\delta q_G)\frac{K}{cn} &= \mathrm{E}G = \mathrm{E}G^*.
\end{aligned}$$

Thus, neither G_t nor $g_{i,t}$ (and hence not R either) can be functions of R_-. At the start of period 1, therefore, $U_R = q_R K/n$, just as before, and U_G cannot be a function of R (since $U_{RG} = 0$). Since EG is a constant, we must have $\varsigma_1 = \overline{y} - (\mathrm{E}G^* - q_G G_0)/n - q_R R_0/n$. Equation (8.13) gives $\partial r_{i,t=1}/\partial G_- = (\partial r_i/\partial g_i)(\partial g_i/\partial\varsigma)(\partial\varsigma/\partial G_-) = q_G/n$. Hence, $U_G = -q_G K/n + \delta U_R q_G = -q_G(1-\delta q_R)K/n$, giving a unique equilibrium. When we substitute that equation into (8.15), we get EG_T =EG^*, just as in the earlier periods. Thus, $g_{i,t} = g_i^*(\mathbf{R}_i^*)$ in all periods.

Proposition 6 follows since, from (3.3), $\partial g_i^*/\partial r_j = -b/(b+cn^2)$, so $g_{i,t} = g_i^*(\mathbf{R}_i^*) = g_i^*(\mathbf{R}_{i,t}^{lt}) - (r_i^* - r_{i,t}^{lt})b/(b+cn^2)$. □

Proof of Proposition 7. The optimal length T balances the cost of underinvestment when T is short and the cost of the uncertain θ is increasing in T. In period T, countries invest suboptimally, not only because of the domestic hold-up problem, but also because of the international one. When all countries invest less, u_i declines. The loss in period T, relative to any period $t < T$, can be written as:

$$\begin{aligned}
H &= -\frac{b}{2}(y_{i,t} - \overline{y}_i)^2 - \frac{b}{2}(y_{i,T} - \overline{y}_i)^2 - K(r_{i,t} - r_{i,T})(1-\delta q_R) \\
&= -\frac{b}{2}\left(\frac{k - \delta q_R K}{b}\right)^2 + \frac{b}{2}\left(\frac{k-z}{b}\right)^2 - K\left(\frac{k-z}{b} - \frac{k-\delta q_R K}{b}\right)(1-\delta q_R) \\
&= \frac{\delta q_R}{b}\left(e + \frac{K}{n}\right)\left[e\left(1 - \frac{\delta q_R}{2}\right) + \frac{\delta q_R K}{2n}\right](n-1)^2.
\end{aligned}$$

Note that H increases in e, n, q_R, and K, but decreases in b.

The cost of a longer-term agreement is associated with θ. Although EC' and thus $\mathrm{E}G_t$, are the same for all periods,

$$\begin{aligned}
\mathrm{E}\frac{c}{2}(G_t)^2 &= \mathrm{E}\frac{c}{2}\left(\mathrm{E}G_t + \sum_{t'=1}^{t}\theta_{t'}q_G^{t-t'}\right)^2 = \frac{c}{2}(\mathrm{E}G_t)^2 + \mathrm{E}\frac{c}{2}\left(\sum_{t'=1}^{t}\theta_{t'}q_G^{t-t'}\right)^2 \\
&= \frac{c}{2}(\mathrm{E}G_t)^2 + \frac{c}{2}\sigma^2\sum_{t'=1}^{t}q_G^{2(t-t')} = \frac{c}{2}(\mathrm{E}G_t)^2 + \frac{c}{2}\sigma^2\left(\frac{1-q_G^{2t}}{1-q_G^2}\right).
\end{aligned}$$

The last term is the loss associated with the uncertainty regarding future marginal costs. For the T periods, the total present discounted value of this loss is given by:

$$\begin{aligned}
L(T) &= \sum_{t=1}^{T}\frac{c}{2}\sigma^2\delta^{t-1}\left(\frac{1-q_G^{2t}}{1-q_G^2}\right) = \frac{c\sigma^2}{2(1-q_G^2)}\sum_{t=1}^{T}\delta^{t-1}\left(1-q_G^{2t}\right) \\
&= \frac{c\sigma^2}{2(1-q_G^2)}\left[\frac{1-\delta^T}{1-\delta} - q_G^2\left(\frac{1-\delta^T q_G^{2T}}{1-\delta q_G^2}\right)\right]. \qquad (8.17)
\end{aligned}$$

If all future agreements last $\widehat{T}$ periods, then the optimal T for this agreement is given by

$$\begin{aligned}
&\min_T L(T) + \left(\delta^{T-1}H + \delta^T L\left(\widehat{T}\right)\right)\left(\sum_{\tau=0}^{\infty}\delta^{\tau\widehat{T}'}\right) \Rightarrow \\
&0 = L'(T) + \delta^T\ln\delta\left(H/\delta + L\left(\widehat{T}\right)\right) = L'(T) + \delta^T\ln\delta\left(H/\delta + L\left(\widehat{T}\right)\right) \\
&= -\delta^T\ln\delta\left[\frac{c\sigma^2/2}{1-q_G^2}\left(\frac{1}{1-\delta} - \frac{q_G^{2T+2}\left(1+\ln q_G^2/\ln\delta\right)}{1-\delta q_G^2}\right) - \frac{H/\delta + L\left(\widehat{T}\right)}{1-\delta^{\widehat{T}'}}\right], \qquad (8.18)
\end{aligned}$$

assuming that some T satisfies (8.18). Since $\left(-\delta^T\ln\delta\right) > 0$ and the term in the brackets increases in T, the loss decreases in T for small T but increases for large T, and there is a unique T minimizing the loss (even if the loss function is not necessarily globally concave). Since G_- and R_- do not appear in (8.18), the T which satisfies (8.18) must equal $\widehat{T}$ – assuming that $\widehat{T}$ will be optimally set. Substituting for $\widehat{T} = T$ and (8.17) in (8.18) gives:

$$\frac{H}{\delta} = \frac{c\sigma^2 q_G^2}{2(1-q_G^2)(1-\delta q_G^2)}\left(\frac{1-\delta^T q_G^{2T}}{1-\delta^T} - q_G^{2T}\left(1+\frac{\ln\left(q_G^2\right)}{\ln\delta}\right)\right), \qquad (8.19)$$

where the r.h.s. increases in T. $T = \infty$ is optimal if the left-hand side of (8.19) is larger than the right-hand side even when $T \to \infty$:

$$\frac{c\sigma^2 q_G^2}{2\left(1-q_G^2\right)\left(1-\delta q_G^2\right)} \leq \frac{H}{\delta}. \tag{8.20}$$

If e and n are large but b is small, then H is large and (8.20) is more likely to hold. If (8.20) does not hold, the T satisfying (8.19) is larger. If c or σ^2 are larger, (8.20) is less likely to hold and, if it does not, (8.19) requires T to decrease. $\square$

Proof of Proposition 8. (i) As noted in Section 3 as well as in the above proofs, in every bargaining situation, the countries happen to be symmetric when they consider $\widetilde{y}_i$ and the (induced) investment costs. Thus, no side transfers would take place (neither on nor off the equilibrium path), regardless of any differences in the R_is or the $\overline{y}_i$s.

(ii) Note that in equilibrium, there is never any trade in permits. Hence, *if* country i invests as predicted in Sections 3-5, the marginal benefit of more technology is the same whether permits are tradable or not. Second, if i deviated by investing more (less), i's marginal utility of a higher technology decreases (increases) not only when permit trade is prohibited, but also when trade is allowed; this is because more (less) technology decreases (increases) the demand for permits and thus the equilibrium price. Hence, such a deviation is not attractive.

(iii) Note that the marginal benefit of being allowed to pollute another marginal unit is equal to $B_i'(\cdot)$ when the total number of permits is fixed. Thus, $B_i'(\cdot)$ must equal the permit price when no country has market power in the permit market. For short-term agreements, (8.9) together with (8.11) implies that the quota price is:

$$\begin{aligned} B_i'(.) &= ncG + nm = nc\theta + nc\mathrm{E}G + nm = nc\theta + n(k - m - z) + nm \\ &= nc\theta + n(k - \delta q_R K/n). \end{aligned}$$

For the last period in long-term agreements, (8.12) implies that $B_i'(y_{i,T}) = k - z = k - \delta q_R K/n$. For earlier periods, (8.14) implies that $B_i'(y_{i,t}) = k - \delta q_R k$, while, at the first-best, $B_i'\left(y_{i,t}^*\right) = K - \delta q_R K$. $\square$

Proof of Proposition 9. (i) Consider first the default setting with no licensing/trade. Country i invests $r_{i,t} = r_{i,t}^A + r_{i,t}^N$, where $r_{i,t}^A$ is the adopted spillover while $r_{i,t}^N$ is investment in new technology. When IPR is modelled as (i), then $r_{i,t}^A = \varphi \sum_{j \neq i} r_{j,t-1}$ with probability $1-\alpha$ and $r_{i,t}^A = 0$ with probability α. The expected cost of $r_{i,t}$ is $\mathrm{E}\left[\underline{k}r_{i,t}^N + (1-\gamma)\,\underline{k}r_{i,t}^A\right] = \underline{k}r_{i,t} - \varphi\gamma\,(1-\alpha)\,\underline{k}\sum_{j\neq i} r_{j,t-1}$. At the moment before i imitates $\varphi r_{j,t-1}$, the additional gain from licensing/trade is $\varphi\gamma\alpha\underline{k}r_{j,t-1}$. If p_t measures the fee per unit of technology, then the bargaining/trade surplus to the purchaser in country i is $[\gamma\alpha\underline{k} - p_t\,(1+\tau)]\,\varphi r_{j,t-1}$, and for the seller j the surplus is $p_t\varphi r_{j,t-1}$.

Maximizing the Nash product $\left([\gamma\alpha\underline{k} - p_t\,(1+\tau)]\,\varphi r_{j,t-1}\right)^{1-\beta}\left(p_t\varphi r_{j,t-1}\right)^{\beta}$ with respect to p_t gives $p_t = \beta\gamma\alpha\underline{k}/\,(1+\tau)$. While the actual purchaser in country i pays $\beta\gamma\alpha\underline{k}$, including the tariff, the tariff revenues are collected by country i, and, as a result, from the country's perspective the price is only p_t, decreasing in the tariff which simply improves the country's terms of trade.

With such licensing agreements, i's cost of $r_{i,t}$ is $\underline{k}r_{i,t} - \varphi\gamma\underline{k}\sum_{j\neq i} r_{j,t-1} + p_t\varphi\sum_{j\neq i} r_{j,t-1}$. The last two terms are constant from period t on, and can be accounted for already in period $t-1$ if we just discount the future benefit by δ by writing the externality as $e = \delta\varphi\,(\gamma\underline{k} - p_t) \Rightarrow (6.2)$.

Regardless of $r_{j,t-1}$, country i's marginal gross cost of $r_{i,t}$ is $\underline{k}$. In the next period, i can expect royalties equal to $p_{t+1}\varphi\,(n-1)\,r_{i,t} = \varphi\gamma\alpha\beta\,(n-1)\,\underline{k}/\,(1+\tau)\,r_{i,t}$. When this revenue is accounted for already in period t, country i's *net* marginal investment cost becomes $[1 - \delta\varphi\gamma\alpha\beta\,(n-1)\,/\,(1+\tau)]\,\underline{k}$, which can be written as $k = K + (n-1)\,e$ where $K = [1-(n-1)\,\delta\varphi\gamma]\,\underline{k}$ as in (6.1).

(ii) When IPR is modelled as in (ii), then with no licensing, i's cost of $r_{i,t}$ is $\underline{k}r_{i,t} - \varphi\gamma\,(1-\alpha)\,\underline{k}\sum_{j\neq i} r_{j,t-1}$. With licensing, i's cost is $\underline{k}r_{i,t} - \varphi\gamma\underline{k}\sum_{j\neq i} r_{j,t-1} - \varphi p_t\sum_{j\neq i} r_{j,t-1}$, where p_t is the royalty fee per unit of technology. The bargaining surplus for i and j is $\varphi\gamma\alpha\underline{k}r_{j,t-1}$, as in case (i), and the rest of the proof is as in case (i).

(iii) When IPR is modelled as in (iii), then with no licensing, i's cost of $r_{i,t}$ is $\underline{k}r_{i,t} - \varphi\gamma\,(1-\alpha)\,\underline{k}\sum_{j\neq i} r_{j,t-1}$, as in case (ii). The rest of the proof is also similar. □

Proof of Proposition 10. Note that, under short-term agreements (as well as business as usual), if interim utility is $W\left(\widetilde{G}, R\right)$, investments are given by $\mathrm{E}W_R = k$ although they should optimally be $\mathrm{E}W_R = K/n$, requiring (6.4). For long-term agreements, investments are optimal in the last period if $k - \delta q_R K/n = K\,(1-\delta q_R)$, which requires (6.5). For earlier periods, the requirement is $k = K$, giving (6.6). □