# Distribution and growth

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## 1 No credit constraints

Consider an economy with a continuum of families with mass 1. Every family consists of a single person living for one period, leaving one offspring (so no population change). The agent in family $i$ living in period $t$ has initial wealth $a_{i,t}$. Using this capital and supplying one unit of labour, she earns $y_{i,t}$, which she spends on consumption $c_{i,t}$ and bequests $b_{i,t}$ for the next generation according to the utility function

$$U = c_{i,t}^{1-s} b_{i,t}^{s},$$

yielding $b_{i,t} = sy_{i,t}$. Hence

$$a_{i,t+1} = b_{i,t} = sy_{i,t}.$$

The distribution of wealth at time $t$ is given by the cummulative distribution function $G_t(\cdot)$, where $G_t(w)$ is the fraction with income below $w$. Average wealth is then

$$w_t = \int w \; dG_t(w).$$

Each agent produces with a Cobb-Douglas technology. When she supplies 1 uint of labour and has access to $k$ units of capital, she produces

$$y = Ak^{\alpha}.$$

The rental rate of capital is $r_t$ (=1 plus the interest rate). An agent then maximizes

$$\max_{k_{i,t}} Ak_{i,t}^{\alpha} - r_t k_{i,t} + r_t a_{i,t},$$

which yields

$$\alpha A k_{i,t}^{\alpha-1} = r_t \;\Rightarrow\; k_{i,t} = \left(\frac{\alpha A}{r_t}\right)^{\frac{1}{1-\alpha}}.$$

Notie that this is the same for all agents, so for all $i$, $k_{i,t} = k_t$. The interest rate $r_t$ is chosen so markets clear:

$$k_t = \int w \; dG_t(w) = w_t.$$

Hence total production is

$$y_t^* = \int y_{i,t} = \int A k_{i,t}^{\alpha} = A(w_t)^{\alpha},$$

which inly depends on average income $w_t$ and not on distribution. So in the standard model with perfect credit markets, distribution has no effect.

## 2 Credit constraints (Piketty REStud 1997)

Consider now a model without credit markets. The only way to save is by investing in own capital. We then get that

$$y_{i,t} = A(k_{i,t})^{\alpha} = A(a_{i,t})^{\alpha}$$

and total production is

$$y_t^{\dagger} = \int A a^{\alpha} \; dG_t(a)$$

As $y(a) = Aa^{\alpha}$ is a concave function, it follows from Jensen's inequality that

$$\int A a^{\alpha} \; dG_t(a) < A\left(\int a \; dG(\alpha)\right)^{\alpha} \;\Rightarrow\; y_t^{\dagger} < y_t^*,$$

so production si lower, and more so the more spread there is in $G_t$. Henc edistribution matters

## 3 A model with occupational choice (Ghatak and Jiang 2002)

Still no credit markets. We now have three classes:

**Subsistence** Wage $\underline{\mathrm{w}}$. Income $y_{i,t}^S = \underline{\mathrm{w}} + r a_{i,t}$

**Worker** Wage $w_t$, works for entrepreneur. Income $y^W_{i,t} = w_t + ra_{i,t}$

**Entrepreneur** Invest $I$, hire one worker. Income $y^E_{i,t} = q - w_t + r(a_{i,t} - I)$

Industrialization efficient: $q - rI > 2w$.

Only agents with $a_{i,t} \geq I$ can become entrepreneurs. Hence $G_t(I)$ cannot become entrepreneurs.

At wage $\bar{w}$, indifferent between worker and entrepreneur:

$$\bar{w} = q - \bar{w} - rI \;\Rightarrow\; \bar{w} = \frac{q - rI}{2}$$

Labour supply to industry:

$$\begin{cases} 0 & \text{if} \quad w_t < \underline{w} \\ [0, G_t(I)] & \text{if} \quad w_t = \underline{w} \\ G_t(I) & \text{if} \quad \underline{w} < w_t < \bar{w} \\ [G_t(I), 1] & \text{if} \quad \bar{w} = w_t \\ 1 & \text{if} \quad \bar{w} < w_t \end{cases}$$

Labour demand from industry:

$$\begin{cases} 1 - G_t(I) & \text{if} \quad w_t < \bar{w} \\ [0, 1 - G_t(I)] & \text{if} \quad w_t = \bar{w} \\ 0 & \text{if} \quad w_t > \bar{w} \end{cases}$$

Only two cases, $w_t = \underline{w}$ or $w_t = \bar{w}$. The first occurs iff $G_t(I) > 1/2$. Then

$$a_{i,t+1} = \begin{cases} = s[ra_{i,t} + \underline{w}] & a_{i,t} < I \quad w_t = \underline{w} \\ = s[r(a_{i,t} - I) + q - \underline{w}] & a_{i,t} > I \quad w_t = \underline{w} \\ = s[ra_{i,t} + \bar{w}] & \forall a_{i,t} \quad w_t = \bar{w} \end{cases}.$$

Assume $sr < 1$: Wealth doesn't grow into heaven.

Stationary wealth distributions ($a_{i,t+1} = a_{i,t}$):

$$\begin{aligned} a^S &= \frac{s\underline{w}}{1 - sr} \\ a^W(\underline{w}) &= \frac{s\underline{w}}{1 - sr} \\ a^W(\bar{w}) &= \frac{s\bar{w}}{1 - sr} = \frac{s(q - rI)}{2(1 - sr)} \\ a^E(\underline{w}) &= \frac{s(q - rI - \underline{w})}{1 - sr} \\ a^E(\bar{w}) &= \frac{s(q - rI)}{2(1 - sr)} \end{aligned}$$

**Case 1** $a^E(\underline{w}) < I \Leftrightarrow s(q - \underline{w}) < I$. Everybody in subsistence in the long run.

**Case 2** $a^E(\bar{w}) < I \leq a^E(\underline{w}) \Leftrightarrow \frac{sq}{2-sr} < I < s(q - \underline{w})$. If initially $G_t(I) > 1/2$, wage always $\underline{w}$, otherwise start in $\bar{w}$, but after a while fewer entrepreneurs and finally $\underline{w}$ reached.

**Case 3** $a^W(\bar{w}) < I \leq a^W(\underline{w}) \Leftrightarrow \frac{s\underline{w}}{1-sr} < I \leq \frac{sq}{2-sr}$. If initially $G_t(I) > 1/2$, wage starts at $\underline{w}$ and satys there forever. If initially $G_t(I) < 1/2$, wage starts at $\bar{w}$ and stays there.

**Case 4** $I \leq a^W(\underline{w}) \Leftrightarrow I \leq \frac{s\underline{w}}{1-sr}$. In all cases the economy converges to the high wage equilibrium.

TABLE 3—REGRESSION RESULTS: ALTERNATE ESTIMATION TECHNIQUES

| Estimation method | Five-year periods: Fixed effects (1) | Five-year periods: Random effects (2) | Five-year periods: Chamberlain's $\pi$-matrix (3) | Five-year periods: Arellano and Bond (4) | Ten-year periods: fixed effects (5) |
|---|---|---|---|---|---|
| *Inequality* | 0.0036 | 0.0013 | 0.0016 | 0.0013 | 0.0013 |
| | (0.0015) | (0.0006) | (0.0002) | (0.0006) | (0.0011) |
| *Income* | −0.076 | 0.017 | −0.027 | −0.047 | −0.071 |
| | (0.020) | (0.006) | (0.004) | (0.008) | (0.016) |
| *Male Education* | −0.014 | 0.047 | 0.018 | −0.008 | −0.002 |
| | (0.031) | (0.015) | (0.010) | (0.022) | (0.028) |
| *Female Education* | 0.070 | −0.038 | 0.054 | 0.074 | 0.031 |
| | (0.032) | (0.016) | (0.006) | (0.018) | (0.030) |
| *PPP* | −0.0008 | −0.0009 | −0.0013 | −0.0013 | −0.0003 |
| | (0.0003) | (0.0002) | (0.0000) | (0.0001) | (0.0003) |
| $R^2$ | 0.67 | 0.49 | | | 0.71 |
| Countries | 45 | 45 | 45 | 45 | 45 |
| Observations | 180 | 180 | 135 | 135 | 112 |
| Period | 1965–1995[a] | 1965–1995[a] | 1970–1995 | 1970–1995 | 1965–1995 |

*Notes:* Dependent variable is average annual per capita growth. Standard errors are in parentheses. $R^2$ is the within-$R^2$ for fixed effects and the overall-$R^2$ for random effects.

[a] Estimates are virtually identical for the period 1970–1995 (with 135 observations).

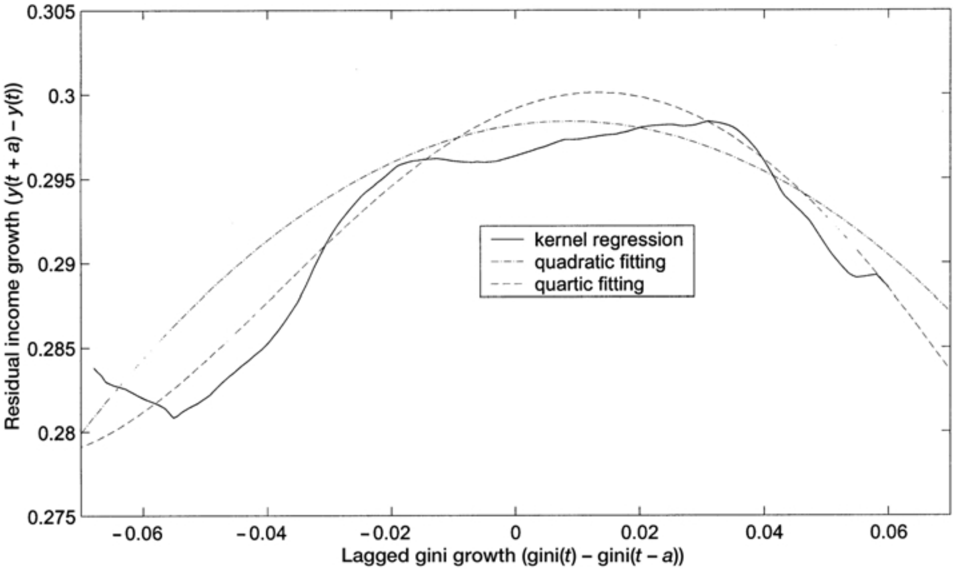


*Figure 2*. Relationship between income growth and lagged gini growth: partially linear model (Barro variables).